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Researching the language for rational explanations in mathematics teaching and learning

Philip C Clarkson

Australian Catholic University

p.clarkson@patrick.acu.edu.au

The paper will explore language associated with the value of rationalism, regarded as a crucial value in learning mathematics. Logical connectives are one set of words and phrases used for reasoning and explanation that are fundamental rational processes in mathematics. However, little or no emphasis is given in the teaching of mathematics to such language, at least in Victoria, Australia, and probably in many other parts of the world. Indeed with the increased use of manipulatives in teaching mathematics in primary classrooms, it is likely that the teaching of descriptive language has increased in such an environment, but with little proactive teaching of language needed for explanations. Hence, there is potentially a dilemma for teachers and students in not having a shared language to embrace this fundamental value of mathematics.

This paper will explore language associated with the value of rationalism, regarded as a crucial value in learning mathematics (Bishop, 1988). Rationalism was one of six values that Bishop chose to arrange in three pairs. Bishop (1988) linked rationalism to objectism, a world-view dominated by images of material objects, and suggested that at present there was an over emphasis on objectism in mathematics curricula. More of a balance is needed, he argued. If one of the fundamental aims of teaching mathematics is to help students develop a relational understanding of the subject, then they will need to have leant the value rationalism. One implication of this will be students will no longer be content to rote learn standard algorithms, be they executed with pen and paper, or mentally, or even on a calculator or computer. Such students will wish to understand why such an algorithm is a correct method, and indeed why it is regarded as 'standard'.

One set of words and phrases that are important in reasoning and explanation, fundamental processes for rationalism in mathematics, are logical connectives. Logical connectives are those words and phrases used in a language to connect ideas. It has been noted in research that an understanding of logical connectives may play an important role as an indicator of higher performance in mathematics (Clarkson, 1983; Dawe, 1983). Although such a result is to be applauded, it is not student performance that is the focus of this paper. The central argument of this paper deals with the learning of a value, that of rationalism. It is hypothesized here that for an appreciation of rationalism to be developed and finally to be embraced by students as at the heart of mathematics, then part of the way towards this is for them to be able to use appropriate language for the associated behaviour.

However there is little or no overt emphasis given to the teaching of such language in mathematics, at least in Victoria, and possibly in many other classrooms elsewhere. With the wide use of manipulative materials used in teaching mathematics in primary classrooms, and hence the increase in talk in these classrooms, an emphasis on the teaching of appropriate descriptive mathematical language has grown in the last twenty-five years. This is as it should be. However it is doubtful whether the language associated with rationalism has also received the same type of emphasis. Hence there is potentially a dilemma for teachers and students in not having a shared language to embrace this fundamental value of mathematics. Ironically the wide introduction of student manipulative material was in part a reaction to a long period of teaching mathematics with an undue emphasis on disembodied rationality, or at least the rote learning of the products of a curriculum that once emphasised rationalism.

There is speculation that inappropriate values learnt when studying mathematics may well lead to life long learning difficulties in this area (Clarkson, Bishop, FitzSimons, & Seah, 2001). It has already been shown that teachers can have difficulty in discussing mathematical values in general because of their lack of realisation that they also teach values whenever they are teaching mathematics (Clarkson & Bishop, 2000). The issue is compounded with the lack of a known and shared language with which mathematics teachers can talk about mathematical values (Clarkson, Bishop, FitzSimons, & Seah, 2000). Such findings give some weight to further investigate the language needed for the value of rationalism in a mathematical context.

It could be assumed that if a mathematical value, such as rationalism, was seen to be at the heart of school mathematics, then such a stance would be reflected in the curriculum documents that governments produced to shape what is taught in classrooms. Clearly incorporation of these ideas in curriculum documents is no guarantee that teachers will emphasise such ideas. However a cursory examination of such documents is enough to show that indeed the value of rationalism is truly embedded in them, but it is often there as an underlying assumption. What are not clearly apparent are the continuing and overt emphases on rationalism, and most other mathematical values. As well the language which students need to discuss such mathematical values is not present in those sections of the curriculum documents which teachers use to guide their day to day teaching. Such an omission from these types of documents lessens the potential of the wide spread planned teaching of this language, and hence the overt and systematic teaching of mathematical values, and in particular rationalism.

