Student teachers' content knowledge and understanding of primary mathematics

Karoline Afamasaga-Fuata'i
University of New England
<kafamasa@une.edu.au>

It is expected that a year twelve level of mathematics is sufficient background for primary student teachers to effectively develop their pedagogical skills to teach primary mathematics. However, for a cohort of primary student teachers at a regional Australian university, results from two mathematics diagnostic tests administered at the beginning of semester one and end of semester two in their first year of university studies showed there were persistent misconceptions and critical skills that required explicit redressing to further enhance their competence in primary mathematics. Student responses from the two Mathematics Diagnostic Tests were analysed using the Dichotomous Rasch Measurement Model to determine a cognitive development scale of mathematical competence with the content of primary mathematics. Analyses of cognitive demands of the items and success rates enabled the identification of persistent misconceptions. Main findings suggest that student teachers find solving word problems the most difficult followed by items on reasoning and operating with fractions, proportions and probability while the basic geometric, algebraic and numeric computations and analysis of visually presented data were the easiest. These have implications for teaching primary mathematics competently and flexibly in ways that can motivate future students to engage meaningfully with mathematics learning.

Introduction

To ensure that primary preservice (PPS) teachers are competent with the content of the primary mathematics curriculum they are expected to teach their future students, a regional Australian university initiated a longitudinal numeracy project to monitor PPS teachers' competence levels at three different times of their four-year bachelor of education program before exit. This paper reports the results of the first two diagnostic tests administered at the beginning of semester one and end of semester two in the first year of studies for a particular cohort. According to Shulman (1986), three of the knowledge requirements for effective teaching are the subject-matter knowledge, curriculum knowledge and pedagogical content knowledge. Given the content and process strands of the primary mathematics curriculum (e.g., NSW BOS, 2002), and prevailing professional teaching standards (e.g., AAMT, 2006) it is crucial that PPS teachers demonstrate competence and mastery of the content of the primary mathematics curriculum if they are to confidently mediate mathematical meaning in the classroom and effectively teach primary students to be successful problem solvers. The main focus questions of this paper, therefore, are: (1) Was there any improvement in the cohort's performance since the first testing? and (2) What are the cohort's persistent misconceptions which require further remediation? PPS teachers' mathematical competence with the content of the primary mathematics curriculum is assessed by a written diagnostic test. The methodology and data analysis are examined next.

Methodology and Analysis

The mathematics diagnostic test (MDT1) comprised of thirty items, selected (Mays, 2005) primarily from the TIMSS 1999-R mathematics paper (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000) as these have reliability and validity data from 15 year olds in 38 different countries, and seven items from the misconception literature on mental computations, fractions, algebraic thinking and proportional reasoning (Heirdsfield, 2002; McIntosh & Dole, 2000;

Thompson & Salandha, 2003) deemed relevant for the preparation of quality teachers of primary mathematics. The 37 items sampled the content areas of the *NSW BOS K-8 Mathematics Curriculum* (NSW BOS, 2002) – fractions and number sense, measurement, algebra, geometry and data presentation, analysis and probability – and five cognitive domains: *knowing, using routine procedures, investigating and problem solving, and mathematical communication* (Mullis et al., 2000). All 37 items were left open-ended to provide access to student errors (Mays, 2005). In the second diagnostic test (MDT2), 11 items were kept common to enable test equating and linking of test results (Bond & Fox, 2001). The rest of the questions remained the same except for changes in numerical values. A total of 177 students took MDT1 and 143 took the second diagnostic test (MDT2). Student responses were categorised *Correct* or *Incorrect* and analysed using the Dichotomous Rasch Measurement Model and QUEST software (Adams & Khoo, 1996).

The Rasch Model examines only one theoretical construct at a time on a hierarchical "more than/less than" logit scale (unidimensionality). Rasch parameters, item difficulty and person ability, are estimated from the natural logarithm of the pass-versus-fail proportion (*calibration of difficulties and abilities*) whereas estimation of fit is measured by mean square (mean squared differences between observed and expected values) and *t*, infit and outfit values (*estimation of fit to the model*) (Bond & Fox, 2001). Fit of the data to the model (infit *t* values (-2, 2)) and reliability of the test (around 1) are examined. The cohort results for MDT1 are presented first followed by those for MDT2. This paper focuses only on the analysis of results from the cohort's perspective.

Results

Psychometric Properties of the First Mathematics Diagnostic Test

The Rasch Model theoretically sets the mean of item estimates at 0 before item and person estimates are calibrated. The MDT1 person ability mean of 0.50 logits suggested the test was relatively easy for this cohort. The item-person map (Figure 1) showed that 12 out of 37 items had difficulty estimates above ability mean. The case and item distributions were spread out between -2.32 and 3.99 logits with the majority of the items (25/37) located below average ability. An item standard deviation of 1.54 (compared to 1.02 for cases) confirmed the items were more spread out with a most difficult item above the top ability estimate and 3 more easiest items than lowest ability estimate. An item fit map (not shown) verified that all items fit the model hence establishing that the 37 items worked together consistently to define a unidimensional scale. The reliability indices for items (0.98) and cases (0.82) were both high (Bond & Fox, 2001) indicating the test produced reliable measures of item difficulty and student teachers' mathematical competence of the primary mathematics content.

