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This paper draws on a one year study of three secondary school classrooms to examine the nature of student-student-technology interaction when working in partnership with computer algebra systems (CAS) on mathematical modelling tasks and the classroom affordances and constraints that influence such interaction. The analysis of these data indicates that CAS enabled technologies have a role to play as *provocateurs* of productive student-student-teacher interaction in both small group and whole class settings. Our research indicates that technologies that incorporate CAS capabilities have the potential to mediate collaborative approaches to mathematical enquiry within life-related mathematical tasks.

There has been interest, by educational researchers and curriculum developers, in the use of both mathematical modelling and digital technologies to enhance the learning experiences of students in secondary mathematics classrooms for at least the past two decades. Mathematical modelling – formulating a mathematical representation of a real world situation, using mathematics to derive results, and interpreting the results in terms of the given situation – is a significant element of the senior mathematics syllabuses in Queensland, Australia and appears, as applications of mathematics, in the curriculum documents of most other Australian states. The need to make use of digital technologies, in learning mathematics, is now included in most state curriculum documents within Australia, and is increasingly apparent in curricula internationally. Recently, Computer Algebra System (CAS) enabled technologies have begun to make an impact on teaching and learning practices; most notably in Australia as the CAS active version of the Victorian Curriculum and Assessment Authority's *Mathematical Methods* (2006). CAS enabled technologies not only have the capability to perform a wide range of mathematical procedures, such as, function graphing, matrix manipulation and symbolic operations, but also, the capacity to provide users with real time advice about errors as mathematics is done. As CAS enabled technologies develop increasing acceptance in mainstream mathematics instruction there is need to explore and understand the synergies that might be developed between CAS and other areas of foci in mathematics education, such as, mathematical modelling and to identify implications of these synergies for classroom practice. This is particularly important as, to date, research has usually lagged behind implementation of CAS active curricula (Zbiek, 2003).

In addition, the current emphasis on the quality of interactions, between students, and between students and teachers, in school mathematics classrooms (for example, see Goos, Galbraith, Renshaw & Geiger, 2003; Manouchehri, 2004), means that any innovation that has to potential to influence these interactions must be of interest to researchers and teachers alike. The synergy that is likely to exist between mathematical modelling and CAS enabled technologies is one such innovation. While there is significant research related to solving contextualized problems through the use of the multiple representational facilities offered by digital technologies (e.g. Doerr & Zangor, 2000; Huntley, Rasmussen, Villarubi, Santong & Fey, 2000; Yerushalmy, 2000) and substantial argument to support the use of CAS to enhance the process of mathematical modelling (e.g., Kissane, 1999, 2001; Thomas, 2001), literature that deals with technology mediated interaction in mathematics classrooms is only just emerging. The project reported upon here, aims to develop a greater understanding of how CAS enabled technologies can support students' learning when they are engaged in mathematical modelling tasks, including ways in which CAS can mediate and support productive social interaction.

## The Role of Technology in the Process of Mathematical Modelling

While there are now significant corpuses of literature on the use of CAS enabled technologies and on mathematical modelling in school contexts, comparatively little has been written in how technology can be used to enhance the processes of mathematical modelling. An examination of the volume produced from the 14th Study of the International Commission for Mathematical Instruction entitled *Modelling and Applications in Mathematics Education* (Blum, Galbraith, Henn & Niss, 2007) for example, contains only one chapter out of 58 which focuses on technology use in mathematical modelling. This is despite acknowledgment from the editors that:

> Many technological devices are highly relevant for applications and modelling. They include calculators, computers, the Internet, and computational or graphical software as well as all kinds of instruments for measuring, for performing experiments etc. These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment. On the other hand, the use of calculators and computers may also bring associated problems and risks.
>
> (Niss, Blum & Galbraith, 2007, p.24)

Niss et al. (2007) go on to list nine questions related to the potential benefits of technology to mathematical modelling and to the possible dangers. These questions are not addressed directly by the authors who propose them and are only responded to, in passing, by other authors in the volume. This is an indication, that while there is acknowledgement of the potential for technology to enhance processes associated with mathematical modelling, there is limited literature that addresses this issue directly. Despite this limitation, a number of models for the role of technology in mathematical modelling have been proposed (e.g., Galbraith, Renshaw, Goos & Geiger, 2003; Confrey & Maloney, 2007). Two of these models are presented below.

### *Technology for Dealing with Routines and Mathematical Processes*

Based on a three year longitudinal case study of a class of students studying mathematics in technologically rich environment Galbraith, Renshaw, Goos & Geiger (2003) provide a description of the role of technology in the process of working with applications of mathematics and mathematical modelling.