In the last decade, four sets of curriculum documents have had a great impact on the teaching of mathematics in primary schools in Victoria. Thy are *Curriculum and Standard Framework - Mathematics* (Board of Studies (BoS), 1995) and the related *Course Advice - Mathematics* (BoS, 1996); *Numeracy Benchmarks Years 3, 5 and 7* (Curriculum Corporation, 2000); and *Curriculum and Standard Framework II - Mathematics* (BoS, 2000). The first, and in some ways the fundamental one of these four documents for Victoria, and the second document in this list, will be examined in the light of the assertion that curriculum documents produced to guide teachers in their teaching of mathematics lack an emphasis on the language that would be needed to teach overtly the mathematical value of rationalism. However before embarking on that task, a discussion more clearly indicating the type of language under discussion is needed.

Logical Connectives

As noted above logical connectives have been seen as important contributors to students' mathematical performances. However what is argued here is that they are important to mathematics in another way as well. They are part of the language that students need to become familiar, and indeed competent with, if they are going to be able to learn clearly and confidently the value of rationalism as it emerges in the context of mathematics.

Logical connectives are those words and short phrases that serve as linkages between ideas in discourse. Hence the word 'and' is obviously one since it is so often used to join ideas. In both science and mathematics education, it has been found that to restrict the notion of connectives to just 'logical operators' in the language, that is, and, not, or, if ... then, etc, is too restrictive. Gardner (1977) in a fundamental work produced a listing of logical connectives that were seen to be important in science education. The listing given in Table 1 was compiled from school science textbooks. It should be noted that these words could be used as logical connectives, but may not always serve that function. Nor is it claimed that this listing is an exhaustive list, although it is clearly fairly comprehensive. It is also noted that this list was compiled in the late 1970s. Hence the frequency of some words may well have changed over time. For example, 'thus' would these days probably not be used as frequently as then. 'Viz' is another connective not often used in more recently published texts. May be a new addition to this listing might be 'implies'. However this listing still seems to be a very useful guide for the language found commonly in texts today. Interestingly the occurrence of these words used as connectives in senior texts were 8-18 per thousand words, rising to 14-45 for senior texts showing that their use can be expected to increase as the ideas become more demanding.

A decade after Gardner's work was published, the importance of logical connectives was acknowledged in the science curriculum documents of Victoria, and teachers were caution to take care of this aspect of English when teaching the prose to be used in Scientific reports. Hence, consequently, therefore and however were examples of logical connectives given (Ministry of Education, 1987). Possibly in mathematics which uses a more concentrated form of language than school science, the rate of use of logical connectives is even higher.

Dealing with connectives like those listed in Table 1 has also been used as a starting point in mathematics education research, although not a very extensive body of work (one exception is Navarra, 1998). Clearly words such as these will be important when trying to encourage students to use the value of rationalism in their study of mathematics. It is hard to imagine a mathematical explanation being formulated by a student that will not draw heavily on this listing of words. However even if a student is familiar with some of these words, the quality of their explanation may be lacking if the student has to be hesitant with the language: If they have to 'fumble in the dark', searching for appropriate expressions that will connect their ideas together. They may be able to use the 'correct' technical language for their age to describe the process they were involved in say, when constructing a polygon. One could image technical words such as parallel lines, right angle, angles, vertices and vertex, sides being used in such a context. And then when it comes to naming their construction as a square or rectangle or pentagon or hexagon, and whether it is indeed a regular one or irregular one, and so on, the student will at least have been overtly introduced to such technical language in school. However some data suggests that a significant number of students who enter university degrees on their way to becoming primary school teachers have difficulty with such language (Clarkson, 1998). It seems however such students are just expected to pick up the language of explanation, one aspect of the value of rationalism. Hence for a student to say with authority "I could not draw a line here *because* that would mean I would go too far. *Hence (So)* I tried this here. *But if* ..." is probably too much to ask of many school students. Actually when we hear a student talking like this, often we immediately start paying attention. We say to ourselves "Ah, now here is a student that knows what they are talking about" because we sense that they have learnt the language for explanation.

TABLE 1.