A Cognitive Developmental Scale and Item Clusters for MDT1

Shown in Figure 2 is the QUEST-generated item locations and distribution including brief item descriptions to facilitate discussions. The vertical interval gaps between item locations indicate the additional cognitive demand and processing to successfully solve the upper item compared to the lower item. Whilst these gaps are larger in some places, there are also psychometrically similar items clustered together along the logit scale. For example, one way of grouping items to facilitate discussion is as shown by the different shadings into six clusters (Figure 2) namely, the *Most*

Difficult (3 items), *Difficult* (7 items), *Average* (2 items), *Below Average* (8 items), *Easy* (11 items) and *Easiest Items* (6 items). It appears that the most difficult items are word problems with multiplicative relationships while the easiest are reading and analysing data from diagrammatic representations and mental computation. The item clusters are discussed further below.

```
                          Mathematics Diagnostic Test 1 (DT1)
-----------------------------------------------------------------------------------------
Item Estimates (Thresholds) (N = 177 L = 37 Probability Level=0.50)
4.0                         |    13
                       X    |
                            |
                            |
                            |
                            |
                            |     8
3.0                XXXXX    |
                            |
                            |
                       X    |
                            |    30
                     XXX    |
                            |
                     XXX    |    26
2.0                         |     5
                 XXXXXXX    |
                            |
                   XXXXX    |    23
                            |     4
               XXXXXXXXX    |
                XXXXXXXX    |     7     17     37
                            |
1.0             XXXXXXXX    |
                    XXXX    |
       XXXXXXXXXXXXXXXXX    |    29
     Case Mean 0.50         |     3
               XXXXXXXXX    |
  XXXXXXXXXXXXXXXXXXXXXX    |    14
                   XXXXX    |     9     32
0.0            XXXXXXXXX    |    16     34   Item Mean 0.00
                            |    31
                   XXXXX    |    20
               XXXXXXXXX    |    21
                XXXXXXXX    |
                    XXXX    |    24     27     28     33
                            |
                    XXXX    |    36
-1.0                        |     6     10     19     25
                  XXXXXX    |    12
                    XXXX    |    18
                            |
                      XX    |
                            |
                            |    35
                            |
-2.0                   X    |    15     22
                            |
                            |
                            |     1      2
                            |    11
                            |
                            |
                            |
 -3.0                       |
-----------------------------------------------------------------------------------------
     Each X represents    1 students
=========================================================================================
```

Note: Items 1-5, 7, 13 and 28 are from the misconception literature

Figure 1. MDT1 Item-Person Map

Most Difficult Items (2.44 – 3.99 logits) - The most difficult items are complex, multi-step word problems on algebraically representing and solving multiplicative relationships. For example, Item 13 on representing "16 students to one professor" using a mathematical expression was the most difficult followed by those on using given information to determine speed, expressed in new units for Item 8 and average weight for Item 30. The gaps (at least 0.70 logits) indicate the relative differences in cognitive processing to successfully solve the higher item.

Mathematics Diagnostic Test 1 (DT1)

```
-------------------------------------------------------------------------------------------------
         Item Estimates (Thresholds) all on all (N = 177 L = 37 Probability Level=0.50)
-------------------------------------------------------------------------------------------------
 4.0  |13   16 students to one professor as a mathematical expression
      |
      |
      |
      |
      |
      |8    Athlete ran 3km in 8mins, find average speed in metre/sec
      |
 3.0  |
      |
      |
      |
      |30   500 crystals weigh 6.5 kg, find average weight of a crystal
      |
      |
      |26   rectangle is twice as long as it is wide, find ratio of width to perimeter
 2.0  | 5   mentally compute 0.3 x 0.3
      |
      |
      |23 one-third given away, one-quarter of remainder given away, 24 left, find initial amount
      | 4  ½ + 1/3
      |
      | 7 asc. order 5/6,7/10,2/3,3/5  17 Elevator problem, floor 3m apart 37 extend table pattern
      |
 1.0  |
      |
      |29 sound travels 330 m/sec, lightning followed 4.5 sec, find distance lightning strike
 CAM  | 3 mental computation of 6013 - 3987
      |
      |14 area of garden path embedded in a 12 x 12 square and 8 x 8 square
      | 9 likely outcome with fifth throw  32 Club with 86 members, 14 more females than male
 0.0  |16 identify 2 similar triangles 34 ratio of nitrate to total weight          Item Mean
      |31 spent five-eighth of $240, find amount left
      |20 12x - 10 = 6x + 32, find value of x
      |21 write three equivalent fractions to 2/3
      |
      |24 prob. multiples of 3 27 simplify nxnxn 28 side of Sim.Δ 33 shaded area?
      |
      |36 find 7th value given diagram of a geometric pattern and table values
-1.0  | 6 algebraic expressions  10 one-third of number 19 missing angle 25 shade a fraction
      |12 5 out of 100 faulty bulbs, find expected number in a batch of 3000 bulbs
      |18 if x=3, find value of (5x+3)/(4x-3)
      |
      |
      |
      |35 extend geometric pattern, 4 figures shown, complete 4th & 5th table values
      |
-2.0  |15 pictograph scale                         22 angle measure closest to 45 degrees
      |
      |
      | 1 mentally computing 8 x 7                  2 mentally computing 25% of 80?
      |11 reading data from a histogram, pupils travelling more than 10 minutes
      |
      |
      |
-3.0  |
-------------------------------------------------------------------------------------------------
  Each X represents    1 student                                 CAM - Case Ability Mean
=================================================================================================
```

Cluster Key

1. Most Difficult	2. Difficult	3. Average	4. Below Average	5. Easy	6. Easiest

Figure 2. MDT1 Items' Brief Descriptions and Item Clusters

Difficult Items (1.19 – 2.14 logits) – Three sub-clusters are apparent. The first one includes a rectangle word problem involving investigation with a multiplicative relationship (length is twice as long as it is wide) and multi-steps to determine a ratio (width:perimeter) followed by 1-digit decimal multiplication. A word problem on operations with fractions and mentally adding simple fractions are the two items within the second sub-cluster. The third sub-cluster includes psychometrically similar items on ordering fractions, an additive relationship (total distance travelled by an elevator) and extension of tabular patterns. Overall, items are less complex word problems requiring interpretation of quantitative relationships, operating with and ordering fractions and extrapolation from table values.

Average Items (0.67 and 0.58 logits) – Of similar cognitive difficulty are a 2-step word problem on finding distance given speed and time and mentally subtracting 4-digit numbers.

Below Average Items (-0.31 – 0.31 logits) – The 8 items are on interpreting a complex diagram, likely outcomes, additive relationships (ratio in a mixture and club membership), knowing similar triangles, finding equivalent fractions, solving a linear equation, and solving a simple word problem on fraction of an amount. Overall, items are mainly on simple problem solving, knowing and using routine procedures.

