Rethinking computers in schools: Calculus in primary

Andrew E. Fluck, University of Tasmania, Andrew.Fluck@utas.edu.au
Dev Ranmuthugala, Australian Maritime College, D.Ranmuthugala@utas.edu.au
Chris K. H. Chin, Australian Maritime College, C.Chin@utas.edu.au
Irene Penesis, Australian Maritime College, I.Penesis@utas.edu.au

Abstract

It has been suggested that computers can transform the nature of schooling, but the process to devise a new Australian curriculum embeds information and communication technology (ICT) as a general capability servicing the needs of traditional subjects. We sought to explore the capacity of ICT to enhance learning of topics considered so complex they are rarely introduced to students under the age of sixteen. This project undertook an investigation in four Australian schools to train laptop-equipped students aged ten to twelve years how to solve problems using integral calculus with computer algebra system software. Using Allen’s (2001) non-template problem solving method, students were introduced to the operational functionality of the software, and then shown how to solve real world problems using it. After eleven lessons the students completed a test constructed from items at the level of a first year engineering degree calculus examination. Mean achievement was at the credit level, and students showed good understanding of the applications of integral calculus. The project is subsequently being trialled in another school with a more conventional computer laboratory infrastructure, and with free software in New Zealand. The inference is that computers can make concrete what is unteachably abstract in the context of pen and paper. It therefore has ripple-on implications for the development of the Australian national curriculum, and suggests treating ICT as a general capability understates what can be achieved with the appropriate equipment and software. It also challenges what we mean when talking about ‘mastery’, since these pupils did not need to remember lists of functions and their integrals, the computer application serving this purpose for them. Lastly, since professional engineers habitually use such mind tool software, we discuss to what degree students should be compelled to do without them.

Introduction

Computers have become an integral part of our lives in the past decade. Children as young as four years of age are able to use the computer and in primary schools most Australian students would have access to a computer provided either by the school or their parents.

With Australia heading towards curriculum renewal, there is also a critical shortage of mathematics graduates capable of teaching mathematics in the primary and secondary schools. In particular, there are less than half as many women as men with postgraduate qualifications in Mathematical Sciences (Edwards & Smith, 2008, p.50), which is significant as the majority of primary school teachers in Australia are women. Improving the uptake of mathematics by all students, and women in particular is therefore an important national aspiration.

Literature

There has been speculation that children using computers can learn concepts at younger ages than hitherto. Seymour Papert suggested this would be possible in the area of mathematics as follows:
If you examine the current 'math' curriculum with an open mind at least 80% of it will be recognized as inferior to the examples of mathematical thinking that are made possible by these three conditions:

1. free access to computers for much more than a few hours a week
2. a level of technological fluency (that has to develop over years) equal to the levels of reading fluency we now regard as basic skill
3. freedom from having to pass tests on nineteenth century knowledge

Many topics that were unteachably abstract in the context of pencil and paper technologies will be considered as appropriate for children in the context of a digital technology that makes the previously formal become concrete. (Papert 2000).

Mitch Resnick demonstrated this idea for the acquisition of systems concepts (1998), and we set out to do so in the area of integral calculus. After the 2007 Australian Federal election, the first item on the new cabinet’s agenda was computers in schools. This evoked great media interest, with the Australian Broadcasting Corporation airing the issue. Mary Gearin’s interview featured the following comment:

Well, I actually think that this is [Prime Minister] Rudd’s education Trojan horse. Getting laptops into the classroom forces the teachers to think about their own skills and their own training and it forces the educational administrators to think about the curriculum and in the end it will force them both to change and that, I think, is the real goal (Pesce 2007).

Principals interviewed on the same program expressed their expectations that the Rudd-Labor digital education initiative (Rudd, Smith & Conroy, 2007) would give a real vision for what education might be in the future, and move the education system forward. The Australian Government committed to a Digital Education Revolution with a focus on schooling in Years 9-12 where students nationwide were to be provided with computer access throughout every school day. The government was concerned about lethargic transformation based on information and communication technology (ICT) in education: “while ICT has fundamentally reshaped whole industries, revolutionized production processes and generated massive improvements in productivity in our workplaces, our education systems have been slower in adapting” (Gillard 2008). Society is being transformed by computers, and learning needs to adapt accordingly (Gosper et al. 2008). This makes it a problem of national significance. To rise to the challenge, the research community can, and should, offer some insights into what school students can realistically achieve with these new tools. Australian institutions aspire to systemic transformative uses of educational computers (Fluck 2003).