List of most frequent logical connectives found by Gardner (1977)
(Words in italics: had a frequency of 20 or more words per 100,000 words
Words in bold: had a frequency of 50 or more words per 100,000 words)

above all	except	on the other hand
according to	finally	once
actually	first(ly)	*only*
after	for	only if
again	**for example**	**or**
also	for instance	other than
alternatively	for this/the same reason	otherwise
although	fortunately	particularly
and	frequently	perhaps
and also	*from*	plus
and so	from his/the/this point of view	possibly
apart from	further	probably
apparently	furthermore	provided that
as	generally	rather than
as a result	*hence*	really
as follows	*here*	recently
as if	**however**	respectively
as long as	i.e.	say
as much as	**if**	second(ly)
as shown by	if/then	similar to/similarly
as soon as	if so	simply
as though	in addition	simultaneously
as to	in case	**since**
as we have seen	in contrast	*so*
as well as	in fact	so also
as you know	in general	so as to
at first	in many/most instances	so far
at least	in order to	so long as
at the same time	in other words	*so that*
at this/that stage	in particular	so to speak
basically	in practice	somehow
because	in terms of	sometimes
before	in that	still
besides	in these examples	*such*
both	in this case	**such as**
but	in turn	such that
but also	in (…) way(s)	suppose (that)
but even	including	that is
but if	indeed	the fact that
by	instead (of)	*then*

TABLE 1.
Continued

by far	*it can be seen that*	thereby
by means of	it follows that	*therefore*
by way of	it seems clear that	third(ly)
certainly	it would appear that/seem that	this means/suggests that
clearly	just as	though
commonly	known as	**thus**
compared with	moreover	*to*
consequently	*(much) like/very much like*	together with
considering that	namely	too
conversely	neither	unfortunately
depending u0pon	neither…nore	unless
despite	nevertheless	unlike
due to	next	until
e.g.	nor	usually
either..or	not only…but	viz
especially	not only…but also	**when**
essentially	notice that	whenever
even	*now*	*where*
even if	nowadays	whereas
even though	obviously	whereby
even when	occasionally	whether
eventually	of course	*while*
evidently	often	whilst
exactly as	on the basis of	*yet*

And of course teachers use this type of language in their explanations with one suspects very little awareness of the heavy dependence on logical connectives and how these may make the explanation difficult to understand unless students have come to an awareness of their importance. For example, the following is part of an explanation given by a teacher to her students when they were dealing with a time and distance problem. The context of the problem was of a number of people on bicycles:

> Either 1 kilometer, or as you just said: 10 kilometers. And now we've got 2 different time designations as well, and if you could now work out how many kilometers Peter cycles in, for example, 10 minutes and then still work out how many kilometers Jens cycles in 10 minutes. Would that work too? (Adele, 1998, p.145)

An examination of these three sentences shows the high frequency of connectives naturally used by the teacher.

The stance taken so far in this paper is that because of the fundamental place of connectives within the language used in doing mathematics, because, in turn, the notion of rationalism is at the heart of mathematics, then connectives should be overtly taught within mathematics. An alternative to this stance is to not ignore the outcomes of this part of language, but try to write and speak in such a way that the frequency of connectives is at a minimum. This can be done for example by using short sentences and there very sequential ordering implies connection of the ideas discussed. However this approach runs the risk of using an inferior type of language that will not do justice to the content being taught. It also begs the question of when the language of mathematics, rich in connectives, needs to be taught, because come what may, this type of language is needed to appreciate mathematics in all its strength and beauty. I take the position that it is best to recognise the centrality of the value of rationalism to mathematics from an early level of schooling. And hence, the necessity to teach overtly this type of language, as appropriate, at the different levels of schooling.

Gardner's compilation then, seems to be a useful list for examining the curriculum documents identified at the end of the last section, and the extent to which the curriculum developers have an emphasis on teaching some of the language needed for an appreciation and use of the value rationalism in a mathematical context.

Curriculum and Standards Framework

For many years through the twentieth century, the curriculum for schools in Victoria had been relatively unchanged, and perhaps many thought unchangeable. However in 1972 the government made a fundamental change. Instead of promulgating a syllabus that defined what schools must teach, they issued for the first time ‘a suggested course of study’ which was to help teachers and schools decide what should be taught (Education Department of Victoria, 1972). In fact this guideline and subsequent documents have been very influential, but still give teachers and schools only a framework within which to work (see Education Department of Victoria, 1985 & 1986; Ministry of Education, Victoria, 1988).

