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DEVELOPING AN INSTRUMENT TO ASSESS THE NUMBER SENSE OF YOUNG CHILDREN

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Abstract

This paper reports on an initial Australian trial of one module, relating to Counting, of an instrument developed in collaboration with a team at the Universiti Sains Malaysia (USM). The instrument consists of four modules assessing various aspects of number sense and applicable to children across grades one to three. The results of the trial revealed some interesting insights into the number sense of the children involved as well as raising a number of possibilities regarding the further enhancement and potential usefulness of the instrument. There appears to be merit in developing English versions applicable in the Australian context, of the remaining three modules.

Background

Many studies have shown that students' experiences related to the learning of number concepts at primary school level are of crucial importance in developing the beliefs and values that they associate with mathematics. If these experiences are meaningful they are likely to lead to positive attitudes, values and beliefs about number concepts (Van de Walle, 2004). In addition children who learn mathematics with understanding tend to be more confident, autonomous and flexible in their learning and use of mathematics, and more likely to persevere in the face of difficulties (National Council of Teachers of Mathematics, NCTM, 2000). Conversely, experiences that are not mathematically meaningful are likely to lead students to believe that mathematics learning consists of memorising activities devoid of meaning and they are often unable to apply their knowledge in new situations.

Mathematics educators have for some time been concerned that many students do indeed demonstrate little understanding of numerical situations in which they have to solve number problems. Many students are good rule followers but unfortunately do not always understand the procedures they learn (Hiebert, 1986; McIntosh, Reys, Reys, Bana & Farrell, 1997). As long ago as 1967 educators were calling for both a widening of the focus of the primary school curriculum beyond computation, and the integration of computation and other mathematical activity in recognition that computation was learned in the same way as other aspects of the discipline (Wheeler, 1967). However the acquisition of computational skill remains at the centre of the mathematics curriculum in many Australian primary schools. The traditional goals of primary school mathematics have been, and often still are, the development of knowledge of basic addition and multiplication facts and the skills of pencil-and-paper addition, subtraction, multiplication and division. These skills are usually developed as a series of rote-learned rules for performing each operation, leading often to misconceptions, miss-remembered procedures, and an inability to use and adapt these procedures flexibly to meet the requirements of everyday real-life situations, resulting, unsurprisingly, in a general attitude to mathematics as a subject dominated by memorisation of facts and rules. This is in spite of the fact that the writers of curriculum documents, such as the Australian Education Council (AEC) (1991) have for over a decade been calling for greater attention to the affective outcomes of mathematics education, and for mathematics teaching that promotes meaningful engagement with numbers. In the National Statement on Mathematics for Australian Schools the AEC (1991, p. 110) state that, “From the earliest years, … A feeling for the way ‘numbers’ work is a major aim and its achievement requires action and reflection by children in investigating patterns in number.” In contrast with this, Wright (1994, p. 24) claimed that “curriculum emphases in early number and related topics have not undergone major changes in the last 30 years and have not changed significantly in the last ten years”, while more recently Stephens (2000) pointed out that negative attitudes to mathematics are present in many students by grade one.

The same scenario is apparent in relation to mathematics in Malaysian schools. Recent findings (Ghazali & Zanzali, 1999) have strongly indicated that most school children do not display good number sense and predictably have an average standing in mathematics achievement compared to other nations. An analysis of the Third International Mathematics and Science Study – Repeat (TIMSS-R) showed that Malaysians students perform well above the international average for questions that require computation but face difficulty and thus perform below the international average for questions that require understanding of basic concepts (Kementerian Pendidikan Malaysia, 1999).

The importance of carefully developing an understanding of number concepts and relationships has long been recognised. One principal aspect of this is emphasising the development of number sense (Trafton, 1994). Although the importance of understanding may well have “long been recognised” in theory, the term number sense is a comparatively new notion in the primary school mathematics curriculum. Number sense has been described as a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations (McIntosh, Reys & Reys, 1993). In the National Statement on Mathematics for Australian Schools number sense is defined as “ease and familiarity with and intuition about numbers” (AEC, 1991, p. 107). Sowder (1992) lists eight behaviours that are indicative (but not proof) of the existence of number sense. These relate to flexible understandings of number representations and the ability to move between them as appropriate, effective use understandings of both the relative and absolute magnitudes of numbers, meaningful connections between operations and symbols, use of number benchmarks and flexible strategies for mental computation and estimation, and an inclination to make sense of numbers. Howden (1989) argues that because number sense builds on students’ intuitive understandings of numbers they do indeed come to believe that mathematics does make sense, are able to evaluate the reasonableness of their answers and develop confidence in their own mathematical ability. She goes on to cite research that claims that such children have more positive self concepts with respect to mathematics and are more likely to choose to study mathematics in the future.