Professor Barry McGaw was appointed Chair of the new Australian Curriculum, Assessment and Reporting Authority (ACARA) to develop a world-class curriculum for all K-12 Australian students. In this endeavour ACARA needs to draw upon practical ways to transform education using ICT.
This is not ICT literacy, nor is ICT solely a support for other learning outcomes previously identified in school curricula. ACARA needs to examine learning outcomes which can only be achieved through student use of ICT. Australia desperately needs clear directions on what teachers can achieve in this area.

In addition to this curriculum renewal, Australia has also been facing a critical shortage of engineering, science, and mathematics graduates (Goldsworthy 2008, p. 2). The shortage has been most acute in engineering with declining enrolments. In Australia only 10% of graduates in 2008 were in engineering; the OECD average was 14% and in China 40% were in engineering. The National Centre for Maritime Engineering and Hydrodynamics was undertaking a research project to identify the effect of school students’ perceptions of engineering and technology careers on university enrolment. Our project supplemented this research, and thus addressed a major issue still faced by many Australian universities. Integral calculus is key to a career route into the engineering profession, and hence our choice for demonstrating a transformation in curriculum.

At this point it is worth clarifying a stark difference between traditional calculus instruction and practical applications by professional engineers. Traditionally students have been taught how to integrate a function using a series of rules which they can understand by looking at the process from first principles. This helps them to understand how to integrate a new function in the future. However, as the catalogue of function integrals grows, the use of poorly memorised results can impede practical calculation, so professional engineers use a variety of specialist software to ‘crunch the numbers’. One might argue that this reliance on computing equipment is analogous to the widening use of word processors in lieu of pens in newspaper offices; there is certainly a discussion to be had about the way these technologies redefine the underlying skills or their acquisition.

The crucial transformational role of ICT in schooling has been underlined by the four types of use identified for the Australian government in *Making Better Connections*:
- **Type A:** encouraging the acquisition of ICT skills as an end themselves;
- **Type B:** using ICTs to enhance students’ abilities within the existing curriculum;
- **Type C:** introducing ICTs as an integral component of broader curricular reforms that are changing not only how learning occurs but what is learned;
- **Type D:** introducing ICTs as an integral component of the reforms that alter the organization and structure of schooling itself (Downes, Fluck *et al.* 2002, p.23).

This project clearly addressed Type C changes, since it radically reconsidered what kind of mathematics might be taught at a far earlier age than hitherto. The power of this exemplar is that it will make necessary a re-consideration of all areas of the curriculum for relevance in a digital age. ICT and associated skills are seen as strategic, and nationally important for economic, pedagogical
and social reasons (Hawkridge 1989). If ICT use in schools is restricted to Types A or B, then the full benefit of high technology investment in schools is limited. However, for uses corresponding to Types C or D, school cultures will need to change markedly. This transformational view of ICT in schools requires a rethink about curriculum content, the applicability of prior learning outcomes and criteria to the future lives of student, and even the structure of schooling itself (Fluck 2003; Fluck 2005; Tinker 2000).

Some major reports into the efficacy of ICT as a support for student attainment in numeracy and literacy show it can be limited when used within an inappropriate curriculum. Dynarski et. al. (2007) conducted a large scale scientifically based study of software efficacy aimed at improving print-based reading and mathematics. The findings showed test scores using conventional items were not improved by the use of this tutorial-style software. Work for the Australian Research Council came to similar conclusions about the minor impact of ICT in improving learning outcomes in traditional curricula (Robertson & Fluck, 2006). Parr (2000) showed ICT is about as effective as other methods for improving education, such as decreasing class sizes, when assessed using non-ICT based tests.

Therefore we argue the impact of ICT as a general capability to assist the teaching of conventional subjects is likely to be severely limited when assessed using conventional methods. This project had students using the computer and specialist software in the assessment phase to provide a context similar to their learning environment.

Method
The project focused on teaching integral calculus to school students aged 10 to 12 years through the use of appropriate computer software and delivery techniques relevant to their experiences. The software selected was MAPLE®, as it is currently used in the first year mathematics units in the Bachelor of Engineering (Maritime) programmes at the University of Tasmania. The intention was for students to use the functionality within MAPLE, of which those relevant to the project are summarised below. The software can:

- accept input of mathematical functions;
- manipulate such functions and solve equations algebraically;
- perform calculus operations such as differentiation or integration;
- calculate the value of an integral between given limits; and
- graph and visualize functions and solutions.