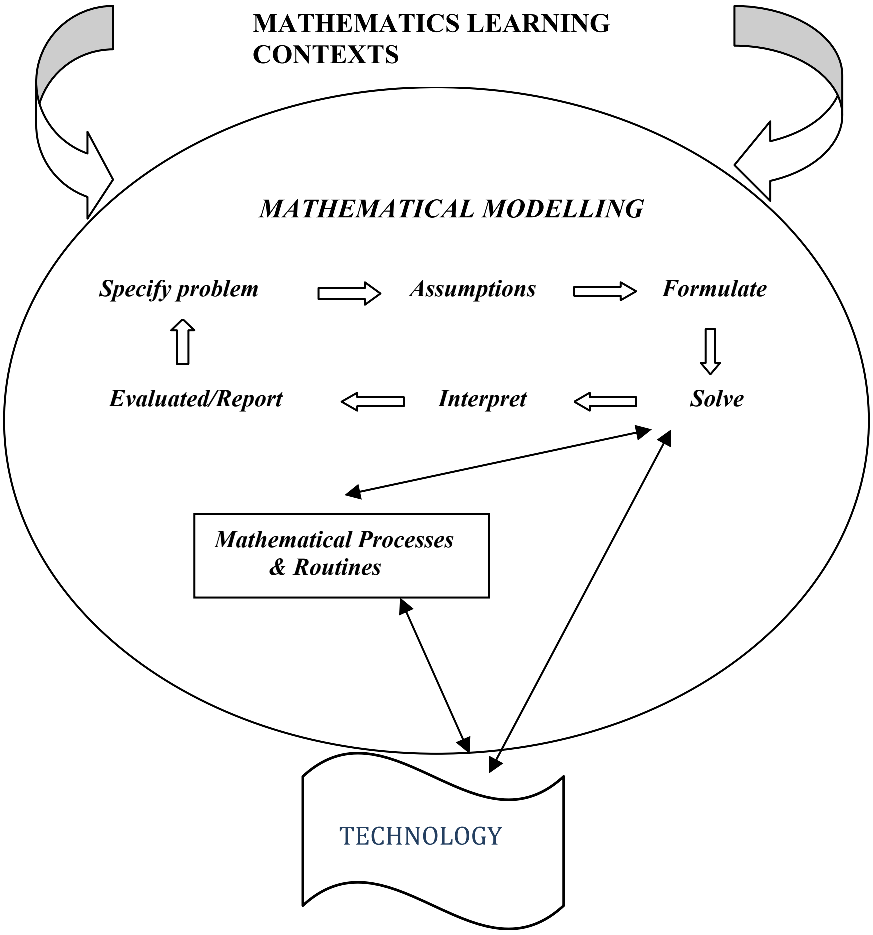


Figure 1. Some technological and mathematical interrelationships from Galbraith, Renshaw, Goos & Geiger (2003, p. 114)

In this description, illustrated in Figure 1 above, mathematical modelling is presented as a cyclic process in which starts with a problem set in a life-related context which is abstracted into a mathematical representation of the contextualised situation and solved through the application of mathematical routines and processes. The solution is then brought into relief against the original problem to consider its fit with the original context. If the fit is not considered sufficient, adjustments are made to the model and the process repeated until a satisfactory fit is achieved. Galbraith et al. argue mathematical routines and processes, students and technology are engaged in partnership during the *Solve* phase of a problem, which follows from the abstraction of a problem from its contextualised state into a mathematical model. This view identifies the conceptualization of a mathematical model as an exclusively human activity while the act of finding a solution to the abstracted model can be enhanced via the incorporation of technology. Thus technology is seen as a tool used to interact with mathematical ideas only after a mathematical model is developed rather than as a tool for the exploration and development of a model or its validation as a reliable representation of a life related situation.

*Modelling as Transforming Indeterminate Situations into Determinate Outcomes*

Confrey and Maloney (2007) identify four approaches to technology in mathematics instruction

1. Teach concepts and skills without computers, and provide these technological tools as resources after master;
2. Introduce technology to make patterns visible more readily, and to support mathematical concepts;
3. Teach new content necessitated by technological enhanced environment (estimation, checking, interactive methods)
4. Focus on applications, problem solving, and modelling, and use the technology as a tool for their solution. (p. 57)

While acknowledging that each of these approaches has its place they regard mathematical modelling as a central goal of mathematics instruction. Drawing on a Deweyian definition of inquiry, they argue that the process of modelling is founded on two activities: inquiry and reasoning. They see inquiry as a means of gaining insight into an indeterminate situation – such as a loosely bound problem in the real world. Reasoning is the process which draws on bodies of knowledge to transform the indeterminate situation into a determinant outcome – a model. In their view:

> Mathematical modelling is the process of encountering an indeterminate situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to bear to transform the situation. The modelling produces an outcome – a model – which is a description of representation of the situation, drawn from the mathematical disciplines, in relation to the person's experience, which itself has changed through the modelling process. (p. 60)

The process of inquiry gives rise to observations, responses, measurements, interactions, indicators, methods of sampling and data collection that are typically mediated by various forms of technology. Confrey and Maloney (2007) claim that is through the coordination of these artefacts and the processes of inquiry, reasoning and experiment, that an indeterminate situation is transformed into a determinate situation.