In the mid 1990s there was again a fundamental change to the curriculum documentation. The emphasis changed in emphasis and structure from the how to teach so that students would learn this or that, to a structure that emphasised what would be learnt using the business developed notions of benchmarks, although the term used in the document was 'learning outcomes' (BoS, 1995). The subsequent curriculum documents for Mathematics in Victoria have followed essentially this same emphasis.

In introducing the learning area of mathematics the Curriculum and Standards Framework (CSF) was rather comprehensive:

> Mathematics is an area of knowledge older than recorded history and has, through the ages, developed into a sophisticated, complex body of knowledge. It has applications in all human activities, crossing cultural and linguistic boundaries to provide a universal way of solving problems in areas such as science, engineering, technology, art, crafts and many everyday activities. (BoS, 1995, p.14)

When it came to what were the goals for mathematics in school, there were the expected ones such as:

- acquire mathematical skills and knowledge so that they can deal confidently and competently with daily life
- develop knowledge and skills in using mathematics for employment, further study and interest
- recognise the fundamental importance of mathematics to the functioning of society

- understand and appreciate the nature of mathematical thinking, the processes by which mathematics changes and its cultural role
- understand the dynamic role of mathematics in social and technological change.
  but also included the following:
- be able to interpret and communicate quantitative and logical ideas accurately (BoS, 1995, p.14)

To fulfilled these goals, the document suggested various categories of "mathematical activities" should be carried out including:

- ability to communicate using clear and precise mathematical language
- ability to conduct investigations using mathematics
- logical reasoning and a conception of the nature of proof

and

> Assessment will cover each type of activity at every level for all students. (BoS, 1995, p.14)

Interestingly, as well as grouping the various outcomes in the more traditional areas of mathematics such as number and algebra, there was also a category of "Mathematical tools and procedures" which included "Communicating mathematics" as well as a category of outcomes which dealt with "Mathematical reasoning" (BoS, 1995, p.15). This in itself shows that language in some sense was to be taken seriously by this curriculum document.

Further in the introductory pages of the curriculum document, various "approaches to teaching and learning in mathematics" (BoS, 1995, pp.17-18, as are all the subsequent quotes in this section), were outlined. These were presumably approaches of which the developers approved. The authors were clear that they felt "While there is no one best way to teach mathematics, many of the most successful approaches share the following characteristics" and proceeded to list what they felt were those that would bring success and insight to students. Teachers were encouraged to allow "discussion and reflection" since this would "strengthen the acquisition of shared understandings." Teachers were also advised that "As students gain access to the power that mathematical thinking offers, they are initiated into a formal system with ingredients that are, at times, counter-intuitive, at least initially. Precise attention to meaning becomes important." With a "variety of modes of classroom activity" encouraged, teachers could expect "successful students will ... sensibly evaluate their work. ... Group discussion and reflective writing are valuable for concept development."

The writers of the curriculum clearly expected that student outcomes would be in part deal with the rational judgement exercised by students. This stance has already been summarised in the above quotes. The writers also expected that appropriate language would be used. Again the above quotes testify to that. The penultimate sub heading for this section "Sensible use of mathematics" begins with the following paragraph:

> Some aspects of mathematical activity do not develop in the same way as the learning outcomes, because they are present at all levels. For example, at every level, students need to judge the adequacy of their answers in the context of the problem being considered. They should choose techniques and tools sensibly. They will pose questions, make and test conjectures, and look for reasons to convince themselves why their ideas work. Although the complexity of the

> problems will vary, students at all levels will engage in a similar broad process of inquiry and evaluation. Consequently, despite the importance of these aspects of doing mathematics and the need for teachers to monitor them, it is not possible to represent them adequately in outcome statements.

In this paragraph the writers' expectations are very clear. Rational judgement by students who are taught well will be an outcome, no matter at what level the students are at. But the use of the word 'consequently' to begin the last sentence is intriguing. It links the notion contained in the last sentence with that of the first sentence. I take these two sentences together to mean that the rational processes stipulated in the four intervening sentences are important and advises that teachers should monitor them. But the two sentences say that these processes fall outside the scope of the curriculum guide because the fundamental structure of the guide based around 'learning outcomes' or 'outcome statements' cannot deal with them appropriately. Some credit must be given to the curriculum developers for them not only to understand this, but also to admit it openly. Interestingly it did not cause them to question their choice of structure for guidelines if such a crucial area of Mathematics had to be set to one side. Or in truth they were not able to do so, the structure for the guidelines having already been stipulated by higher Departmental officials to be the frame within they needed to work. However they could have presumably explored other techniques as additions to the fundamental structure as a possible supplement. But in fact with this omission and lack of an alternative, the so-called 'importance' to use the developers' word, which they placed on this area, may be questioned.