Easy Items (-1.16 - -0.57 logits) – Two sub-clusters are apparent with the first sub-cluster including psychometrically similar items on probability of a multiple of 3, basic algebraic computation, proportional reasoning and area of a shaded rectangle. In the second sub-cluster are items involving interpreting and extending patterns (numeric and geometric), algebraically representing a relationship, simple problems on geometric properties, proportions, representing and operating with fractions and evaluation. Overall, items are mainly one-step word problems involving basic algebraic and geometric properties and routine procedures.

Easiest Items (-2.32 - -2.53 logits) – Most items are on reading and interpreting data from diagrammatic representations, extending a geometric pattern and mentally computing product and percentage.

Overall, the six clusters highlight the developmental scale of mathematical competence in terms of the mathematics content areas and types of cognitive demands required to successfully solve items ranging from the most difficult complex word problems on representing and solving multiplicative relationships through to relatively less complex problems involving additive relationships, ordering, operating with and representing fractions, using routine procedures, analysing visually presented data and basic computations.

Item Success Rates and Mastery Level for MDT1

An examination of the item analysis data showed items with at least 75% success rates were those on mentally computing 8 x 7 (Item 1) and 25% of 80 (Item 2), reading data from a histogram (Item 11), determining a pictograph scale (Item 15), proportional reasoning (Item 12), evaluation (Item 18), recognizing a 45° angle (Item 22), and extending a geometric pattern (Item 35). These items are basic computations, routine procedure, reading of graphs and extension of visual patterns, which correspond to items in the Easiest Item Cluster and bottom end of the Easy Item Cluster.

In contrast, items with less than 50% success rate involve mentally computing 6013 - 3987 (Item 03), ½ + 1/3 (Item 4) and 0.3 x 0.3 (Item 5), determining total distance traveled by an elevator (Item 17), operating with fractions (Item 23), finding distance sound traveled given time and speed (Item 29) and extrapolating values from a table (Item 37). These items correspond to those in the Difficult and Average Item Clusters.

Items with success rates less than 20% included items that are complex word problems involving investigation, multi-steps and representation of multiplicative relationships such as average speed and unit conversion (Item 8), student:professor relationship and mathematical expression (Item 13), length is twice as long as it is wide and ratio of width: perimeter (Item 26) and average weight of a salt crystal (Item 30). These items correspond to those in the Most Difficult Item Cluster and one from the top-end of the Difficult Item Cluster.

If mastery competence level of the content of primary mathematics is defined as 75% success rate, then the majority of items (29/37) were not mastered by this cohort. Individually, a total score of at least 75% on the test would indicate mastery competence of the content of primary mathematics as assessed by MDT1. Accordingly, the total score results showed that only 18.6% (33/177) of the cohort achieved mastery. This implied that the rest of the PPS teachers urgently required follow-up remediation before the second testing.

Psychometric Properties of the Second Mathematics Diagnostic Test

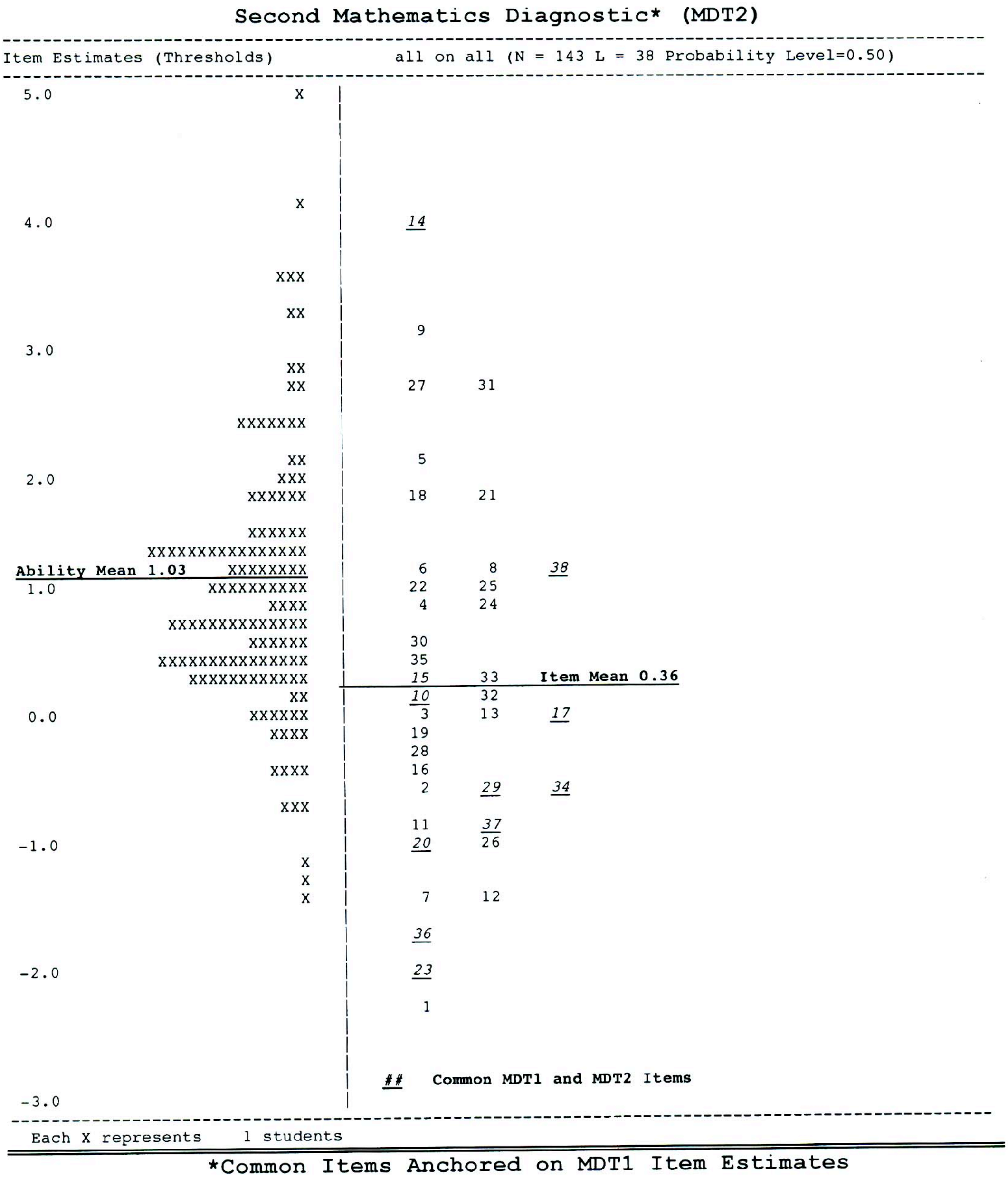
Responses from MDT2 were Rasch analysed with item estimates of the 11 common items anchored on MDT1 values to enable comparison of student performance. The item difficulty mean for MDT2 was higher at 0.36 logits with a narrower standard deviation of 1.41 (compared to MDT1's 1.54 logits, as expected given that this group included students that did not attain mastery competence level in MDT1).