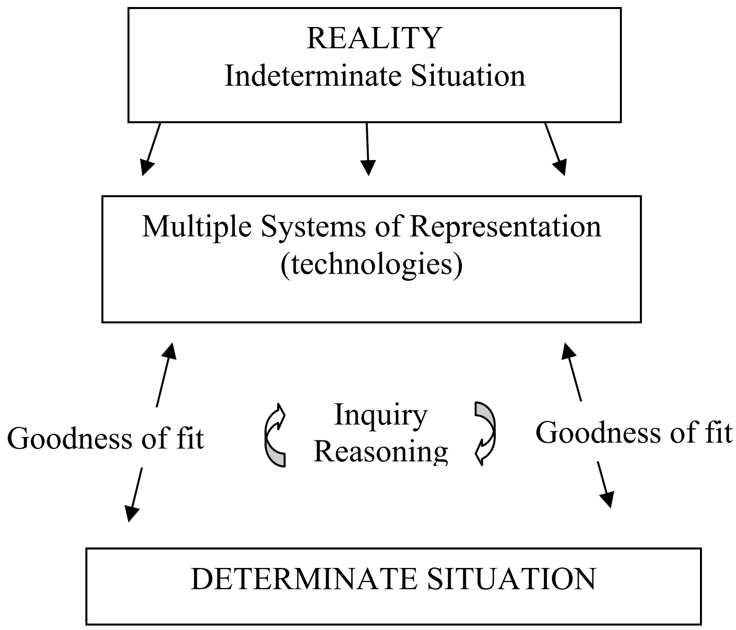


Figure 2. An inquiry and reasoning approach to mathematical modelling
from Confrey and Maloney (2007), p. 67

The role of technology in this model is many-fold. It can incorporate and generate representations, which can assist in transforming an indeterminate to a determinate situation. Technology also plays a central role in coordinating the inquiry, reasoning, and systematizing that lead to a determinate situation.

### *Commentary on the Role of Technology in Mathematical Modelling*

Both of these descriptions of the role of technology, within technology rich settings, recognise the cyclic nature of the modelling process and the important interplay between exploration, conjecture, reasoning, the use of mathematical procedures and process, and validation. In both of these models technology can be used to deal with mathematical procedures and processes, however, Confrey and Maloney (2007), suggest technology has a role to play in the coordination of inquiry and reasoning, and by association, interaction. What is not explored, in this approach, is the nature of the interaction between human participants and technology in the cycle of inquiry and reasoning. The section which follows provides an outline of research into the role of technology in mediating productive student interaction in mathematics classrooms.

## The Role of Technology in Promoting Social Interaction in Technologically Rich Mathematics Classrooms

While there is now an extensive body of literature related to the use of technology to learn mathematics, this has been principally concerned with the acquisition of mathematical knowledge and the advantages offered by the capacity of technology to present multiple representations of mathematical ideas as an aid to learning. Other authors (e.g., Burrill, 1992; Goos, Galbraith, Renshaw & Geiger, 2000b), however, have suggested that the most significant changes related to the introduction of technology into mathematics classrooms will be in the ways students and teachers interact. From this perspective, questions such as the role technology can have in mediating social interaction, or how technology is entwined into the fabric of a learning discourse in collaborative learning environments, receive greater primacy. There is now a growing interest in social perspectives of learning with technology and a number of authors have attempted to define the territory. Simonsen and Dick (1997), for example, in a study of teachers' perceptions of students' use of graphics calculators, conclude that this technology has a role to play in shifting the orientation of the classroom towards more student centred, discursive and exploratory approaches. The availability of technology alone, however, will not ensure the development of collaborative practices in a learning environment and so the classroom teacher, or, in other circumstances, the designer of the virtual learning environment, has a vital role to play in mediating the type of social interaction that is regarded as collaborative within a community of learners.

Willis and Kissane (1989) have introduced the notion of *Computer as a Catalyst*. In this mode the computing environment is used as a means of provoking mathematical explorations and discussion or to invoke the use of problem solving skills. This recognises the potential of technology to support learning focused interaction between students and suggests a mediating role for technology in learning.

The metaphor of *Computer as a Catalyst* is further extended, by Goos and Cretchley (2004) in a review of the role of technology in education in the Australasian region. Their development of the metaphor refines the view of the computer as a tool and catalyst for visualization; higher order thinking; and collaboration.

