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ENHANCING MENTAL COMPUTATION TEACHING AND LEARNING IN YEAR 3

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The purpose of the study was to develop and investigate the effectiveness of a short instructional program to enhance mental computation strategies (addition and subtraction) in two classes of Year 3 students (approximately 8 years of age). Outcomes of the project were aimed at benefiting both the teachers and the students. The short instructional program made use of two models (100 board and empty number line) to support students' development of mental strategies. Pre-instruction and post-instruction interviews were conducted to monitor students' progress.

While international literature (e.g., Klein & Beishuizen, 1994; Maclellan, 2001; McIntosh, 1998; Reys, Reys, Nohda, & Emori, 1995) has argued the importance of including mental computation in mathematics curricula that promote number sense, Queensland has been slow to pick up the challenge. Mental computation does not feature in the present syllabus (Department of Education, Queensland, 1987); although it will feature in new syllabus that will be mandated in 2007 (Queensland Studies Authority (QSA), 2003). For instance, "[Students] use a range of computation methods including mental, written and calculator to solve problems" (QSA, 2003, p. 20; extract from Level Statement from Level 2). In the Core Content (QSA, 2003, p. 44), computation methods are explained as mental computations, written recordings (student-generated and traditional methods), and calculators or computers. However, student-generated *mental* strategies are not mentioned. This is in contrast to recommendations elsewhere (e.g., Anghileri, 2001, Blöte, Klein, & Beishuizen, 2000; Buzeika, 1999).

At present, formal written procedures for addition and subtraction are introduced at an early stage (Year 2), resulting in students' resorting to these written procedures regardless of efficiency (e.g., Cooper, Heirdsfield, & Irons, 1996). In contrast, some research (e.g., Buzeika, 1999) has shown that students can and do formulate their own efficient computational strategies. The incongruity between formal and self-developed strategies is seen as a hindrance to understanding in mathematics and to the development of number sense (e.g., McIntosh, Reys, & Reys, 1992).

One focus of mental computation research has been teaching experiments. These have taken a variety of directions. In The Netherlands, where mental computation strategies are taught specifically, teaching experiments have focused on investigations of best models for students to use to successfully employ efficient mental strategies (Beishuizen, 1999; Blöte, Klein, & Beishuizen, 2000). However, Treffers (1998) has suggested that *structured* calculating (calculating with the help of suitable models) is only an intermediary step to *formal* calculating, where numbers are used as mental objects and calculations are performed without the aid of structured materials or models. Buzeika (1999) focused on encouraging students to develop their own computational strategies (both mental and written) from a constructivist perspective. McIntosh (2002) reported positive outcomes in a project aimed at developing students' informal written strategies, firstly by concentrating on developing students' invented or informal mental computation strategies.

The purpose of the study reported here was to investigate the effect of an instructional program where students were encouraged to formulate and discuss mental computation strategies. Models were used to assist students in the development of some mental computation strategies. The main approach was *arithmetic for exploring procedures* "where the purpose is to generate understanding of mathematical relationships underlying the processes involved in finding a solution" (Anghileri, 2001, p. 93). Outcomes of the project were aimed at benefiting both teachers and students. The development of the principles of an instructional program designed to encourage the development of students' strategic mental computation was to contribute to teacher knowledge about mental computation strategies and teaching methods to promote these strategies. Thus, it was envisaged that students would benefit from effective mathematics teaching. DeCorte, Verschaffel and Greer (1996) endorsed the contribution of this type of research:

> *..there is a strong need for additional theoretical and empirical work aiming at a better understanding and fine-grained analysis of the acquisition processes that this type of learning environment[students developing their own strategies] elicits in students.* (p. 537)

THE STUDY

Overview and background: The focus of this research was enhancing students' mental computation performance through the implementation a teaching experiment, in which the researcher and the classroom teacher collaboratively planned an instructional program. The teacher assumed responsibility for implementing this plan. The researcher was a participant observer who acted as a critical friend to the teacher. Discussion and reflection between the teacher and researcher resulted in responsive and intuitive interaction in the instructional program, through ongoing retrospective analysis, consistent with the methodology of Steffe and Thompson (2000). To track individual student learning, pre- and post-instruction mental computation interviews were conducted. In Table 1, the stages of the project are presented.