Perhaps the above points to another issue as well. It may be that the developers were not aware of some preliminary ideas that learners need to come to some understanding of before, or at least at the same time as, the complex rational processes come into play. One such issue is coming to have some facility with an adequate language for these processes. Such a facility presumably needs to be part of the teaching process, and could be contained within the structure of 'learning outcomes'. The next sub section will show how inadequate this document was in providing guidance to the teaching of such a language facility.

Language in Some of the Different Strands of the CSF

The format of the CSF was based around five content, and one other strand. The content division into areas such as number, space, measurement, chance and data, and algebra is not unusual. The first four of these strands were developed from the first year that children entered school through to year 10. The curriculum for the final two years of schooling (11 and 12) was contained in a separate but articulated document. The sixth strand termed 'Mathematical Tools and Procedures' was developed to emphasise particular areas that were to be integrated across all content strands. The content of the strands were 'learning outcomes' which were written to give guidance to teachers about what and to some extent how they should be teaching at different levels. The six levels into which the learning outcomes were clustered did not coincide with the year levels of schooling. This was done to de-emphasise a strict one to one set of outcomes for each year level. Level 1 was to coincide with the preparatory year of schooling. Level 2 with years 1/2, and so on till level 6 was reached which gave guidance for years 9/10. An additional level 7 gave guidance for the better students in year 10. The Algebra Strand was an exception being developed for levels 5 and 6 only. Each strand was also sub divided into various sub strands, with the number varying depending on the strand.

A full analysis of the six strands for their language content is not attempted here. An analysis of sections of two content strands, Algebra, and Chance and Data, and the 'Mathematical Tools and Procedures Strand' will suffice.

Table 2 shows the Algebra Stand with some explicit references to language highlighted. There are no other references to natural or verbal language in this strand. The types of learning outcomes for algebra in Table 2 are the expected ones to be found expressed in many curriculum guides, although they may be expressed in other ways. The language references also are probably of no surprise. What these highlights do show is there is some emphasis on using language in the learning of algebra. It would be expected for these outcomes to be achieved that the students would need to use a range of logical connectives such as if, but, and, therefore, etc.

Table 2.
Level 5 from the Algebra Strand with Some Language Emphases Highlighted

Substrand	Expressing generality	Equations and inequalities	Function
Level			
5	Introducing algebraic notation including the use of pronumerals to represent variables. Using and devising rules, both verbal and symbolic, to generate and specify sequences, to produce tables of values and to specify relationships between variables. Simplifying and demonstrating equivalence between simple algebraic expressions.	Using algebraic notation to establish and solve equations in one unknown. Finding numbers and number pairs which satisfy a single constraint stated in natural language.	Using the coordinate system to represent data. Plotting, sketching and interpreting graphs of linear functions. Interpreting and sketching informal graphs of the relationship between two variables in everyday situations.

The Chance and Data Strand has probably the most overt emphasis on language of all the strands in the CSF. Various language references have been highlighted in Table 3 for this strand. Indeed with two of the sub strands called 'Posing questions and collecting data' and 'Interpreting data' there was already a clear emphasis on language in this strand. Within such a strand, the posing of questions will be heavily reliant on logical connectives. The 'description' of the data also implies in such a strand that the language will be articulating relationships between elements, ideas, variables, and the like. Such language must use logical connectives.

In Table 4 there are sections of the 'Mathematical Tools and Procedures Strand'. As would be expected there is a heavy language emphasis. Indeed every level of one of the sub strands has been highlighted because of the clear language emphasis in each. So at levels 1 and 2 the outcomes indicate that students should be able to use "natural language" in dealing with mathematical ideas. By level 4 (years 5/6) students are to be expected to be able to use "mathematical terms, symbols and notations to describe mathematical objects, situations and relationships". This progressed so that by level 6 (years 9/10) it would be expected that students could use "conventional mathematical language" so that "clear and logical accounts of mathematical activity" could be communicated. There is no doubt that such outcomes as these imply the use of logical connectives. However there is doubt as to whether the outcome at level 4 of "mathematical terms, symbols and notations to describe mathematical objects, situations and relationships" will include such logical language. Nor the outcome of level 3 which has an emphasis on the use of mathematical terms to describe and compare.