It is interesting to note that none of these descriptions of number sense mention counting. However, competence with counting that includes facility with: both forward and backward number sequences; forward and backward counting by twos and fives particularly across decades; meaningfully applying such counting sequences to both fixed and movable collections of objects; as well as the possession of clear mental images of the positions of numbers in relation to benchmarks, are all aspects of the flexible mastery of numbers that is inherent in the various characterisations of number sense.

A team in the Faculty of Education at Universiti Sains Malaysia (USM), Penang, under the leadership of Dr. Munirah Ghazali, with the aid of a USM grant, is developing an interview-based assessment instrument to assess the number sense of Malaysian children in grades one to three. These grade levels cater for children of the same age as children in the corresponding grades in Tasmanian primary schools where this study was conducted. The instrument comprises four modules aimed at assessing number sense in relation to each of Counting, Place Value, Addition and Subtraction, and Multiplication and Division. The third author of this paper is international consultant for this project and has been intimately involved in the development of both the Malaysian and English versions of the instruments in association with the other authors of this paper.

It was thought that trialling this instrument in a country other than Malaysia would be of potential value in determining whether the instrument could have a wider value in aiding international comparisons. Furthermore, since Malaysia is moving towards instruction in English for mathematics at the relevant grade levels, there was added value in developing English language equivalents of the tests. From an Australian perspective, assessing the value of an instrument developed for use in Malaysian schools had two aspects of interest. First, international comparisons have tended to focus on countries that have performed exceptionally well in TIMSS testing. It seemed of equal interest to develop instruments with potential use in comparing the number sense of children in two countries, Australia and Malaysia, which although culturally disparate have populations of approximately equal size, and which have performed somewhat similarly on TIMSS tests. Second, although assessment instruments exist for assessing mathematical ability and numeracy of young children (e.g. assessment protocols used in Mathematics Recovery, and Count Me In Too (Wright, Martland & Stafford, 2000) and in the Early Years Numeracy Research Project (Clarke, Sullivan, Cheeseman & Clarke, 2000), tests that concentrate on number sense and attempt to cover the full range of students in grades one to three do not currently exist. The Malaysian instrument was deemed to have potential for adaptation and use in the Australian context.

Research questions

This study involved a trial in Tasmanian schools of the English version of the Counting module of the number sense instrument. Specifically it aimed to answer the following questions:

1. To what extent do Tasmanian children in grades one, two and three display number sense in relation to counting?
2. How applicable to the Australian context is the English version of the Counting module of the instrument?
3. In what ways could the module potentially be used by Australian teachers?

Methodology

Sample

The three government primary schools selected included one large urban school, one small urban school, and one small rural school. Each school was asked to provide three students in each of grades one, two, and three, comprising (in the view of the classroom teacher) one ‘average’ student, one ‘somewhat above average’ and one 'somewhat below average' student. A total of 27 students was thus selected. In Tasmania, grade one is the second year of compulsory schooling during which students typically turn seven.

The Counting Module

The Counting module consists of an interview assessment schedule that can assess across grades one to three rather than only for a single age group, and concentrates on items that assess number sense relevant to these grades. The module contains a total of 25 assessment items. Each item constitutes one assessment task, presented either in relation to a picture, in relation to a collection of objects, or abstractly. Table 1 shows details of the clusters of items in the module.

Table 1
Counting module items by cluster

ITEMS	TASK TYPE
1 to 3	Counting small number of objects in picture
4 to 6	Counting larger numbers of objects in picture
7 to 9b	Counting up and down in ones (abstract)
10a to 12	Counting up and down in twos (abstract)
13 to 16	Counting up and down in fives (abstract)
17 to 18	Removing real objects by counting
19 to 20	Counting real objects in 2s and 5s
21 to 23	Placing numbers on a blank number line

Assessment of number sense involves not only an indication of whether the answer is 'correct' or 'incorrect' but also knowledge of the strategies used and their relative sophistication, as well as indications of hesitancy and the exact nature of errors and misunderstandings revealed by the student. This provides insight into the extent to which children are 'in charge of' the numbers 1-100 which is perhaps the essence of number sense as it relates to counting. To facilitate the efficient recording of this detail, each item in the module has a carefully constructed assessment grid that allows the interviewer to indicate, simply by ticking the relevant box, the level of proficiency shown on the item and the solution strategy used. In addition space is given for notes regarding unexpected strategies and non-verbal actions of the child.