Table 1
Stages of Enhancing Mental Computation Project

March	Initial meeting – researcher and teacher met to discuss project. Background reading, etc. were provided to familiarise teacher with the philosophy of mental computation.
May, June	Pre-instruction interviews – informed the teacher of students' base knowledge. Planning – researcher and teacher discussed the base knowledge of the students and designed the instructional program. Additional support material provided.
July, August	Implementation of instructional program – 1 lesson per week for 6 weeks (videotaped). Development of mental computation strategies supported by models and discussion of students' strategies. Discussion between researcher and teacher after each lesson.
Sept	Post-instruction interviews conducted.
Nov	Final meeting – discuss students' growth or otherwise, debriefing.

Participants: The participants in this study were two classes of Year 3 students (58 students) and their classroom teachers. According to the current mathematics syllabus documents (Department of Education, Queensland, 1990, 1991), students in Year 2 should be able to solve addition examples involving single-digit and 2-digit numbers, without regrouping (in written form); and students in Year 3 should be able to solve addition and subtraction examples involving 2-digit numbers, with regrouping, and 3-digit numbers without regrouping (all in written form). Students are taught written algorithms to solve these examples. Mental computation of multidigit calculations does not feature. In both year levels, students are encouraged to develop number facts strategies to help them learn their number facts by recall. The students in the two classes had been introduced to the written algorithms for 2-digit addition and subtraction, with regrouping, and 3-digit addition and subtraction without regrouping. With respect to number facts strategies, the Year 3 teachers saw no evidence of the students having been taught number facts strategies in the previous year (most students counted to solve number facts), so the teachers had endeavoured to develop some number facts strategies (e.g., doubles and near doubles, through 10). The students were at varying levels of proficiency with both the written algorithms and the number facts strategies. The students had not been taught any mental computation strategies.

Pre- and post-instruction interviews: Pre- and post-instruction interviews were conducted with individual students. They consisted of one-, two-, and three-digit addition and subtraction mental computation items (10 addition and 10 subtraction), presented in picture form and accompanied by a verbal question (e.g., "What is the total cost of the two computer games?" – $46 and $28). The student solved the problem mentally (without external aids), and verbalised the answer and the solution strategy. The researcher, teachers, and research assistant administered the interviews. In this way, the teachers were able to identify some students' spontaneous strategies.

Instructional Program: Students' prior knowledge of mental computation was identified in the pre-instruction interviews. At the first meeting with the teachers, the researcher provided background information for mental computation (project summary, relevant web sites, journal articles, the draft syllabus, pre-instruction interview tasks, written explanation of a variety of mental strategies).

After the pre-instruction interviews and the documentation of the students' strategies, the teachers designed an instructional program based on students' base knowledge and with the assistance of the researcher. The teachers were provided with additional support material for developing the instructional program (additional web sites, activities to develop flexibility in numeration and the effect of operation on number – number facts strategies appeared to be well developed in most of the students, and examples illustrating the use of the hundred board, 99 board and the empty number line as models to support the development of mental computation strategies). While the teachers worked together to develop the program, it was implemented slightly differently in the two classes, because of individual teacher differences.

The instructional program was conducted over a six-week period, with students engaging in this program once per week for approximately 40 minutes (although not all students were involved in each lesson – students were placed in ability groups for some of these lessons). The teachers decided to use two models to support the students' development of mental computation – the hundred board (because it supported a *build to 10* strategy for interim calculations for addition) and the empty number line (ENL). The teacher in one of the classes stated that most students were familiar with the ENL, but not as familiar with the hundred board – finding numbers quickly and efficiently, jumping forwards and backwards in ones and tens. In the other class, the students were more familiar with the hundred board, so more work was needed with the ENL. In this class, the teacher provided the students with "almost empty" number lines. They were lines, with the tens marked on them. The students were directed to point to numbers anywhere along the line, using the tens as guides. Very quickly, these marks were removed and the students used the ENL. To familiarise the students with both models, the first lesson for each model was spent in locating numbers (c.f., Menne, 2001).