Goos' and Cretchley's (2004) metaphor is consistent with a socio-cultural perspective on learning where learning takes place via social interaction and is supported by cultural artefacts or physical tools. The inseparability of cognitive activity from both the process of learning within the group and from the tools that help mediate the activity are consistent with a Vygotskian view of the social nature of learning and Pea's (1985, 1987) description of the role of cognitive tools in distributed cognition. Other authors (Geiger and Beatty, in press; Geiger, 1998; Geiger & Goos, 1996; Goos, Galbraith, Renshaw & Geiger, 2000a, 2000b; Trouche, 2005) have claimed social interactions can take place in learning environments that make use of technological tools which have not necessarily been specifically designed for collaborative activity. Studies which exemplify the use of technologies designed primarily as mathematical tools to mediate collaborative practice include those of Geiger and Goos (1996) and Manouchehri (2004).

In a case study designed to investigate the social and material mediation of computer-based learning in an upper secondary mathematics classroom, Geiger and Goos (1996) found that interaction was both tool and task dependent. The computer was intended to act as both a tool, in enabling students to generate and manipulate data in a spreadsheet, and as a catalyst, in provoking exploration of the patterns that emerged from the data. However, the extent to which such exploration occurred depended on the type of task the students were given. Differences in the social organisation of students' work, identified in the function of their talk and the structure of their interaction, were associated with differences in task focus, with a focus on process, rather than products or means, producing collaborative discussion. Results implied that computer environments do not automatically facilitate peer interaction and that careful attention needed to be given to the structure of tasks if they are to elicit high level verbal reasoning. Students were most likely to interact if there was a genuine problem to be solved. This finding is of particular relevance to the current study because the nature of mathematical modelling ensures students will be challenged by authentic, loosely bound problems.

In a study involving undergraduate preservice teachers using NuCalc, an interactive algebra application, Manouchehri (2004) observed students' mathematical discussions displayed greater complexity while using NuCalc than when they used no mathematical application. Manouchehri identified the following four ways that the software supported discourse:

1. by assisting peers in constructing more sophisticated mathematical explanations;
2. by motivating engagement and increased participation in group inquiry;
3. by mediating discourse, resulting in a significant increase in the number of collaborative explanations constructed;
4. by shifting the pattern of interaction from teacher directed to peer driven.

Further, Manouchehri concluded that because of the immediacy of feedback to students, the software also supported a culture of conjecturing, testing and verifying, formalizing mathematics and collaboration and shifted the locus of power from the teacher to the students.

These studies offer support for the premise that technology can play a role in the mediation of collaborative learning processes. The immediacy of the feedback offered by technology can offer enhanced possibilities for classrooms where conjecturing, testing and verifying mathematical argumentation is viewed as important aspects of learning and doing mathematics. These aspects are also consistent with the model for the use of technology in mathematical modelling proposed by Confrey and Maloney (2007) and outlined earlier in this paper.

## Methodology and Research Design

The data described below is sourced from a 12 month study of the use of CAS enabled technologies in senior secondary classroom settings. Participants consisted of three secondary mathematics teachers and one class of students for each teacher. Teachers were selected because of their interest in exploring the use of CAS enabled technologies in teaching senior mathematics and because of a history of effective use of mathematical modelling tasks in their teaching practice. In addition, all three teachers were known to be supportive of collaborative approaches to learning mathematics. Two teachers were identified via their participation in teacher professional development events as presenters who made use of digital technologies in their classroom practice. The third was invited to participate in the study because of a prominent teaching role within a school which had a reputation for innovation in teaching mathematics – particularly in the use of technology to promote effective learning. While each teacher had developed considerable experience and expertise in teaching mathematical modelling and in the use of non-CAS technologies, such as graphing calculators, because of mandated curriculum requirements and because of their personal professionalism, there were differences in their facility with using CAS enabled technologies. One teacher had significant experience with the use of CAS in his teaching over a number of years, the second teacher had begun to trial the use of CAS in his teaching during the year in which this study is situated, and the third teacher had made use of CAS during professional development opportunities but has not attempted to use it in his own classrooms.

Three cohorts of students, one for each teacher, consisted of one Year 12 class and two Year 11 classes. All classes were studying Mathematics B, a subject that includes substantial elements of calculus and statistics. Each class was equipped with a set of Texas Instruments CAS enabled *Nspire* handheld devices (at least one for each student and the teacher) and one licence for software that mirrored the facilities of the *Nspire* handheld device. These technologies possess all of the features of a typical graphing calculator, such as function and graph plotting modules, but also included a CAS capability that is highly integrated with other calculator facilities. Other features include a fully functional spreadsheet (again with CAS integrated capability) and a well developed feedback mechanism for reporting on input errors.

The class groups came from one government school and two non-government colleges. These classes were nominated by their respective teachers because their teachers believed curriculum requirements to engage with mathematical modelling would be enhanced though the use of CAS enabled technologies. Students' experience in the use of technology to learn mathematics varied across the three classes. While none of the students had used the *Nspire* handhelds before the beginning of the year in which the study was situated, two groups received substantial previous use by the time they were first observed by the researcher, the other group receiving very limited exposure.