For example a plot for the curve \( y = x^2 + 1 \) produced by a student is shown in Figure 1.
In short, the software removes the need to memorise dozens of integration techniques and accurately calculates the value of a definite integral for a given range. The efficacy of similar software has been demonstrated at undergraduate level, reducing the time to learn calculus by half (Palmiter, 1991).

The basic methodology was an intervention study involving one class (about 25 students) in each of four government schools. The latter educate around 66 percent of students in Australia (Australian Bureau of Statistics, 2009). These schools were drawn using purposeful sampling from four separate states: Tasmania, Victoria, New South Wales and Queensland, where the targeted students had already been assigned individual laptop or netbook computers. Each school provided a designated facilitator to deliver the programme over 11 sessions and manage the assessment component at its culmination. This ensured accessibility and acceptance to the students and the school, including meeting all statutory requirements stipulated to those dealing with children. In most cases, the facilitator was the mathematics or class teacher of the students designated for the project.

The research team trained these facilitators through a dedicated one-day training workshop conducted at the University of Tasmania in Launceston aimed at introducing the project goals, mathematical concepts, software (including MAPLE, which was new to all participants), suggested delivery techniques, and the delivery and assessment tools. This included instructional sessions on how to use our problem series and Allen’s non-template problem solving method to teach the underlying concepts and their application. For example, the explanations covering the equation of a curve and its integration must be in terms that the students can understand and relate to. The delivery was based on a series of simple PowerPoint presentations (as seen in Figure 2) by the facilitator to explain the concepts, followed by an interactive MAPLE worksheet that took the
students progressively through a series of examples and exercises, gradually introducing them to new mathematical concepts and MAPLE functions.

![Integration Example](image.png)

Figure 2: A snapshot of a PowerPoint slide showing how to perform integration.

Exercises were first introduced entirely through MAPLE, followed by a number of relevant problems on printed paper (albeit with sketches if required), with an associated MAPLE template to assist with their solution process. The students were allowed to progress through the problem series at their own pace, and encouraged to experiment with the various functions and capabilities of MAPLE.

Recognising that students of this age have short memory spans (Cestari, Lucidi, Pieroni, & Rossi-Arnaud, 2005), the programme had inbuilt revision exercises that endeavoured to maintain the competencies acquired during previous sessions. A final assessment of achievement and understanding (another hour, making 12 in total) culminated the intervention and provided data for analysis. In addition to the data gathered from these tests, students and facilitators were interviewed to ascertain their views on the programme and their perceptions on the outcomes.

Each problem contained a scenario from which a function equation could be derived and an area or rotational volume deduced. It was important students saw the relevance of each scenario to their lives, thus catching their interest and providing engagement with the learning of the relevant mathematical concepts. An example problem addresses painting a decorative wall shown in Figure 3. The student is asked to calculate the area in order to obtain sufficient paint for it.
Students chose a curve as the boundary, and then calculated the integral to find the area to be covered and thus the amount of paint required. The problem series were refined in line with the non-template problem solving method of Allen (2001) and a realistic mathematics education approach (Gravemeijer & Doorman, 1999) with the aid of a teacher advisor to ensure it fitted students in the targeted age range. Students utilised MAPLE (installed on their computer) for algebraic manipulation and calculation of definite integrals. The problem series ensured they also mastered relevant concepts.

The intervention was then conducted at the four schools. During this period, the facilitators gradually introduced each problem, the mathematical concepts, the software tool, and ways to solve the problem using it. For the integral problem dealing with the area of a surface bounded by a curve (the decorative wall problem in Figure 3), the process was presented as follows. In the first phase the facilitator used the problem to introduce the concepts and solution techniques related to curves and equations. The next phase introduced integration and its solution, including the use of MAPLE to solve the problem. Thus, the facilitators guided the students through the solution process, but were also able to ensure they imparted the underlying concepts to the students.

During this intervention stage, members of the research team visited each school to ensure that the programme was on track, required outcomes were being met, assist with any difficulties faced by the facilitator, and obtain relevant feedback. An intervention lesson was observed and focus group interviews conducted with students where possible. The research team also monitored the progress at each school via telephone conferences and email communication with the facilitators.

In the final stage, the students undertook a test based on questions drawn from first year engineering calculus examination papers to assess the knowledge and skills gained through the programme. The questions were provided on a printed sheet of paper, with a MAPLE template for
students to carry out the solutions. The latter did not have any of the mathematical material in it, thus the students were required to demonstrate their capacity to input mathematical functions through the MAPLE user interface. The completed MAPLE files were corrected and analysed by members of the research team.