The two strategies which leant themselves to development with the ENL and the hundred board were *N10* (e.g., 46+28: 46+20=66, 66+8=74) and *compensating* (e.g., 46+28: 46+30=76, 76-2=74). While it was expected that these two strategies would be developed by the students, there was also an expectation that students would employ any mental strategy they thought useful. The students were constantly asked to share their strategies with the class. In one class, the students were also encouraged to evaluate the strategies, and identify likenesses and differences in the strategies, used by individual students.

Computational examples were presented in both real world contexts and in horizontal number sentences. Students solved examples using their personal hundred board or empty number line. Because of the short duration of the project, students relied on models to support computation over the period of the program; that is, they did not engage in computation without the aid of external models. In other words, the students were engaged in *structured* calculating, only (Treffers, 1998). Ongoing meetings between the researcher and teacher also informed the development of these activities. As a result of ongoing retrospective analysis, teachers were provided with additional support; in particular, a sequence of presentation of number combinations was suggested to the teachers.

In one class, the teacher recorded the jumps (that the students suggested) in number sentences on the blackboard, and in the other class, the teacher recorded the jumps on the ENL on the blackboard. The researcher suggested to the teachers that a combination of both methods of recording would be beneficial. However, because of the short duration of the program, this was not accomplished.

Data collection and analysis: Data comprised of videotaped observations of students engaged in addition and subtraction mental computation tasks (pre- and post-instruction interviews), videotaped classroom observations of lessons, teachers' unit plans, field notes, and ongoing reflection and discussions between the researcher and the teachers. The lessons were analysed for sequencing of examples presented by the teachers, and the strategies the students discussed and documented. The pre- and post-instruction interviews were analysed for variety of strategies, level of strategies (e.g., low level *1010* strategies – e.g., 46+28: 6+8=14, 4+2=6, 60+14=74; or higher level *N10* or *compensating* strategies), efficiency of strategies, and accuracy.

FINDINGS AND DISCUSSION

As the teachers were treated as co researchers, they were encouraged to provide feedback on the project, student learning, and suggestions for future instructional programs.

Teacher learning: The teachers believed that they had benefited from the professional development in several ways. They felt they had become "better" teachers, offering more interesting and engaging lessons to the students, as a result of having to plan for the unit and lessons, and being provided with appropriate background readings, resources, and feedback from the researcher. While the teachers were accustomed to planning together, and discussing progress of their mathematics programs, the additional emphasis on reflection and discussion among the teachers and the researcher promoted further reflection on their practice.

The teachers also felt that a positive outcome of having to write lesson plans was that they became more aware of the sequencing required to develop mental computation strategies; however, they also felt that they required more time to develop the sequencing more effectively. Additional assistance from the researcher was required to develop appropriate sequencing.

The "sequence" for introducing number combinations for each of the models used (100 board and ENL), formulated by the researcher reflected what students could complete easily (e.g., counting forward and backward in tens from multiples of tens – 60 jump on in tens), through more difficult jumps (e.g., jumping forwards and backwards in tens and ones – 68 jump back 25), to examples bridging the tens (e.g., 75 jump back 28). The suggested sequence for both the 100 board and the ENL was:

1. jumping in tens forwards and backwards from multiples of ten (e.g., start with 40 – jump forwards or backwards in tens);
2. jumping in tens forwards and backwards (e.g., start with 43 – jump forwards or backwards in tens);
3. relate the previous to addition and subtraction (e.g., start with 43 – add 10, add 20, add 30; take away 10, take away 20, etc);
4. further addition and subtraction, without bridging tens (e.g., 43±20);
5. further addition and subtraction, bridging tens (e.g.,47±28; 47±19 as a special case). For an example of the type 47-28, only the ENL might be used.