Table 3.
Sections from the Chance and Data Strand with Some Language Emphases Highlighted

Substrand	Chance	Posing questions and collecting data	Summarising and presenting data	Interpreting data
Level				
1	Recognising elements of chance in familiar situations.	Posing questions about collected objects and information.	Classifying and sequencing. Representing data concretely and pictorially.	Describing data orally.
2	Classifying events as certain, possible or impossible. Comparing possible events.	Framing questions. Collecting data in the form of objects or information.	Making decisions about classifying and sequencing data. Representing data visually.	Describing and interpreting visual representations.
3	Determining outcomes from simple experiments. Making random selections. Ordering more or less likely situations.	Collecting data to answer questions posed.	Organising data to answer posed questions. Making tallies and obtaining frequencies. Displaying discrete data including many-to-one correspondence. Representing measurement data.	Interpreting information in tables and graphs.
4	Ordering the likelihood of everyday situations involving chance. Using a numerical scale for chance events. Using the common (everyday) language of chance.	Collecting and recording data systematically.	Presenting graphical displays using simple scales on axes. Representing and summarising data. Obtaining simple summary statistics.	Interpreting information presented in tables, graphs and diagrams. Interpreting simple summary statistics.

Table 3.
Continued

Substrand	Chance	Posing questions and collecting data	Summarising and presenting data	Interpreting data
Level				
5	Determining equally likely outcomes. Assigning probabilities in one-step experiments. Using repeated trials and simulation.	Collecting a variety of data types.	Organising collected data. Plotting univariate data. Determining measures of location and spread.	Interpreting univariate plots and simple measures of location and spread. Reporting on data collection and results.
6	Estimating probabilities and proportions. Determining probability of mutually exclusive, complementary and compound events. Assigning and using odds and subjective probabilities.	Planning experiments, simulations and surveys.	Organising data systematically. Presenting grouped univariate data. Summarising location and spread. Plotting bivariate data.	Interpreting collected and published data. Making comparisons between two data sets. Interpreting bivariate relationships.
7	Using probability models and simulation. Determining probability of dependent and independent events.	Further planning of experiments, simulations and surveys.	Organising data systematically. Displaying and summarising data to show location, variability and association.	Interpreting collected and published data to construct and evaluate arguments.

Interestingly none of the outcomes are explicit in this matter as might have been expected. For that one needs to turn to the accompanying document published for teachers to flesh out and interpret the learning outcomes. I leave to one side why two documents were needed.

Course Advice - Mathematics

As noted above, the *Course Advice* was published as an additional guide and help for teachers, interpreting the CSF for them. Essentially the document consisted of a series of detailed unit outlines from which teachers could develop individual lessons for their own classrooms. The unit outlines, although focussed on one strand at a particular level, clearly indicated overlap with other strands, including in particular the Mathematical Tools and Procedures Strand. In fact there were no units developed for this non content strand since the

Table 4.
Sections from the Mathematical Tools and Procedures Strand with Some Language Emphases Highlighted

Substrand Level	Communicating mathematics	Strategies for mathematical investigation
1	Using natural language to convey mathematical ideas appropriate to this level.	Using teacher-generated situations and stories to pose and answer questions which require mathematical processes appropriate to this level, such as classifying, matching, ordering, counting, moving or placing.
2	Using natural language and symbols appropriate to this level to convey mathematical ideas in the space, number, measurement and chance and data strands at this level.	Considering mathematical questions arising in everyday situations and stories, including questions that are posed by the students.
3	Integrating terms and notations from space, number, measurement and chance and data into descriptions and comparisons of mathematical situations.	Using problem-solving strategies to obtain answers to non-routine mathematical questions arising from both mathematical and everyday situations. Investigating familiar mathematical and everyday situations to generate further mathematical questions.
4	Using mathematical terms, symbols and notations to describe mathematical objects, situations and relationships.	Solving problems using known mathematical concepts and the application of a developing repertoire of problem-solving strategies. Identifying further mathematical questions arising from known mathematical concepts and investigating everyday situations in order to formulate problems.
5	Using mathematical terms, symbols and notation to record and communicate the results of mathematical activity.	Conducting investigations and solving problems in both mathematical and everyday situations; developing further problem-solving strategies.
6	Using conventional mathematical language and symbolic expression to communicate clear and logical accounts of mathematical activity.	Conducting investigations and solving problems involving the identification of related investigations and sub problems and relating the results obtained for sub problems. Generalising from results of investigations.
7	Using correct mathematical language and notation in solving problems and presenting mathematical arguments using conventional forms and symbols.	Conducting investigations and solving problems which involve generalising from one situation to another and changing the initial constraints of a situation in order to investigate further.

intent of the CSF was always for the learning outcomes of this strand to be fulfilled when teaching of one or other of the content strands was in progress.