Research into interactions between multiple participants and between participants and digital tools, in an authentic classroom setting, must employ a methodology with the capacity to accommodate educational phenomena that are situated, temporal and complex. Further, the nature of the classroom environment brings with it, in the case of this study, the prospect of unanticipated or emergent outcomes (Ramsden, 1997) in terms of both the usage of digital technologies and in the type and quality of the interactions between participants and technologies – thus a naturalistic research design was employed.

Data collection instruments included observational field notes, video and audio recording of small groups of students working on specified tasks and video and audio recording of episodes of whole class activity. Each class group was observed on three different occasions, each time for periods ranging from 45 minutes through to 90 minutes. On the majority of occasions, students and teachers worked on tasks that incorporated some element of mathematical modelling. In addition, individual student and teacher interviews were conducted after each class session in order to ascertain their perceptions of the benefits offered by CAS enabled technologies to learning mathematics, in general, and more specifically to working on mathematical modelling tasks. Finally, a focus group interview was conducted with the three teachers involved in the project and two of the principal researchers after all classroom observations had been completed. The purpose of the focus group interview was to document the participant teachers' perceptions on the benefits offered by CAS to mathematical modelling and also to record their responses to observations, reflections and elements of theory building offered by the researcher.

The analysis which follows is based on data drawn from two classroom episodes which are reported as vignettes. The vignettes are developed from observational field notes on whole class activity, audio and video recordings of students and teachers working together in both small group and whole class settings and follow-up interviews of the two teachers.

## Analysis - The Role of Emergence

Research into interactions between multiple participants and between participants and digital tools, in an authentic classroom setting, must employ a methodology with the capacity to accommodate educational phenomena that are situated, temporal and complex. Further, the nature of the classroom environment brings with it, in the case of this study, the prospect of unanticipated or emergent outcomes in terms of both the usage of digital technologies and in the type and quality of the interactions between participants and technologies. As emergent uses of technology can sometimes provide the most exciting outcomes and point the way to more innovative and creative uses of a technology than for which it was designed (Ramsden, 1997), emergent uses have been actively sought after as part of the data gathering processes for this study.

Ramsden (1997) argues that while we should not seek to use a technology for a purpose that it is patently unsuited to, emergent uses should be productively sought, including uses that no one (including the designers) could have predicted. This study seeks to document and interpret emergent patterns of behaviour evident in interactions between humans and technological tools. Because of the emergent nature of this study a naturalistic research design was chosen in order to accommodate and build theory around classroom events as they occurred *in situ*. Consistent with a naturalistic methodology, data collection and analysis were conducted simultaneously with theory building. Patterns of emergent behaviour were documented and categorised. Where emergent phenomena were noted and documented, the researcher made use of follow-up interviews with the relevant teacher to feed these back to the relevant teacher in order to triangulate the occurrence of the identified phenomena and to validate the researcher's interpretation of what was observed by ascertaining the participant teachers perspective on the event, occurrence or episode. Once confirmed, the research incorporated the observed phenomena into developing theory while attempting to incorporate the perspectives of all participants.

## Two Vignettes Where CAS Promotes Contention

The two vignettes reported below come from classrooms in two different schools – one a government school and the other a private college. The teacher in the government school had personal experience with the use of CAS but had not used it previously in his teaching. His students had begun to make use of CAS from the beginning of the year; approximately two months before the first vignette was documented. The second vignette records an episode from the classroom of a teacher from a private college. This teacher had extensive experience with the use of CAS and his students had been using CAS, in a least a limited way, since Year 9, although that had been using the *Nspire* handheld for about two months. In both cases, the teachers challenged students to make use of CAS based technology as an aid to working with mathematical modelling problems.

### *Vignette 1*

The vignette described below took place in a Year 12 (final year of secondary school) mathematics classroom where students were investigating the nature of population decay towards extinction. During one lesson observation the teacher set the students the following question.

When will a population of 50 000 bacteria become extinct if the decay rate is 4% per day?

One pair of students developed an initial exponential model for the population y at any time x, $y = 50000 \times (0.96)^x$, and equated this to zero in the belief that the solution to this equation would give the number of periods, and hence the time it would take for the population to become extinct. When students entered this equation into their handhelds, however, the device unexpectedly responded with a *false* message, as illustrated in Figure 3.