Further assistance for making links in the lesson was also offered to one of the teachers, as it was perceived that she was not focusing on the mathematical intent of the task. The students were instructed to "jump" on the ENL from one number to another (e.g., from 28 to 63). While they were asked to share and verbalise the series of jumps that they made, their attention was never drawn to the total of these jumps. As a result, some students were finding the most "creative" ways of getting from one number to another, rather than focusing on the total distance between the two numbers, and how best to calculate this. Therefore, the teacher was encouraged to ask the students to start on one number, jump to another number and then explain how they might get there, in other words, how far is it altogether? This idea was shared with the other teacher.

The teachers saw the value of encouraging the students to develop their own strategies and share these strategies with the class. "The children need to work it out for themselves… They need to talk more." However, while "talk more" became a feature in both classrooms, one teacher merely focused on asking the students to explain their strategies, while the other teacher also focussed on the similarities and differences of the strategies. This led to the students in the second class making judgments about the suitability of the strategies. In feedback to the first teacher, the researcher suggested some appropriate questions to be incorporated in the lessons to encourage more reflection by the students.

With an emphasis on process (rather than product), the students appeared to develop more confidence in their ability to "do" mathematics, and more competence in using a variety of appropriate strategies. The students employed strategies appropriate to the numbers involved. They also appeared to enjoy mathematics more than before. This was evident in their enthusiasm to solve problems and share strategies. It was suggested that for a future instructional program, an affective pre- and post-survey be administered to the students to gauge any increase in enjoyment. The teachers suggested that the students were becoming "more mathematically inclined" and were engaging more in mathematics. They also felt that the students started to see a purpose in more of their mathematics; for instance, the students could see a purpose in learning number facts strategies, place value concepts, counting in tens, etc. Not only had the students developed a variety of mental strategies, they also recognised there are different methods of recording their calculations.

The teachers were aware that some of the students were still using pen and paper algorithms mentally. However, when the researcher suggested that all reference to the pen and paper algorithm be eliminated during any future instructional program, both teachers were hesitant as they believed the students still needed to learn the pen and paper algorithm, and that most parents would also expect this.

Apart from the support provided to the teachers in background reading and feedback during and after lessons, the teachers appreciated the opportunity to interview some of their own students (pre- and post-instruction individual interviews). As a result of interviewing the students, they became more aware of individual student's understandings and possible short-falls in their own teaching strategies. In fact, one teacher exclaimed (after interviewing a small number of her students), "What have I been doing all these years?" She was referring to the over-emphasis of teaching written algorithms, when students rarely used them, and if they did, little understanding was exhibited. Most understanding was exhibited by those students who had developed their own efficient strategies. There was also a gap between what students could do with pen and paper (possibly without understanding) and what they could do mentally. One teacher was amazed that some students experienced no difficulty completing pages of written algorithms in class, but had been unsuccessful in completing examples mentally in the interviews.

One teacher documented students' strategies on the ENL, while the other documented the students' strategies as written equations. It would appear that the teacher's documentation as equations would lead students to developing their own informal written strategies. By the teacher's documenting the strategies on the ENL, students who were a little slower than the others were better able to understand the more advanced/efficient strategies, as they sometimes experienced difficulties when trying to follow another student's verbal explanations. It would appear, then, that a combination of both types of documentation by the teacher would be beneficial.

Student learning: Most students (41) improved in accuracy and efficiency of strategy choice; that is, they employed higher level strategies (e.g., *compensating* – 46+28 is solved by: 46+30=76; 76-2). While there is an expectation that the students' accuracy would have improved on the period of the project, simply because of maturation, it cannot be assumed that students would develop efficient mental computation strategies without some form of "intervention". Although many students already used some mental strategies (evidenced in the pre-instruction interviews), they developed higher level strategies during the instructional program and applied them to more examples (evidenced in the post-instruction interviews). Four students only employed *1010* strategies (e.g., 46+23 solved by 6+3=9; 40+20=60; 69) or a mental strategy that reflected the pen and paper algorithm in the pre-instruction interviews. Yet, they employed *compensating* and *N10* (e.g., 46+28 is solved by: 46+20=66; 66+8=74) for a variety of examples in the post-instruction interviews.