Three levels of proficiency were distinguished for each item: 'Beginning', 'Emergent' and 'Competent'. 'Beginning' indicated a wrong answer, often together with hesitation or lack of strategy. 'Emergent' and 'Competent' both indicated a correct answer: 'Emergent' was associated with hesitation or an inefficient strategy, whereas 'Competent' denoted efficiency and fluency. The notion of fluency in this context is similar to that described by Griffin (2003) in relation to early computation in that fluency represents the culmination of a developmental sequence that involves progression in the development of number sense reflected in the use of increasingly efficient strategies. An example of an item, involving counting down in ones, and its associated summary matrix are shown in Figure 1.

The response categories Beginning and Emergent are distinguished on the basis of whether or not the child can successfully perform the task. Emergent and Competent categories are distinguished on the basis of the extent to which the child's responses are 'fluent' or 'hesitant'. This involves a subjective judgment on the part of the interviewer but is aimed at distinguishing between children who are clearly comfortable with the numbers involved and their order, and those whose grasp could be described as tenuous. This distinction is at the heart of number sense as it relates to counting.

Question C**Q9b**

> I am going to count down to a number, then, you continue counting.
> 105, 104, 103 ………… Now you continue counting
>
> Student's answer:____________________________________
>
> Answer: 102, 101, 100, 99, 98
>
> *(stop the student after he/she has counted five numbers)*

Student's response		***Category (tick)***	**Notes**
1. Cannot count five numbers correctly		Beginning	
2. Counts on fingers but incorrect			
1. Correct, counts on fingers, begins with number other than 102 [record starting number in Notes]		Emergent	
2. Correct, counts on fingers, begins with 102, hesitant			
3. Correct, does not use fingers, begins with number other than 102, hesitant [record starting number in Notes]			
4. Correct, does not use fingers, begins with 102, hesitant			
1. Correct, counts on fingers, begins with number other than 102, fluent [record starting number in Notes]		Competent	
2. Correct, counts on fingers, begins with 102, fluent			
3. Correct, does not use finger, begins with number other than 102, fluent [record starting number in Notes]			
4. Correct, does not use fingers, begins with 102, fluent			

Figure 1

An example of a question, without its response matrix, that involved children placing a number on a blank number line is shown in Figure 2.

Question ***C***Q23

0______________**100**

> Put a mark where you think 85 might be on this number line.
>
> Student's answer:____________________________________
>
> How did you decide that? ______________________________

Figure 2

The number line on which the children wrote was on a separate page. In this case the response categories are distinguished on the basis of both the accuracy of the child's response and the nature of the explanation that they offered. No relevant response or a response outside the range of 65 to 95 were classified as Beginning; responses in between either 65 and 75 or 95 and 99 with an adequate explanation, and responses between 75 and 95 but without an adequate explanation were classified as Emergent; responses in the range 75-95 accompanied by an adequate explanation were deemed Competent.

Procedure

Because number sense is a more difficult and subtle quality to assess than the skills of paper-and-pencil computation, an interview-based approach is necessary to allow exploration of children's thinking and use of strategies. Thus twenty-seven semi-structured interviews were conducted using the same Counting module with all children. All data were recorded in the summary matrices designed and refined in collaboration with the USM team and the data from the individual student summary matrices were collated and analysed for trends within the Tasmanian sample.

The three interviewers, comprising two of the researchers and a research assistant who was also an experienced teacher and research student, participated in a series of trial interviews of appropriately aged children who were not participants in the study. In each case, one interviewer conducted the interview and all three scored the child's responses and made notes on the summary matrix as they would in an actual interview. After three such interviews and subsequent discussions the researchers were confident that both their presentation of the items and their interpretations and ratings of students' responses were very closely aligned. Inter-rater reliability was also enhanced by the use of space for notes on the summary matrix for each item to record any relevant aspects of the students' responses not specifically mentioned in the matrix or details of any responses that were potentially problematic in terms of scoring.