For the case estimates, the MDT2 mean ability was 1.03 (compared to MDT1's 0.50 logits) with a standard deviation of 0.94 (compared to MDT1's 1.02). One student got a perfect score. The item-person map (Figure 3) showed the two distributions were not aligned in terms of mean and spread. Two ability estimates were above the most difficult item with 3 easiest items below the lowest ability estimate. Overall, only 14 out of 38 items had difficulty estimates above ability mean with the majority of the items (24/38) located below average ability. An item fit map (not shown) verified that all items fit the model hence establishing that the 38 items worked together consistently to define a unidimensional scale. The reliability indices for items (0.98) and cases (0.83) (similar to those of MDT1) were both high (Bond & Fox, 2001) indicating the test produced reliable measures of item difficulty and ability measures (i.e., student teachers' mathematical competence of the primary mathematics content). The cohort's performance as assessed by MDT1 and MDT2 ability estimates showed a statistical significant difference ($p = 0.00$).

A Cognitive Developmental Scale and Item Clusters for MDT2

Shown in Figure 4 is the QUEST-generated item locations and distribution including brief item descriptions to facilitate comparisons with Figure 2. The clustering of MDT2 items is slightly different from Figure 2 with a few of the items shifting difficulty locations as a result of changing numerical values in the MDT2 items, in some cases, from integers to rational numbers. Figure 4 shows 6 clusters namely, the *Most Difficult* (increased from 3 to 4 items), *Difficult* (reduced from 7 to 3 items), *Average* (increased from 2 to 7 items), *Below Average* (increased from 8 to 15 items), *Easy* (reduced from 11 to 4 items) and *Easiest Items* (reduced from 6 to 5 items). Like MDT1, the most difficult items were word problems with multiplicative relationships and involving operations with decimals while the easiest remained reading data from diagrammatic representations, geometric pattern extension, angle recognition and mental multiplication, and a new addition to the cluster, an item on basic algebraic representation. The item clusters are discussed below in terms of changes in their success rates and/or difficulty locations.

Most Difficult Items (2.61 – 3.99 logits) – The most difficult items are still complex word problems involving multi-steps on algebraically representing and solving multiplicative relationships with an added level of difficulty when operating with rational numbers. For example, while the student:professor (Item 14) and average speed and unit conversion (Item 9) items were still the most difficult, the next two items on average weight (Item 31) and ratio of width to perimeter (Item 27, Width) had increased difficulty estimates as a result of changes from “500 crystals weighing 6.5g” to “600 crystals weighing 7.8g” and “length is twice as long as it is wide” to “length is one and a half its width”. Item analysis outputs (in MDT1 and MDT2) are shown in Figure 5 in terms of success and error rates. In general, there are increases in success rates for all four items by the second testing but Item 27 (Width) had the least increase. Results indicate that a significant majority of PPS teachers still experienced difficulties solving complex word problems.



Note: Items 1-6, 8, 14 and 29 are from the misconception literature

Figure 3. MDT2 Item-Person Map

Difficult Items (1.86 – 2.10 logits) – The three items involve 1-digit decimal multiplication, solving a linear equation and a word problem on an additive relationship (total distance travelled by an elevator). While slight to no change were noted with the success rates of two of the items (Figure 6), the most marked change (increased difficulty level or reduced success rate) occurred when the equation in Item

21 (Equatn) changed from “$12x-10=6x+32$” to “$11-10x=6x-52$” in which the revised item has a non-integer solution. Evidently, the majority of PPS teachers still demonstrated conceptual and computational difficulties with these items.

Second Mathematics Diagnostic* (MDT2)

```
--------------------------------------------------------------------------------------------
       Item Estimates (Thresholds)      all on all (N = 143 L = 38 Probability Level=0.50)
--------------------------------------------------------------------------------------------
5.0    |
       |
       |
       |
       |
       |
       |
4.0    | 14 Express 16 students to one professor as a mathematical equation
       |
       |
       |
       |
       |
       | 9 Athlete ran 4.5km in 12mins, find average speed in metre/sec
3.0    |
       |
       | 27 length is one-and-half width    31 600 crystals weigh 7.8g, average weight
       |
       |
       |
       |  5 mentally computing 0.2 x 0.3
2.0    |
       | 18 elevator problem, floors 2.8 m apart    21 value of x in 11 - 10x = 6x - 52
       |
       |
CAM    | 6 asc. decimals   8 asc. fractions 5/7,4/9,3/4,7/10 38 extend table pattern
1.0    | 22 equivalent fractions to 3/7    25 probability of chip being even and a multiple of 3
       |  4 mentally computing ¼ + 2/5    24  operations with fractions, find initial amt
       |
       | 30 sounds travels 330 m/s, after 5.2 s, find distance
       | 35 ratio of nitrate to total weight in a mixture
       | 15 garden path area    33 club membership, 16 more females than males    Item Mean 0.36
       | 10 likely outcome, 5th throw of coin 32 spent five-ninths of $270, amount left
0.0    |  3 4-digit subtraction 13 proportion faulty bulbs 17 identify similar triangles
       | 19 if x=5, find value of (7x-3)/(3x-2)
       | 28 write in its simplest form 4 x n x n x n
       | 16 pictograph scale
       | 2 mentally computing 20% of 70    29 side of Sim.Δ    34 area of shaded rectangle
       |
       | 11 one-third of number    37 extrapolation from numeric values
-1.0   | 20 missing interior angle of a quadrilateral    26 shade 2/9 of a 6 x 6 grid
       |
       |
       |  7 algebraically representation    12 reading data from a histogram
       |
       | 36 extending a geometric pattern
       |
-2.0   | 23 angle measure closest to 45 degrees
       |
       |  1 mentally computing 9 x 7
       |
       |
       |
       |
-3.0   |
--------------------------------------------------------------------------------------------
                 Each X represents    1 student          CAM - Case Ability Mean
```