Results
To provide consistency, facilitators e-mailed the MAPLE documents produced by students in the culminating assessment to the research team for marking. These were printed, marked and returned to the facilitator with a community report for reproduction in the local school newsletter. Table 1 contains a summary of the class and school characteristics and the mean test scores.

Table 1: Demographics, location, school advantage and mean test score for project classes.

<table>
<thead>
<tr>
<th>School</th>
<th>State</th>
<th>Males</th>
<th>Females</th>
<th>Youngest</th>
<th>Oldest</th>
<th>Location</th>
<th>ICSEA*</th>
<th>Mean test score</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>NSW</td>
<td>13</td>
<td>16</td>
<td>10.2</td>
<td>13.3</td>
<td>Metropolitan</td>
<td>993</td>
<td>64%</td>
</tr>
<tr>
<td>School 2</td>
<td>QLD</td>
<td>14</td>
<td>14</td>
<td>11.5</td>
<td>12.7</td>
<td>Metropolitan</td>
<td>1099</td>
<td>83%</td>
</tr>
<tr>
<td>School 3</td>
<td>TAS</td>
<td>12</td>
<td>12</td>
<td>12.6</td>
<td>13.5</td>
<td>Provincial</td>
<td>982</td>
<td>66%</td>
</tr>
<tr>
<td>School 4</td>
<td>VIC</td>
<td>8</td>
<td>8</td>
<td>11.4</td>
<td>12.2</td>
<td>Provincial</td>
<td>988</td>
<td>64%</td>
</tr>
</tbody>
</table>

*Index of Community Socio-Educational Advantage (ICSEA): The mean ICSEA value is 1000 with a standard deviation of 100. Values below the mean indicate schools with fewer advantages.

Unsurprisingly, the mean score of the more advantaged school was higher, but this is an expected result. More importantly, the mean scores are all above the 50% required to pass the engineering examination, and only eleven of the ninety seven students scored below this level. The greatest age range was in School 1 but the correlation between age and score was virtually zero (r<0.1). Two of the youngest students in this class scored above 75% and the eldest scored below 50%.

The results are very encouraging. These students working with computers were not selected for their academic prowess and came from schools with average or lower socio-economic advantages. Their performance was similar to that of students some eight or more years older selected for a university engineering course, but who are tested in a conventional way (using handwriting only).

The project succeeded in showing how students as young as ten years old in government schools could learn to use integral calculus as well as first year university engineering students. The research team feels that our original intent to show learning of complex concepts at a younger age
through ICT has been fully vindicated and Resnick’s work in systems has been repeated in this area of mathematics.

In addition to the quantitative results we also examined the interviews with students and their responses to the question “what is calculus good for?” on the test paper. Students at School 1 showed a good understanding of calculus, stating it was useful because:

- To work out maths for work or school. It can be used for working out how many kilometres to somewhere or to measure a garden.
- Because it helps you measure and calculate the distance and area of places. eg. how far you throw a ball. … thank you…
- It is useful because you can go home and do calculus. Went [sic] you start it looks hard.

These students used 15” screen Acer and Dell laptops financed by the school and by parents and shared their work at home. They celebrated their learning because they were able “to outwit Mum & Dad” or more seriously, “measure my house - figure out how much building material went into it”. They enjoyed graphing functions (see Figure 1).

School 2 students showed a good understanding of calculus, stating it was useful because:

- It helps us work out real life shapes and equations. You can work out the area of a swimming pool or a skateboard ramp.
- It is helpful because you can use it in real life situations. It’s good software and lets kids know what you will be using if you become an engineer. If someone knows how to work with all the expressions and what they mean, MAPLE 13 can be really easy.

One girl said “My dad's an architect, so I showed him MAPLE. He had a go with it. He said he wished he'd had it when he was studying! My mum doesn't like it because she doesn't understand it. And she's a teacher at this school!” Students went on to design mathematical cups using the software (see Figure 2).

In School 3, students liked calculus because:

- you can work out the area of a building or something like that.
• it can help you figure out hard questions easily.
• some of the questions look hard but if you use this it’s really easy.
• for when you get older and you might get calculus in college or university.
• It helps you understand things better and using MAPLE it was easier than I thought it would be.
• if you’re painting a house or a skate ramp it tells you how much paint or whatever it is you need to get. It can save you money and time.

Students in School 4 said:
• Integral maths is useful because it can help measure things that are curved. It can also be used to measure things that are on different angles on different areas. It can be useful for example to find out how much cement you need to make a skate ramp.
• I find MAPLE fun to learn and I would recommend it to all the schools. P.S. It’s a good way to get out of maths!!