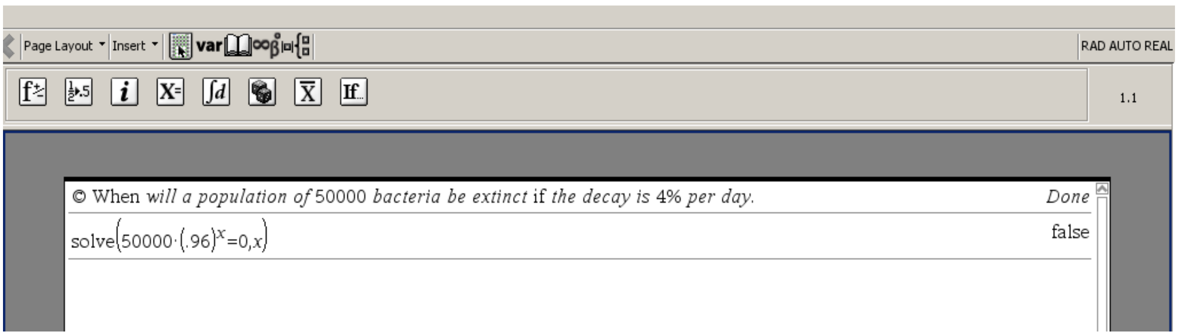


Figure 3. *Nspire* display for the problem $y = 50000 \times (0.96)^x$

The students were initially concerned that this response had been generated because they had made a mistake with the syntax of their command. They re-entered the instruction several times and tried a number of variations to the structure of the command but did not consider that there was anything at fault with the parameters they had entered. When the students asked their teacher for assistance, he looked at the display and stated that there was nothing wrong with the technical side of what they had done but they should think harder about their assumptions.

After further consideration, and no progress, the teacher directed the problem to the whole class. One student indicated that the difficulty being experienced was because "you can't have an exponential equal to zero". This resulted in a whole class discussion of the assumption that extinction meant a population of zero. The discussion identified reason for the unsatisfactory calculator output as equating an exponential model to zero and then considered the possible alternatives. Eventually the class adapted the original assumption to accommodate the limitations of the abstracted model by accepting the position that extinction was “any number less than one”. Students then made this adjustment to their entries on the handheld and a satisfactory result was returned.

In a follow-up interview, directly after the lesson, the researcher asked the teacher about the episode.

> Researcher: I saw an element of what we just talked about today when conflict that was generated by an interpretation of the question about bacteria. Students developed an equation and then, because no bacteria were left, they equated it to zero. The calculator responded with a false message. In some ways you could think it was a distraction and that the procedure didn’t work; some kids might just give up. But on the other hand, what it provoked in your class was an opportunity to discuss. “Did you push the wrong buttons? Oh, you think you did – let’s look at the maths. Well your maths is right! Do you understand why it couldn’t be? Let’s talk about the assumption”.
>
> Teacher 1: Simon was one of those, he said – “no way you could get that to equal zero”, without necessarily understanding why. Not that he couldn’t solve it when it equalled zero, it was that concept he couldn’t see; that population couldn’t become zero.
>
> Researcher: Yes didn’t need CAS to understand that, they just understood it because they knew their maths well enough.
>
> Teacher 1: Yeah we actually use the CAS to create the confrontation.

In this excerpt, the teacher identifies the message created by the CAS – that the equation was *false* – as a mechanism for confronting a students’ lack of understanding of the interplay between the demands of their mathematical model and valid mathematical expression. The context indicated the model should be equal to 0 to represent the extinction of the bacteria while, from a purely mathematical perspective, this was not valid. Interestingly, this conflict or “confrontation” was viewed by the teacher as an opportunity to promote productive interaction among the class, which ultimately lead to the resolution of the problem and a broader understanding of the role of assumption in the mathematical modelling process.

### *Vignette 2*

In this second vignette the teacher was working with a Year 11 class on a unit about a variety of mathematical functions including linear, quadratic, cubic, exponential and power functions. During the observed session they were asked to work on the following task.

> The CSIRO has been monitoring the rate at which Carbon Dioxide is produced in a section of the Darling River. Over a 20 day period they recoded the rate of CO2 production in the river. The averages of these measurements appear in the table below.
>
> The CO2 concentration [CO2] of the water is of concern because an excessive difference between the [CO2] at night and the [CO2] used during the day through photosynthesis can result in algal blooms which then results in oxygen deprivation and death of the resulting animal population and sunlight deprivation leading to death of the plant life and the subsequent death of that section of the river.

From experience it is known that a difference of greater than 5% between the [CO2] of a water sample at night and the [CO2] during the day can signal an algal bloom is imminent.

Table 1. Rate of CO2 Production versus time

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Time in Hours | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Rate of $CO_2$ Production | 0 | -0.042 | -0.044 | -0.041 | -0.039 | -0.038 | -0.035 | -0.03 | -0.026 | -0.023 |
| Time in Hours | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| Rate of $CO_2$ Production | -0.02 | -0.008 | 0 | 0.054 | 0.045 | 0.04 | 0.035 | 0.03 | 0.027 | 0.023 |
| Time in Hours | 20 | 21 | 22 | 23 | 24 | | | | | |
| Rate of $CO_2$ Production | 0.02 | 0.015 | 0.012 | 0.005 | 0 | | | | | |

Is there cause for concern by the CSIRO researchers?

Identify any assumptions and the limitations of your mathematical model.