While the students were engaged in *structured* calculating, many were able to move to *formal* calculating (Treffers, 1998). Because of the short duration of this project, students did not engage in mental computation without models; therefore, neither the researcher nor the teachers expected much change when the students were probed in the post-instruction interviews, as they were not presented with any models to support their mental computation in the interviews. However, there were changes noticed in both accuracy and employment of higher level mental strategies. All students who were able to employ mental strategies at the beginning of the project improved in their use of these strategies and the development of others, but it is not certain that the use of the number line and the hundred board were the sole contributors to this improvement. Rather, the validation of alternative strategies in class might have been sufficient to encourage these students to use alternative, efficient strategies. For instance, six students developed *transforming* (e.g., 246+199=245+200), but this was not taught. Others developed *mixed method* (e.g., 46+28: 40+20=60, 60+6=66, 66+8=74). Again, these strategies were not taught. Therefore, by merely being involved in the program, some students developed efficient mental strategies.

Although the students were presented with the models of the 100 chart and the ENL to support their mental computation, they were not shown specific strategies to use. As a result, students developed a variety of strategies using the models. For instance, for the example, 47+38, some students jumped in ones first and then tens; while others jumped in tens first and then ones. As can be seen in Figure 1, some students solved some subtraction examples by *adding up* (see Buys, 2001); while used subtractive jumps.

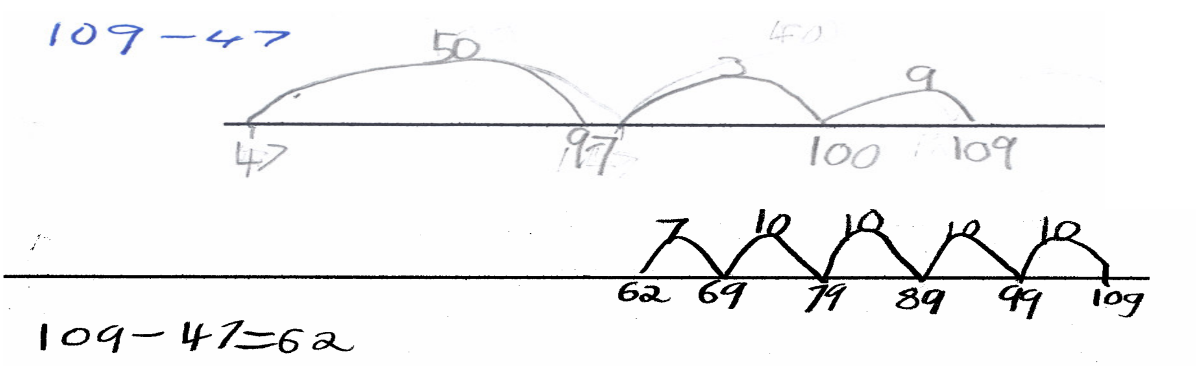

Figure 1. *Two solution strategies for 109-47*

While it was encouraging to witness most students attempting to use *compensating* strategies at least once in the second interview, it was disconcerting to identify their lack of understanding. For instance, in attempting to solve 25+19, a student stated, "25 plus 20 is ....5 plus 0 equals 5. 2 plus 2 equals 4. 45. Now take…can't remember…take 1." So, although the student understood the principle of taking 19 up to 20, she then proceeded with the pen and paper algorithm, mentally. Similarly, other students solved 246+199 by taking 199 up to 200, but then proceeded with the pen and paper algorithm mentally. They experienced great difficulties as they had to remember all the interim calculations, yet very few of the numbers were visible.

Six students improved (in accuracy), but they employed a mental image of the pen and paper procedure (teacher-taught algorithm) both in the pre-instruction interviews and in the post-instruction interviews. During the instructional program there was concern voiced by both the teacher and the researcher that many students were still tending to employ the pen and paper procedure mentally. Reasons for this could have been that all students had been taught this strategy to automation, and they were comfortable with the procedure. They might have decided that there was little point in learning a new strategy when an old one would suffice. Also, the pen and paper algorithm was constantly practised in the classroom, and might have interfered with learning new procedures. In hindsight, it would have been beneficial to avoid all mention of the pen and paper algorithm while the mental computation instructional program was being conducted.