Following the interviews and recording of the data, a meeting was held at each of the schools involved, with the Principal and the classroom teachers of the three classes taking part. At this meeting, which was videotaped for later analysis, the data for that particular school were given to the teachers, together with the interview questions, and teachers were asked to comment on the relevance of the assessment instruments to their class, on the value of the resulting data in providing new or unexpected information regarding the strengths and weaknesses of their children, and on any other features of the instruments or the process.

Results

Table 2 shows the number of student responses classified as 'Competent', 'Emergent' and 'Beginning' at each grade for each cluster of items. It shows, for example, that for the four items (Items 10a to 12) assessing 'Counting up and down in twos (abstract)', the nine grade three students gave responses of which 25 were classified as 'Competent, eight as 'Emergent' and three as 'Beginning': the numbers for responses in each category from grade one students for the same cluster of questions were respectively 10, 3 and 23.

Questions 1 – 3 asked students to count two, three and four objects in a picture. Although all children except one answered correctly, as the numbers of objects increased, the number of children who pointed or counted with their fingers increased from one to six.

Question 4 asked students to count the number of windows arranged in a four by five array. Twenty-two children answered correctly: correct strategies employed included counting in ones (often helped by nodding the head), counting rows of four, and multiplying four by five. Others did nothing, or counted incorrectly with their fingers.

Table 2
Number of responses at each proficiency level by Task Type and Grade

TASK TYPE	ITEMS	NUMBER OF ITEMS	COMPETENT			EMERGENT			BEGINNING		
			Gr 3	Gr 2	Gr 1	Gr 3	Gr 2	Gr 1	Gr 3	Gr 2	Gr 1
Counting small number of objects in picture	1 to 3	3	26	26	26	1	0	1	0	1	0
Counting larger number of objects in picture	4 to 6	3	23	15	19	1	1	1	3	11	7
Counting up and down in ones (abstract)	7 to 9b	4	35	27	25	0	4	3	1	5	8
Counting up and down in twos(abstract)	10a to 12	4	25	10	10	8	12	3	3	14	23
Counting up and down in fives (abstract)	13 to 16	4	30	19	13	5	7	1	1	10	22
Removing real objects by counting	17 to 18	2	18	18	18	0	0	0	0	0	0
Counting real objects in 2s and 5s	19 to 20	2	15	12	6	2	1	1	1	5	11
Placing numbers on a blank number line	21 to 23	3	13	4	0	8	10	13	6	13	14
TOTALS (% of responses)		(25 items)	82	58	52	11	16	10	7	26	38

Question 5 asked students to count tyres on four vehicles, having established that all had four tyres each, though not all were visible. Twenty children answered correctly, with correct strategies included counting in ones, counting eight wheels and then doubling, counting in fours, and multiplying four by four.

Question 6 asked students to count all the people in a picture. This was quite challenging as the people (18 in all) were scattered round the picture in a variety of clothing and postures. Those who counted incorrectly often relied on observing the picture and nodding their head, without using their fingers to point to individual people. Several of the children who obtained the correct total in fact stopped during their initial counting and began again in a more systematic way. These responses were classified as Emergent.

Questions 7 to 16 asked students to count up and down in ones, twos and fives, with questions 7 to 9b involving counting up and down in ones. These questions caused few problems for the grade three children. However, counting down caused problems for half of the grade two and grade one students, particularly when it involved reaching a new decade. For example, four students counted down 42, 41, 30 …

Questions 10a to 12 asked students to count up and down in twos. Counting up in twos from a small even number (six) caused no problems, but many of the grade two and one students, and some grade three students, were clearly uncomfortable counting down in twos or counting up or down in twos from an odd number. For example, in responding to question 10b which required children to continue the sequence 17, 19, 21, … for five more numbers just one response from a grade two child and one from a grade one child were classified as Competent. Several grade one and grade two children simply began with 22 or in one case 24, and proceeded to count fluently in twos. Others counted in ones from 22. One grade one child declared that "You can't do that". All of these responses were classified as Beginning. Emergent responses involved hesitation, usually initially or after 27 or 29.