***Common Items Anchored on MDT1 Item Estimates**

Cluster Key

1. Most Difficult	2. Difficult	3. Average	4 Below Average	5. Easy	6. Easiest

Figure 4. MDT2 Items’ Brief Descriptions and Item Clusters

Average Items (1.8 – 1.38 logits) – This cluster included psychometrically similar items on ascending decimals and fractions (AscFrac), equivalent fractions (EquivFrac) and operating with fractions (OperFrac), extending a tabular pattern (ExtendP) and finding probability (Probab). Item analysis outputs (Figure 7) showed success rates increased for the pattern extension item and all fraction items except two items; Item 22 on generating 3 equivalent fractions when changing from "$\frac{2}{3}$" to "$\frac{3}{7}$" and Item 25 on probability when the favourable outcome changed from "multiple of 3" to "multiple of 3 and even" given eleven chips labelled "2, 3, 5, 6, 8, 10, 11, 12, 14, 18 and 20". As evidenced by the last two items, multiplicative reasoning, interpretation and synthesis of conceptual understanding and procedural proficiency appeared the most problematic. Overall, the majority of PPS teachers demonstrated conceptual and computational difficulties with most items shown in Figure 7.

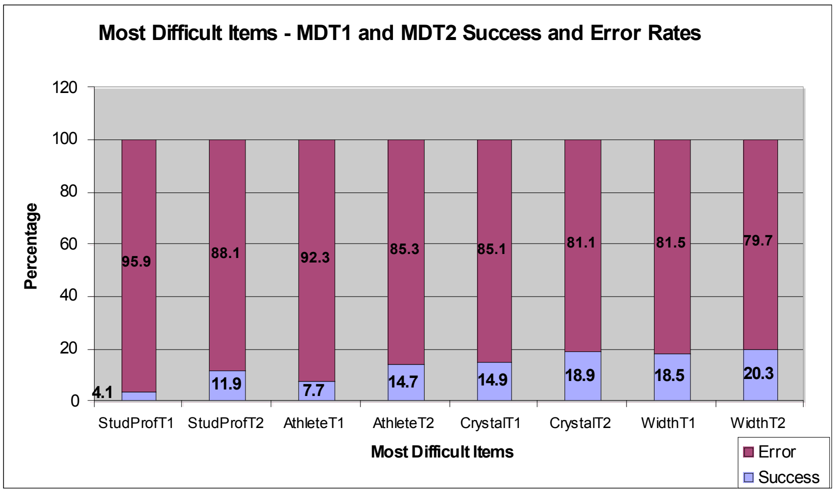


Figure 5. Most Difficult Items – MDT1 and MDT2 Success and Error Rates

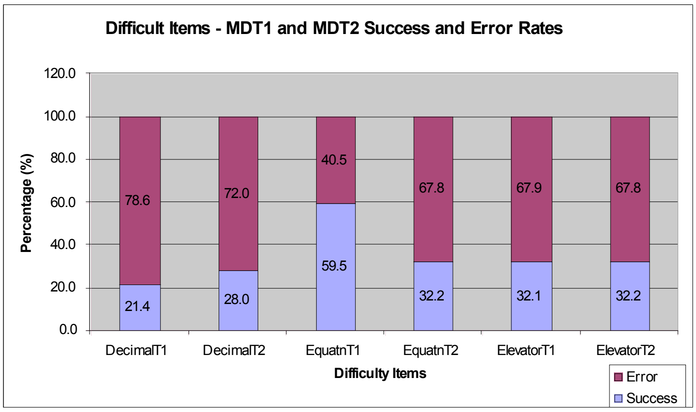


Figure 6. Difficult Items – MDT1 and MDT2 Success and Error Rates

Below Average Items (-0.54 – 0.67 logits) - The cluster included items on solving simple word problems on speed, ratio, additive relationship (club membership), fraction of an amount, likely outcome, proportional reasoning, basic geometric properties (interior angles and similar triangles), basic algebraic representation, interpreting diagrams and mentally computing percentage and 4-digit subtraction.

Item analysis outputs (Figure 8) showed most of the items had increased success rates by the second testing. The exceptions are Item 30 (Sound) on finding distance given speed of sound and time when time changed from "4.5 sec" to "5.2 sec" and Item 13 on proportion of faulty bulbs (PropBu) when "5 faulty bulbs in a sample of 100 from a batch of 3000" changed to "13 informal votes in a sample of 200 from an electorate of 6000 voters". Reduced success rates were noted for Item 19 (EvalEq) when “$\frac{5x+3}{4x-3}$” changed to “$\frac{7x-3}{3x-2}$”and for Item 2 (Percent) when “25% of 80” changed to “20% of 70”. It appeared that multiplicative and proportional reasoning and operating with decimals and percentage are associated with reduced success rates. Although Item 16 (PictSca) remained exactly the same in both tests, it showed a reduced MDT2 success rate. Overall, items involved solving simple word problems, multiplicative reasoning, basic algebraic, numeric, and geometric computations, analysing visual data, and mental computation.