Students took their netbooks home and shared what they were doing. Some had shown it to friends, and seven had shown it to parents. A few had used MAPLE to do their other homework and saw the potential of using it for other studies (especially homework) and applications. Students had found new mathematical functions on their own and most felt they had learnt new things through the project, such as cubic equations and graph plotting.

Conclusion
This project was timely and urgent. It demonstrated the principle that ICT can facilitate learning at a much earlier age than conventionally accepted in traditional curricula. The project also vindicated Papert’s idea that ‘unteachably abstract’ topics in the context of pencil and paper can become normal in a 1:1 computer classroom (2000). It also transforms our understanding of ‘teaching integral calculus’. A conventional course in calculus contains several elements which were also considered in this project: function graphs, quadratics, mathematical notation and solving problems using integral calculus. However, our course did not cover factorization, manual integration of polynomials, maxima, minima, derivatives or turning points. Students in the project classes did not need these skills to achieve what we set out to teach them: to solve problems using integral calculus. They demonstrated their mastery of these skills using the test instruments used in conventional assessment – the engineering degree examination questions. They did not need factorization to facilitate manual integrations – the MAPLE software did this for them. They did not need to remember a library of integral transforms – the computer did this for them. We need a public debate amongst mathematicians and mathematics educators to understand the power of this approach, and where it can lead to shortcomings for those students who excel in this discipline. Such a debate will be informed by the progress of student cohorts that skip ahead of their age-mates using the new technologies.
As a pilot in miniature, the study has shown what a digital curriculum could look like, what sorts of learning outcomes can be addressed, and how this might be done. As a nation we cannot keep tacking computers onto existing learning goals, but we need to revolutionize the teaching process (Gillard, 2008) using methods like this. Frankly, it is obvious that keyboarding is unlikely to improve handwriting. This is not an argument to discard existing curriculum outcomes, but to envisage new ones so that choices can be made. Schools can then choose to mix handwriting with computer-based skills of keyboarding and voice recognition dictation. They may also want to include in curricula new ICT-based learning outcomes in computational chemistry, weather forecasting and other learnings which cannot be realistically achieved without a computer.

Furthermore, this was a significant project because it linked ICT-based transformation with the development of a love of learning mathematics as used practically in science and engineering. With our new approach showing how integral calculus can solve real world problems, we used scientific and engineering examples to engage students. By promoting an interest in the mathematics which lies at the heart of these difficult disciplines, we hoped to address the shortage of engineering, science, and mathematics graduates. This long term aim is yet to be fully realized, because it will take several years for a critical mass of schools to adopt our methods and for their students to apply to undertake tertiary studies. Our method is innovative, because we implemented the project with ICT equipment already available in the final years of primary schooling. So we are hopeful that this early intervention is likely to be effective and sustainable.

The implications for curriculum design are salutary. Australia is about to adopt a nineteenth century curriculum in the emerging face of a twenty-first century paradigm shift. The debate about how to leverage student access to personal computers needs to take into account the potential demonstrated by this project.

We intend to carry the project further in three ways. Firstly, we will analyse our data to a greater extent to examine any significant age and gender related factors through the use of Rasch methods. Secondly, we are repeating the project in a fifth school using more conventional equipment infrastructure. This cohort attends a session in a computer laboratory once a week, which is similar to the situation in many primary schools. This is a risk, but it is necessary to see the effect of the personal laptop provisions in the previous project schools. Thirdly, we are hoping to negotiate the use of the approach in some New Zealand schools, with the possibility that free or open source software may be used. For instance, Maxima is free and open-source software that has similar functionality to MAPLE. However, this is causing some debate amongst the project team, because Maxima does not have a user interface using conventional mathematics notation.
Integrals can be created using either command line syntax or a dialogue box wizard. We know that students can use computers to solve problems with integral calculus – but will this learning be affected by the use of a non-traditional notation system? Another alternative is an emulation of a calculator on the computer screen, Microsoft Mathematics 4.0\textsuperscript{1}. This can solve complicated algebraic equations, geometry calculations and handle advanced calculus formulae and graphs like integrals and derivatives. It offers a template-based equation editor similar to MAPLE, with command-line characteristics and alternatives.

Finally, we will extend the work by delivering and assessing other advanced concepts beyond integral calculus through improved techniques based on the findings of this project. Our further investigations will probe this aspect with the intent of making our methods more accessible to schools by looking at lighthouse school propagation methods and deepening the learning to the point where students can devise their own formulae.

**References**


\textsuperscript{1}http://www.educational-freeware.com/freeware/microsoft-mathematics.aspx