Students were expected to build a mathematical model by inspecting a scatterplot of the data which was then used to determine the general form of function that would best fit the data. This general form was then to be adapted for the specific data presented in the question. The questions at the end of the task were to be answered using the resulting model. Students had earlier studied strategies for determining if a particular function type was most suited to a data set. Most recently, students were introduced to a technique where ln versus ln plots of data sets was used to determine if a power function was an appropriate basis on which to build a mathematical model. This appears to have influenced the actions of two students as the transcript below indicates.

Researcher: So you are up to building the model are you?

Student 1: Well we worked out a plan of what we are going to do, we are just putting it on paper.

Researcher: So do you want to tell me what the plan is?

Student 1: The plan is to do the Log/Log plot of both the data to see if they are modelled by a power function. We have previously seen that the………..

Researcher: So that is something you have learnt to do over time? Whenever you see data look like that, you check if it's a power function by using Log/Log.

These students experienced problems with this approach, however, as the technique, in this case, meant the students tried to find the natural logarithm of 0.

Student 1: 0.44 zero… (entering information into the Nspire device)). Don't tell me I have done something wrong. Dammit. Mummbles… Start at zero is it possible to do a power aggression? I don't think so!

This comment was in response to the display which resulted when the students attempted to find the natural logarithm of both *Time* and *CO2 output* data using the spreadsheet facility of their handheld device (Figure 4). Students were surprised by the outputs they received for both sets of calculations, that is, the #UNDEF against the 0 entry in the *Time* column and the lack of any entries in the *CO2* column. In addition, an error message was produced indicating the results of the students' entries were problematic for the handheld device.

After a little more thought students realised where the problem lay.

Student 1: L N time is going to be equal to the L N of actually time. Tim…. oh is that undefined cause it's zero?

Student 2: Yep.

Student 1: Right now if I go back to my graph… Enter

Student 2: If you try zero fit, it will just go crazy.

Students eventually identified the problem with their approach and realised their initial assumption, that is, the best model for whole data set as power function, was at fault. Eventually, they realise it was best to model the data with two separate functions.

Student 1: So we have fitted a linear model for the top data and then we fitted a power function to the bottom data given we take the absolute value of those the question asks, the difference greater than 5% we need to look at the actual CO2 produced, now what we have got is the rate, to go back to the actual CO2 absorbed we need to integrate the model or both models and then use the percentage difference formula – predicted minus actual divided by actual or in this case night minus day divide by day x 100 to look at whether for any x or any t there is any percentage difference greater than .05.

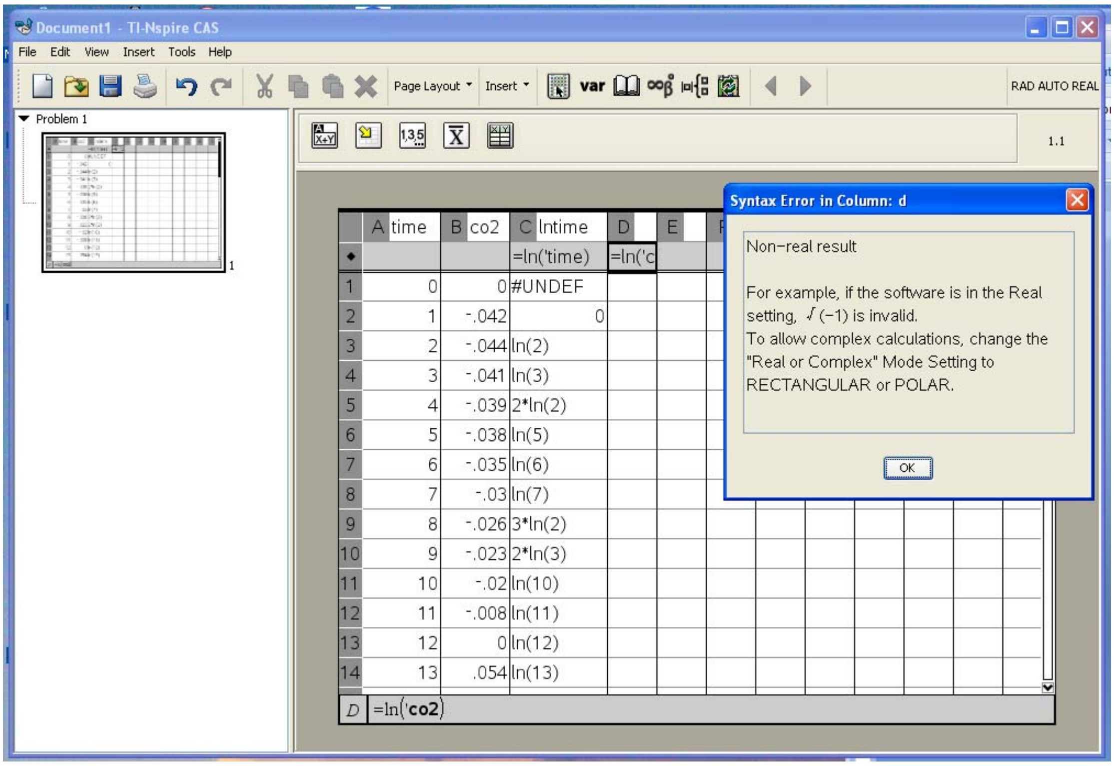


Figure 4. Nspire display of spreadsheet for natural logarithm of time and CO2 output data.