Most errors in calculation could be attributed to lack of understanding of place value and effect of operation on number (in particular, the effect of changing the subtrahend), and slow number facts strategies (this is discussed below). While it was intended to include learning activities to enhance understanding of place value and effect of operation on number, this did not eventuate, possibly because of time constraints.

Feedback from the teachers indicated that the use of the models (100 board and empty number line) resulted in increased use of a *build to 10* stratgey for both number facts and interim calculations in mental computation; although, the 100 chart could not be easily used for subtraction where jumping across ten was required. They believed that this was a positive outcome, as many students were using more efficient calculation strategies than they had before. It is certainly true that more students used this strategy in the post-instruction interviews than in the pre-instruction interviews.

CONCLUDING COMMENTS

The teachers saw the project as a very beneficial professional development program, in which it was important to provide teacher-release time for access to the literature, websites, etc., discussion and planning, pre- and post-instruction interviews, and reflection on the progress of the project. Without this release, the teachers felt they would not have been able to spend the time that they did in the project, and they would not have achieved as much as they did. Both teachers have readily volunteered for participation in another project in 2004. They hope to gain further knowledge about students' mathematical learning.

The teachers gained insight into effective teaching of mental computation, as a result of incorporating ideas from the literature and from the researcher, and from reflecting on their own practice. Also, the researcher gained insight into what actually works in a real classroom, and what guidance teachers might require. As a result of the researcher's interaction in the classroom, a suggested "sequence" for the introduction of number combinations was formulated, aspects that need focus were identified, and practical ideas from the teachers were noted.

Because of the short duration of this project, students did not engage in mental computation without models; therefore, neither the researcher nor the teacher expected much improvement when the students were probed in the post-instruction interviews, as they were not presented with any models to support their mental computation in the interviews. However, there was improvement. While many students employed more efficient mental strategies more often, the success was not completely attributable to the use of models, as some strategies that were evident in the interviews could not be developed with these models. It is posited that the validation of alternative strategies in class might have been sufficient to encourage the students to use alternative, efficient strategies. In a study to be conducted in 2004, including Years 1, 2, and 3, teachers will be asked to encourage students to discuss their strategies and make judgments about the efficiency, etc. of these strategies, with and without models. While models/materials (e.g., 100 chart, empty number line, multibase arithmetic blocks, tens frames) will be presented to the students, it is acknowledged that not all students will (1) want/need to use models/materials, and (2) not all/no model(s) will be useful to all students. The "big picture" is the development of number sense; therefore, encouraging the self development of strategies is more important than learning how to use the models.

What is important is the development of abstract thinking on different levels that will progress and become more curtailed, otherwise, the expected advantages of an earlier start on fostering the general aspects of mental strategies might get lost in the prolonged use of 'modelling strategies', even on the empty number line.

(Beishuizen, 2001, p. 130)

There was a perception (more by the researcher than by the teacher) that practice of the pen and paper algorithm during the project interfered with the development of mental computation strategies. Therefore, in 2004, teachers will be required to avoid the teaching of the vertical algorithm for the duration of the project (c.f., McIntosh, 2002). Vertical algorithms dictate a rigid procedure, and do not lend themselves to encouraging students to manipulate numbers flexibly. In contrast, it is suggested that written procedures be permitted, as long as they are used by the students to explain their thinking/support memory. That is, the written documentation will not be the algorithm.

Finally, six weeks was a very short period in which to engage students in developing mental computation strategies, when they had not previously been encouraged to compute mentally. There was some success, but it is hoped that a longer period of immersion and additional supporting lessons (to develop numeration and effect of operation on number) will result in yet further success. Success should also be gauged by students' engaging in mental computation without the use of models (c.f., Beishuizen, 2001).

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