Questions 13 to 16 asked students to count up and down in fives, always starting from a multiple of five. Counting up in fives caused relatively few problems. However counting down in fives proved particularly difficult for grade one students, although almost all attempts recognised that the units digits would be either zero or five. For example, all of the responses of grade one students to question 15 which involved continuing the sequence 85, 80, 75, … for five more numbers were classified as Beginning. Their responses included 65, 50, 55, 40, 45, 30 which is clearly a descending pattern; 75, 60, 65, 70, 60, 65, 70 consisting of a repeating upward sequence; and 65, 35, 30, 50, 5 which is an apparently random list of multiples of five. In contrast with the grade ones, all of the responses from grade three students were classified as Competent with grade two responses falling across all three categories.

Questions 17 and 18 asked students to remove, from a collection of objects, a number of objects stated orally (Question 17) or indicated by numerals on a card (Question 18). Neither question caused problems at any grade. However, grade three students were more likely than others to remove the objects in twos and threes rather than singly.

Questions 19 and 20 asked students to count a collection of objects in twos (16 objects and in fives (20 objects). Counting by twos caused a surprising number of problems for grade two and grade one students. Some counted in twos (two, four, six, …) while moving only one counter at a time. One student counted the 20 counters provided in question 20 by fives in this way finishing with 100. When asked, "So how many counters are there?" she responded, "100". Others counted incorrectly in twos, even though they had previously (without objects) counted in twos correctly. Grade two and grade one students clearly found counting objects in fives an unfamiliar task, choosing often to count the fives in twos and ones, or first to arrange the objects in fives and then count them.

Questions 21 to 23 involved locating or estimating the position of a number on a blank number line. Although the responses of children suggested that blank number lines were unfamiliar to them, they also indicated that the majority of students were quite comfortable with the problems, and indeed their oral responses to these items were among the most interesting in the test. Students whose responses were classified as Beginning either said that they guessed or used strategies that involved starting at the number given at one end or other of the number line and then counting backwards or forwards by ones, or in one case by twos, moving their finger along the line until they arrived at the desired number. One child, located the position of 85 in relation to 0 and 100 quite accurately but did so by counting in ones from zero and moving his finger along the line approximately a finger width with every number. At the end of the line (which was labelled 100) he simply returned to the zero position and continued counting as before until after several counts along the full line he reached 85 fortuitously close to the correct position. Responses that were classified as Emergent or Competent were arrived at by strategies that involved looking at the line as a whole and locating other relevant points such as the number that would be in the middle or the positions of multiples of ten and then narrowing down to the required number. Overall though, despite being somewhat inaccurate, most of the students' answers, particularly from grades two and three, showed a reasonable understanding of the relative positions of numbers up to 100.

Interviews were held at each of the three schools with the school principal and the teachers of the three classes involved. All teachers found the level and content of the items relevant to their children (for example, a teacher of grade three found counting forward and back in fives very relevant to work on time and money), and all found useful information relating to some of their children in the data. They saw the material as a valuable item bank, and expressed interest in using the other modules. Although they were unlikely to have the time to assess all the children using the entire instrument, they saw several valuable and practical uses:

- Using one cluster of items with a small group, to help determine how to proceed;
- To separate out children lacking understanding from those who needed practice;
- As professional development for themselves in becoming aware of the different strategies used by children;
- In enabling them to observe a child closely without the need to write many notes at the same time;
- To pinpoint the exact areas of weakness with less competent children.

As the principal of one of the schools said:

> *The individual questions are really good for pinpointing particular areas you are uncertain about, and how the children are coming up with solutions; because we tend to concentrate on right/wrong answers and don't notice if two children both got the right answer but one was doing it much more efficiently.*

Discussion and Implications

Research question 1: The number sense of young Tasmanian children

The totals in Table 2 indicate that the number sense in relation to counting of the children in this study was clearly related in the expected way to grade level. However there appears to be scope for improvement at all grade levels. It is surely of concern that some students, albeit a minority, seem to have formed beliefs such as that it is only possible to count by twos from an even number, or that the cardinality of a set of objects depends upon whether one counts them by ones or by fives. On the basis of this very limited trial of the Counting module, we were able to make a number of comments and suggestions to the teachers involved that would be likely to lead to improvement in their students' number sense in the context of counting. It is likely that they are also of relevance to other Australian teachers of grades one to three.

Firstly, almost all children had reliable strategies for counting objects in ones, but it might be valuable to have children sometimes count objects, such as objects in photographs, that are not arranged neatly and that cannot be moved. This would assist some children to develop systematic and effective ways of counting in these situations.