The second teacher was interviewed approximately two weeks after the episode described above.

Reseacher: One of the things that's come out quite strongly in your classes, and also I noticed it with John's as well, is there have been quite a few occasions where something has happened with the technology and it's really provoked a discussion.

The last time I was here the students tried to put in an logarithmic model and make it equal to zero and it just said "No - not doing it!", which provoked a whole discussion about where the problem lay.

Today, there was a problem about the difference between a model developed using integration, either on the calculator or by hand, verses the regression model that was produced by doing the "area-so-far curve" which also provoked a discussion. Do you consciously do that or are these just incidental things that you just run with as they come up?

Teacher 2: Just as they come up, yeah.

Researcher: That last one looked to me like you deliberately did it, but you are just saying it just happened and that it was a good thing to follow-up.

Trevor: Yeah basically, I mean they crop up and we call them teachable moments. There are things that crop up and you work through and you sort of think yeah that's pretty cool. I mean yeah it worked out alright today.

In this except the teacher acknowledges the value of discussion which flows from unexpected outcomes but makes the point that he has not attempted to deliberately catalyse the collaborative discourse which developed during the lesson by in-building a "confrontation" into the lesson design. Despite the benefit that this teacher believes is an outcome of debate around a blockage to students' progress, and the apparent prevalence of this type of discourse in is classroom, he does not believe the type of scenario, described above, should be contrived, but simply embraced when they occur.

## Discussion and Conclusion

In each of the vignettes described above, teachers found students were experiencing blockages to their progress and used this as a catalyst for a whole class discussion in which the problematic issue was explored and then resolved. While these blockages were the result of erroneous input by the students, the unexpected set back catalysed discussion in a way consistent with Goos' and Cretchley's (2004) commentary. It is important to note, also, that during both small group and whole class discussions students themselves contributed to the improvement of knowledge and understanding. Technology has therefore played a role in catalysing student participation in their own learning through small group and more public interactions with the teacher and their peers.

In contrast to the role attributed to technology in mathematical modelling by Galbraith, Renshaw, Goos and Geiger (2003) the electronic output forced students to revaluate fundamental assumptions they had made within the context of the described problems. This means that technology related activity takes place during the assumptions phase rather than only at the solve juncture outlined in Figure 1. Consequently, this assigns a role to technology in the conceptualisation of the model rather than simply as a tool which is used to solve a mathematical problem after it has been abstracted – a position more consistent with that of Confrey and Maloney (2007) who acknowledge a role for technology in the inquiry/reasoning cycle.

The unexpected output on the handheld devices in both vignettes influenced the inquiry/reasoning cycle by confronting students with an unanticipated result which, in turn, provoked the rethinking of their original assumptions, sometimes with the guidance of their teacher, and adjustment to their approach solving these problems. This rethinking was characterised by highly collaborative modes of discourse in which interactions between students and between students and the teacher focused on the processes of conjecture, knowledge testing and validation by a classroom community that included all classroom participants – both students and the teacher. As a result, students were forced to reshape their early thinking to satisfy the demands of both the context and the limitations of their abstracted model. Thus technology, in this case, has fulfilled a more interactive role than simply that of a powerful computational tool – it has mediated interaction by producing an initial unexpected result and also played a part in the final resolution of the conflict through its use to validate alternative solutions. The way in which technology mediated discourse, facilitated collaborative interaction and shifted the locus of interaction from the teacher to the students is consistent with the findings of Manouchehri (2004).

The provocations also represent opportunities for teachers to gain an awareness of students' misconceptions and then to provide appropriate scaffolding in order to move students forward in their understanding of the issue that was proving problematic. As reported above, both teachers used the consternation generated by the error messages recorded on students' handhelds to structure a forum in which student-student-teacher interaction played an important role in resolving the issue of concern. Despite the value teachers placed on this process, neither teacher believed that this could be implemented in a contrived way – rather they indicated that such opportunities are by nature serendipitous and that it was part of a teacher's repertoire to accommodate and take advantage of such events as they occurred.

The role of CAS based technologies as a *provocateur* of productive student-student-teacher interaction, in both small group and whole class settings, appears to have potential to mediate collaborative discussion, and within the particular context of this paper, provides possibilies for enhancing the teaching and learning of mathematical modelling, and is therefore an area worthy of further research.

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