As has been shown for the CSF, the value of rationalism was also embedded in the *Course Advice*, although again often implicitly rather than in overt declarations. Interestingly there was some overt attention to language issues in the *Course Advice*. Under the early heading of "Variety of modes of classroom activity" it was noted that:

> Relevant problems (based in our world or in fantasy) and intriguing situations will motivate mathematical thinking. ... Group discussion and reflective writing are valuable for concept development.

What was marked out early in the advice to teachers and indeed quite explicitly was that the learning of mathematics relies to a large degree on language, in both verbal and written modes, and hence this consideration needs to be clearly focussed on in the teaching of mathematics. This message was reinforced with a later sub heading of "Emphasising the role of language" where teachers were reminded that:

> Language is vital in the learning process. Students need to discuss and share experiences and ideas and to describe, explain and record mathematics in their own language. Reflecting on learning and recording mathematical ideas in written language can clarify thinking, demonstrate understanding and prompt new thoughts. In time the language being used will become more mathematical as students assimilate the terms being modelled.

Here the message is that students need to reflect at length on what they are learning together. Hence attention to them learning appropriate verbal language needs to be catered for by teachers explicitly teaching for this outcome. Further teachers are encouraged to have students start journals and use written modes of language as well. This point is reinforced in a further sub section headed "Mathematics journal entries" when teachers are advised that:

> Regular writing in a journal can encourage student reflection on work occurring in class, and also provides a useful form of summary of ideas or key points. ... As students' writing matures and moves from simply recording 'what we did in class today' towards entries like 'this is why I have difficulty with these types of problems', or 'what if I apply the method I used to solve last week's problem to this one?' ... (p.24)

In summary there is a clear an unequivocal theme running throughout this introduction to the *Course Advice* that students need to use language to communicate with themselves as individual learners, with their peers, as well as with the teacher as they reflect on their learning. The heart of this learning is not the regurgitation of facts heard in class. But this learning is about understanding the linkage between ideas, seeing new relationships between facts already known, and exploring new possible connections perhaps with facts or notions not yet fully understood or even not yet discovered or uncovered. This whole enterprise rests on the assumption that the enterprise is a rational exercise. It is underpinned by the value of rationalism. It can only be known fully if a facility with language that can express the connection between ideas is acquired. One would therefore expect to see that logical connectives were embedded in the language that was to be overtly taught.

To explore in more detail the emphasis that the *Course Advice* did have, the units were examined for the language they overtly advised teachers to teach. Each unit had a 'suggested vocabulary list' in the introduction section to the unit. Teachers were advised to give due consideration to this list in their preparation of lessons. Such lists are important. Clearly there is an expectation from the curriculum developers that teachers may ensure that students know the meaning and spelling of these important words. But there is also a clear message to teachers that they should use these sets of terms in their ongoing classroom discourse so this vocabulary is embedded into that of the students.

This exploration is restricted to only those strands of the CSF that have been reviewed in the previous sub section, Algebra at level 5, and Chance and Data levels 1 though 5. As noted above, there were no units written for the Mathematical Tools and Procedures Strand.

Algebra Strand, Level 5

In the introduction to the course advise for the strand of Algebra at level 5, the first time algebra appears in the curriculum document (at years 7/8), it is noted that:

> [Algebra] is a new language for students, a language that is used to describe generalised number relationships. For many students the idea of a generalised number relationship is not yet developed, although it is begun in Level 4 through pattern work. The steps involved in this generalising process include: creating patterns (numerical and possibly also geometrical), extending and describing the patterns, generalising in words and, eventually, as a kind of 'shorthand', using symbols to represent the generalisation. In the process, two important concepts are developed: variables and formulas (or rules).