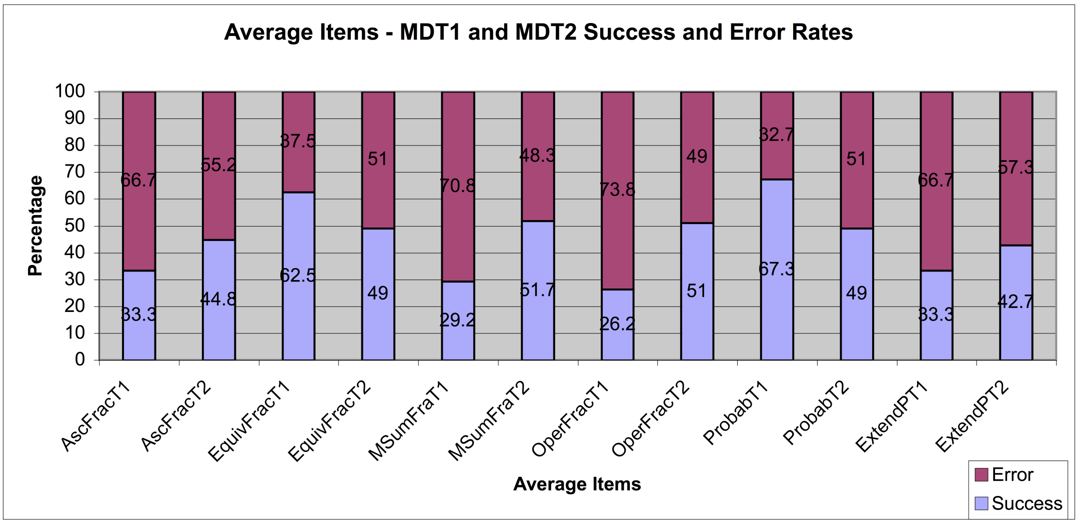


Figure 7. Average Items – MDT1 and MDT2 Success and Error Rates

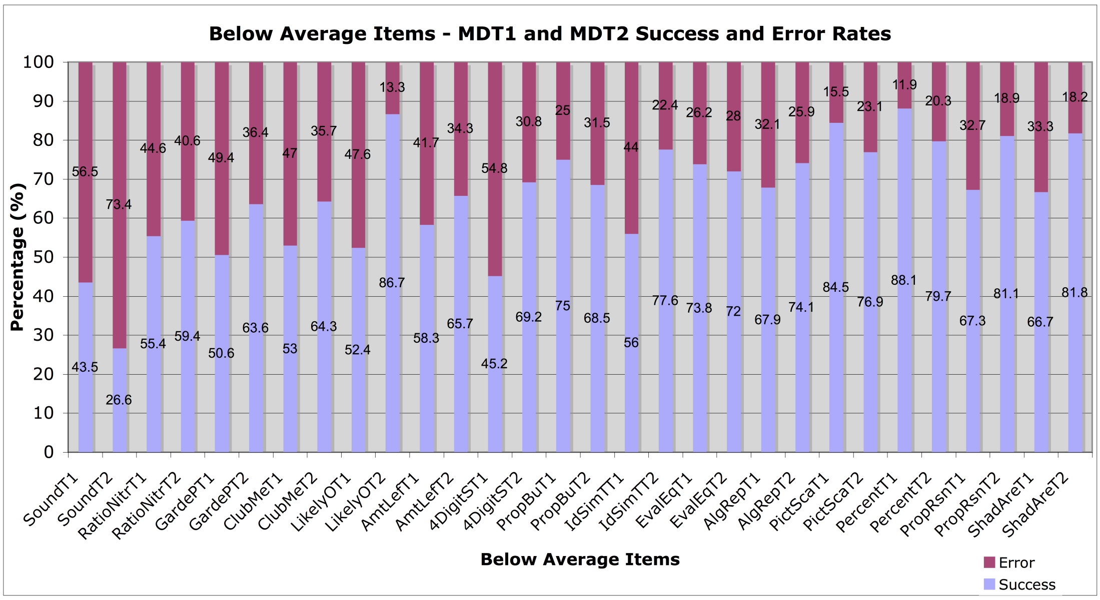


Figure 8. Below Average Items – MDT1 and MDT2 Success and Error Rates

Easy Items (-0.98 - -0.75 logits) – The psychometrically similar items involve operating with and representing fractions, extending a numeric pattern, and applying geometric properties. The item analyses in Figure 9 showed increases in success rates. In general, the majority of PPS teachers were successful with these items.

Easiest Items (-2.22 - -1.38 logits) – Items are on reading and interpreting data from diagrammatic representations, extending a geometric pattern and mentally multiplying 1-digit whole numbers. A new addition to the cluster is an algebraic representation item while the pictograph scale item had shifted up one cluster. Item analyses (Figure 10) showed that the majority of the PPS teachers were successful with these items.