It seems that many children might benefit from more attention to each of counting down, particularly over decades (e.g. 42, 41, 40, 39, 38 …), counting down in twos, counting up and down in twos starting from an odd number, and counting up and down in fives, starting from a multiple of five. Activities of this sort would assist children to construct the rich and flexible understandings of the number sequence and the relative sizes of numbers that are essential to number sense.

Some of the children who were able to count abstractly in twos (two, four, six …) appeared not to relate this to the counting of objects two by two. A similar problem was evident with some children in relation to counting objects in fives. It would therefore be valuable to ensure that children count collections of objects in twos and fives as well as reciting two, four, six … It is clearly not sufficient to provide concrete materials only when children are first learning to count upwards by ones. Rather, skip counting sequences need also to be linked to real objects in order that children learn them meaningfully. One to 100 boards used in conjuction with counters, as well as calculators for skip counting are useful tools to this end. Similarly, while rational counting includes the understanding that the last number of a counting sequence (by ones) gives the cardinality of the set, this study provides evidence that some children incorrectly extend this notion to sets counted in twos or fives but with one object moved at a time. The periodic use of concrete materials at later stages in children's counting development could reduce the likelihood of this occurring.

Finally, most children appeared not to find the idea of the blank number line difficult even though they had not previously met it. This suggests that even though many, especially younger children, have some difficulty with ordering numbers, particularly downwards, most do have a reasonable idea of the relative proximity of numbers to benchmarks such as 0, 20, 50 and 100. In addition, valuable estimation and computation activities can be built upon the blank number line (Beishuizen, 1993). For example, children could use pegs to hang numbers on a clothesline with end points marked, perhaps with zero and 20. The blank number line can also be used to record the steps of a mental calculation. For example, 36 + 7 could be calculated as shown in Figure 3.

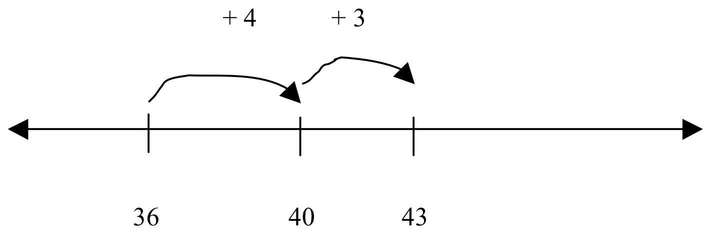

Figure 3

Research questions 2 and 3: Applicability to and use of the instrument in the Australian context

On the basis of this trial we are confident that the Counting module is applicable to the Australian context with potential to be used in all of the ways identified by the teachers of the children involved in this study. The range of items included some that the youngest and least able (as perceived by their teachers) were able to complete successfully and others that the older and most capable students found somewhat challenging. The high rates of success achieved in relation to items 1-3 suggest that some or all of these items may be redundant for children in grades one to three. Alternatively the instrument may be of value for assessing the number sense of prep grade children as well.

The items along with the criteria used to classify students' responses as Beginning, Emergent or Competent were largely effective in distinguishing between those responses that were indicative of well developed number sense in relation to counting, and those that were not. Crucial to this was the focus on strategies, explanations, and fluency of the children's responses. These elements, rather than just the correctness or otherwise of the answer get to the heart of what number sense is about in identifying the extent to which children are at ease with and in charge of numbers.

The teachers' idea of using discrete clusters of items with small groups of children whom they suspected were unsure of a particular aspect of counting seems particularly worth pursuing. All of the teachers agreed that such clusters of items could provide meaningful and very useful insights into their students' thinking and yet be quick to administer in the context of a busy classroom.

Conclusion

We have some confidence that the materials trialled in this study have the potential to provide very valuable information at classroom, school and system level. Initial English versions of the remaining three modules have been produced but these are in need of further refinement prior to trialling in the Australian context. We also see potential for this and other modules to be formatted more economically in terms of the number of pages required for each child. We are particularly encouraged by the positive feedback from the teachers involved, especially in relation to the module's value in helping teachers to better understand their students' thinking and thereby improve their teaching.

Compared with many existing instruments these materials offer the advantage of requiring minimal teacher training due in part to the simplicity and consistency of the basis of classifying students' responses, their applicability across a relatively broad range of grade levels, and the potential for sub-sections of the instrument to be used to quickly assess specific aspects of students' number sense.

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