There is nothing revolutionary in this introduction. It is the type of note that could be expected in most curriculum documents. The point of interest in this discussion is the study is to be understood as "a language that ... describes ... relationships". Here one would expect to find language that connects ideas, notions, etc if relationship is at the heart of this study. One would think then there would be a high proportion of logical connectives. Of course the language will also become specialised with equations and inequations and then to variable, formula, function and so on to the host of specialist terms associated with algebra. But these need to be discussed using language that the students are familiar with and such language drawn in the first place from ordinary English.

Six units were developed for the Course Advice at this level to give guidance to teachers. An examination of the suggested vocabulary lists for these six units showed that there were 160 terms listed, of which 112 were distinct terms. Hence there was some re-emphasis of what were considered key terms. The 112 terms were compared to the list of logical connectives in Table 1. There was one match, the word 'even'. Although this term was probably used to describe a quality of number, it is still included here since at times it would be used in its role as a logical connective leading from one idea to the next. The vocabulary lists were examined again to see whether there were other terms, perhaps more decidedly mathematical in nature, that did not appear in Table 1 but nevertheless had the quality of a logical connective. No other extra terms were found that met this criterion.

The paucity of logical connectives in the vocabulary lists for these six units is a surprise given the stated description of how algebra is understood.

Chance and Data, Levels 1 to 5

As noted in the previous sub section dealing with the CSF, this strand had the most references to language of the four content strands. There were many learning outcomes which assumed students would be asking questions and discussing their ideas. Table 5 is a summary of the analysis of the 19 units published in the *Course Advice* for this strand. The same procedures were used as for the Algebra strand.

Table 5.
Analysis of the Units from the Course Advice Written for the Chance and Data Strand

Level	**No. of units**	**No. of terms**	**No. of distinct terms**	**Overlap with listing of L.C. in Table 1**	**New L.C.**
1	2	43	43	compare with, like, possibly, probably, sometimes	maybe (?), probably not
2	2	50	48	compare with, like, possibly, probably, sometimes	maybe (?), probably not
3	3	61	55	compare with, like, possibly, probably, sometimes	maybe (?), probably not
4	4	74	63	compare with, like, possibly, probably	
5	8	128	89	and, or, possibly	
Total	19	356	190*		

** This figure is not the sum of the column. The overlap between levels has been accounted for.*

Again it is strikingly clear that there were few logical connectives included in the various lists of suggested vocabulary. This is for a strand it will be remembered when students are expected to conjecture about possibilities given certain conditions. If there is any time that the use of logical connectives are clearly heard in classrooms it is when ideas of chance and data are being discussed, argued about and made sense of. This is not to say that what is contained in the vocabulary lists are not pertinent to the learning of these ideas. Important technical terms that would be expected to be seen there are included such as; average, bar graph, biased, generalisation, intuition, and so on. It is also good to note that less mathematically technical words are there too such as; being lucky, a sure thing, fifty fifty, might happen, no way, odds on, and so on. However it might have been thought that logical connectives may have been more prominent in these particular vocabulary lists.

Summary

Most recent learning theories for mathematics have at their centre rationalism. For example, whatever version of constructivism has been embraced, a crucial feature is the student having to make sense of perturbations in their thinking either individually as radical constructivism would have it, or in conjunction with others if the social constructivism path is taken (Cobb, 1988; von Glassersfeld, 1995; Yackel, Cobb, Wood, Wheatley & Merkel, 1990). Criticisms of constructivism do not normally question this tenet. Enactivism, for example, in all the shortcomings it sees in constructivism, does not question the notion of students having to make sense in a rational manner of mathematical propositions. Rather it suggests that other ways of knowing are important as well, and are not replacements for rationalism.

But we have not exploited to the full the notions of rationalism in the way we have taught in schools. For many years it has been clear that in schooling we are often content in teaching low level cognition. Clearly there has been an ongoing theme on the need to have teachers ask higher order questions in their teaching, perhaps starting with Bloom's cognitive taxonomy and the recognition that the higher order thinking skills are only stimulated when teachers teach for this. But this has not been all that successful. It is conjectured here that in order to have a realistic teaching pattern that drives at the higher order thinking skills, then students need to be exposed to and encouraged to gain facility with the relevant language. In part this includes a command of logical connectives. This will not happen until curriculum documents are broaden to take this type of language seriously, and in turn make the values that they want taught, including rationalism, more explicit.

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