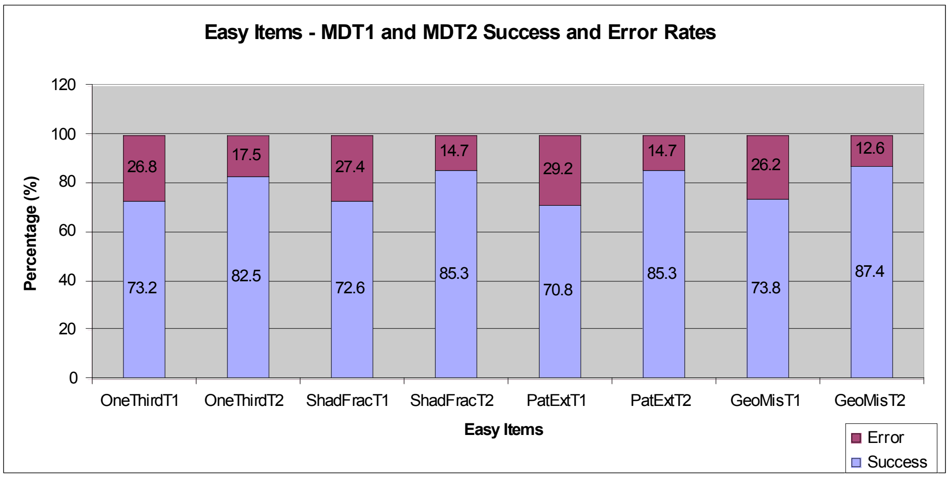


Figure 9. Easy Items – MDT1 and MDT2 Success and Error Rates

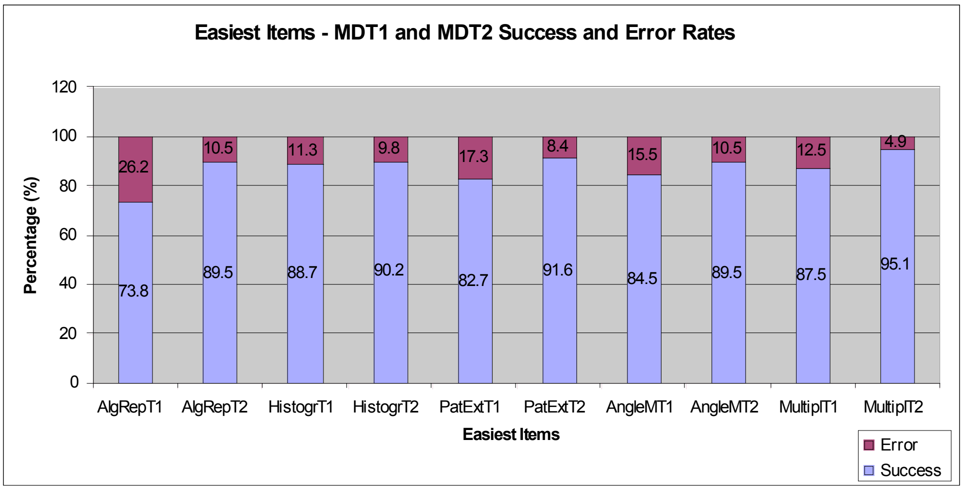


Figure 10. Easiest Items – MDT1 and MDT2 Success and Error Rates

Overall, the six clusters highlight the developmental scale of mathematical competence in terms of the mathematics content and types of cognitive demands required to successfully solve items ranging from the most difficult complex word problems on representing and solving and reasoning with multiplicative relationships through to additive relationships, ordering, operating with and representing fractions, using routine procedures, analysing visually presented data, and basic (algebraic, numeric and geometric) computations.

Item Success Rates and Mastery Level for MDT2

An examination of the item analysis data showed 15 items have at least 75% success rates. Items were on mentally computing 9 x 7 (Item 1) and 20% of 70 (Item 2), recognizing a 45° angle (Item 23), likely outcome (Item 10), reading data from a histogram (Item 12) and pictograph (Item 16), extension of visual and numeric patterns (Items 36 and 37), algebraic representation (Item 7), missing interior angle (Item 20) and knowing similar triangles (Item 17), representing and operating with fractions (Items 26 and 11), shaded area (Item 34) and proportional reasoning with similar triangles (Item 29). These items are basic algebraic, numeric and geometric computations, basic reading of visual representations and extrapolations from geometric and numeric patterns. These items correspond to all those in the Easiest and Easy Item Clusters and six from the Below Average Item Cluster.

In contrast, items with less than 50% success rate (10 items) involved mentally computing 0.2 x 0.3 (Item 5), ascending decimals and fractions (Items 6 and 8), determining total distance traveled by an elevator (Item 18), solving a linear equation (Item 21), generating equivalent fractions (Item 22), probability (Item 25), word problems on multiplicative relationships (Items 27 and 30), and mathematical communication and extrapolation from table values (Item 38). These items correspond to those in the Average and Difficult Clusters with one item from each of the adjacent clusters.

Items with success rates less than 20% are complex word problems involving investigation, multi-steps and representation of multiplicative relationships such as average speed and unit conversion (Item 9), student:professor relationship and mathematical expression (Item 14), and average weight of a salt crystal (Item 31). These items are all in the Most Difficult Item Cluster.

The item success rates showed that the majority of the items (23/38) were not mastered by this cohort. On the basis of total test scores, only 19.5% (28/143) of the cohort achieved mastery at at least 75%. This implies that the majority of the PPS teachers were subsequently advised to enrol in a mathematics unit specifically designed to address their mathematics misconceptions before the third testing and before exit. It appeared that the idea of a year twelve mathematics background, as sufficient preparation for PPS teachers to cope with the content of primary mathematics, needs revisiting given the results reported here.

Discussion

Reliability of the Diagnostic Test

Statistics from the Rasch analyses of student responses confirmed the diagnostic test produced reliable measures of item difficulty and PPS teachers' competence of the mathematics content of primary mathematics along a unidimensional scale. Items were spread out in a hierarchical order from the most difficult to the easiest to provide a cognitive developmental scale of mathematical competence as evident from the two item-person maps. QUEST-generated item and ability locations along a common logit scale enabled the identification of item clusters that were psychometrically similar and different to facilitate identification of the cohort's mathematical misconceptions across the two tests.

Cohort Performance

The cohort's performances in the two tests were significantly different indicating that overall there was improvement in cohort performance. However, as indicated by individual item success rates, a majority of the items did not achieve mastery success rate suggesting that the majority of students require specific remediation with their knowledge and understanding of the content of primary mathematics. Item analyses indicated there were improvements (increased success rates) with most of the items, with some showing reduced success rates. While the cohort's performance in the second test was significantly different from that in the first test, the cognitive developmental scale as described by item locations and clusters remained more or less the same across the two tests, with some variations with a few of the items.

Cognitive Developmental Scale of Competence

Further analysis of the cognitive developmental scale of items resulted in a more refined hierarchical framework of item difficulty, which ranged from the most difficult to the easiest item clusters.

By the second testing, the most difficult items tended to be *complex word problems* requiring investigation and critical interpretation of *multiplicative relationships* (ratio, rate, and average) and involving multi-steps and operations with decimals.

Difficult items were less complex word problems on *additive relationships* (total distance) and solving equations which involved *operating with decimals* whereas average items were mainly on *ordering, representing and operating with fractions*, reasoning and extrapolation from tabular patterns and identifying favourable outcomes.

Below average items were simple, one or two-step word problems on rate, ratio, additive relationships (e.g., 16 more females than males), likely outcome, operating with fractions, *proportional reasoning*, knowledge of similar triangles, *analysis of visually presented data* (complex diagrams, pictograph), and basic algebraic (evaluation, index notation) and numeric (percentage) computations.

Whilst easy items were increasingly simple word problems on fractions, extrapolation from numerical patterns, and knowing interior angles and area-model of fractions, easiest items involved basic (algebraic, geometric and numeric) computations and analysis of visually presented data/patterns.

Cohort's Mathematical Misconceptions

The difficulty item-cluster framework implied that solving word problems is a critical skill, which requires explicit teaching and remediation for the PPS teachers particularly as they are expected to cultivate the same in their future students. Also the hierarchical, decreasing order of difficulty from multiplicative reasoning and relationships through to additive reasoning and relationships and basic computation with fractions and numbers emerged from the Rasch analysis of student responses reported here. This same cognitive trend has being reported elsewhere particularly those working in the area of multiplicative reasoning (Thompson & Salandha, 2003) and mental computations (Heirdsfield, 2002; McIntosh, & Dole, 2000).

An examination of item success rates further confirmed the hierarchical framework of item clusters as a viable indicator of general areas of misconceptions from the cohort's perspective. For example, while the Easy and Easiest Item Clusters had items with more or less 75% success rates, lower rates for items in the higher clusters (Below Average up to Most Difficult) revealed content specific conceptual and procedural difficulties, which require specific remediation. The extremely low success rates of the most difficult items suggested that critical problem solving skills require explicit remediation.

Gaps in the PPS teachers' knowledge and understanding of some of the content areas of primary mathematics particularly fractions and number sense, measurement, geometry, and data presentation and analysis were evident through the high difficulty estimates and low success rates of the items above the Easy Item Cluster.

Findings reported here contribute to the literature on teacher education particularly to the debate on how much mathematics content PPS teachers ought to have as part of their formal preservice programs (Burgess, & Bicknell, 2003). For example, is it feasible to graduate PPS teachers knowing that there are gaps in their own content knowledge and understanding of the curriculum they are expected to teach? Is a year twelve mathematics background appropriate preparation for those planning to teach primary mathematics? These are questions and concerns that continue to plague the community of mathematics educators and teachers of mathematics. There are no easy answers but the questions continue to be asked particularly within the prevailing climate to certify the numeracy competence of PPS teachers before exit.

Conclusions and Implications

Main findings which emerged from the empirical data analysed and reported here provide possible answers to the two focus questions for this paper, namely, (1) Was there any improvement in the cohort's performance since the first testing and (2) What are the cohort's persistent misconceptions which require further remediation? First, there was a demonstrable, significant improvement in the performance of the cohort over time as evident by the improvement in mean ability estimates and item analyses. Second, a refined hierarchical framework of item difficulty provides guidance to the cohort's general areas of persistent misconceptions. For example, solving complex word problems that involve reasoning with multiplicative relationships (e.g., average, rate, ratio and proportion) continue to plague these PPS teachers. Also ordering, representing and operating with fractions and decimals are other areas the cohort found particularly difficult. The identified misconceptions of these PPS teachers also parallel those experienced by primary students themselves. Therefore, PPS teachers' misconceptions need remediation before exit.

The main findings of this paper imply that the PPS teachers would need to be competent and confident in their content knowledge and understanding of the primary mathematics curriculum before they can hope to effectively mediate mathematical discussions and dialogue in the classroom. Since pedagogical content knowledge is dependent on subject-matter knowledge and curriculum knowledge, PPS teachers need to know the mathematics first as learners before they can teach others to know (Huckstep, Rowland, & Thwaites, 2003). Accordingly, the impact of mastery competence on PPS teachers' pedagogical mediation of meaning in the classroom is an area worthy of further research for this cohort.

References

Adams, R. J., & Khoo, S. T. (1996). *QUEST*: *Interactive Item Analysis*. Melbourne: Australian Council for Education and Research.

Australian Association of Mathematics Teachers (AAMT). (2006). *Professional Teaching Standards*. AAMT.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex Publishing.

Bond, T. & Fox, C. (2001). *Applying the Rasch Model Fundamental Measurement in the Human Sciences*. Lawrence Erlbaum Associates.

Burgess, T., & Bicknell, B. (2003). Subject matter knowledge in preservice teacher education. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley, (Ed.), *Mathematics Education Research: Innovation, Networking, & Opportunity*. Proceedings of the 26 th Annual Conference of the Mathematics Education Research Group of Australasia (pp. 176-182), Sydney, MERGA.

Heirdsfield, A. M. (2002). Inaccurate mental addition and subtraction: causes and compensation. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.) *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp. 334-341). Sydney: MERGA.

Huckstep, P., Rowland, T., & Thwaites, A. (2003). Primary teachers' mathematics content knowledge: what does it look like in the classroom? Retrieved on February 16, 2007 from http://www.leeds.ac.uk/educol/documents/00002534.htm.

Mays, H. (2005). Mathematical knowledge of some entrants to a pre-service education course. In M. Coupland, J. Anderson, & T. Spencer (Eds.), *Making Mathematics Vital*. Proceedings of the

Twentieth Biennial Conference of the Australian Association of Mathematics Teachers (AAMT) (pp. 43-52). Sydney: University of Technology.

McIntosh, A., & Dole, S. (2000). Mental computation, number sense and general mathematics ability: Are they linked? In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000.* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia Incorporated, pp. 401-408). Perth: MERGA.

Mullis, I., Martin, M., Gonzalez, E., Gregory, K., Garden, R., O'Connor, K., Chrostowski, S., & Smith, T. (2000). *TIMSS 1999 International Mathematics Report.* Retrieved on February 16, 2007 from http://timss.bc.edu/timss1999/math_achievement_report.html.

New South Wales Board of Studies (NSWBOS) (2002). *K-8 Mathematics Syllabus*. Sydney, Australia: NSWBOS.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15* (2 February), 4-14.

Thompson, P. W., & Salandha, L. A. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for Schools* (pp. 95-113). Reston, VA: NCTM.