Posing and solving problems in open-ended investigations: Authentic tasks with Grade 1 children

One way to provide young children with the opportunity to engage in more diverse and flexible thinking is to encourage them to pose their own problems. Problem-posing activities reveal much about the understandings, skills and attitudes the problem poser brings to a given situation. This investigation explored the way in which a Grade 1 child (6 year old) posed and solved open-ended investigations over a three-week period. The participant was able to identify and discuss the type of mathematics knowledge and related processes that would needed to complete the task. Moreover, she was able to recognise aspects of the task that would be difficult to solve and was able to propose alternate pathways when it was apparent that her initial approaches were not appropriate. It is argued that the meaningful and empowering nature of the problem-posing environment was influential in the success achieved by the participant.

Problem posing is associated with an individual’s capacity to generate problems for either themselves or others to solve. Problem posing can occur before, during and after the process of solving a problem (Silver, 1995). Although reports such as those produced by the National Council of Teachers of Mathematics (NCTM, 2000) have called for an increased emphasis on problem-posing activities in the mathematics classroom, such activities are not promoted on a regular basis. As Silver (1994) argued, “students are rarely, if ever, given the opportunity to pose in some public way their own mathematics problems” (p. 19). One way to provide young children with the opportunity to engage in more diverse and flexible thinking is to encourage them to pose their own problems.

PROBLEM-POSING CONTEXTS

There are many benefits gained from fostering a problem-posing classroom. English (1997a) argued that problem posing allowed students to generate more diverse and flexible thinking in ways that not only enhanced problem solving but also reinforced and enriched basic mathematics concepts. As Silver (1994, p. 25) commented:

Problem-posing tasks provide…a window through which to view students’ mathematical thinking and a mirror in which to see a reflection of students’ mathematical experiences. Second, problem-posing experiences provide a potentially rich area in which to explore the interplay between the cognitive and affective dimensions of students’ mathematical thinking.

From a teaching perspective, problem-posing activities reveal much about the understandings, skills and attitudes the problem poser brings to a given situation and thus become a powerful assessment tool (English, 1997a; Lowrie, 1999). Not surprisingly, reports such as those produced by the National Council of Teachers of Mathematics (NCTM, 2001) have called for an increased emphasis on problem-posing activities in the mathematics classroom.

Recent studies (English, 1997a, 1997b; Kilpatrick, 1987; Leung, 1996; Silver, 1993) have found that broadening children’s perceptions of mathematical situations enhances problem-posing development. English (1997b) suggested that children with strong number sense were more likely to be able to able to pose appropriate problems than children with limited number sense because they had a better understanding of problem structure. In these studies, the children who had conceptually sound mathematical ideas tended to pose problems that were well structured and had clear
goals. Since an ability to pose problems is linked to metacognitive thought, it is not surprising that “more able” students are more successful in generating problems (Ellerton, 1986; Lowrie, 1998). Lowrie (1999), for example, found that talented Grade 3 students were able to modify two-step word problems into problems that were open-ended in nature.

Lowrie and Whitland (2000) found that all of the Grade 3 students investigated in their study were able to generate problems for others to solve. Decisions concerning the appropriateness of content for a particular task were often related to number magnitude, operation complexity or the type of mathematics concepts thought to be taught at a grade level above or below that of the problem poser. Other problems were set in contexts that were related to perceived interests or real-life experiences problem solvers had encountered. Interestingly, the tasks that were related to these realistic experiences tended to be more challenging and mathematically rich than the “traditional” text-book like problems usually posed by young children (Lowrie, 1999). The present investigation moves beyond such studies by challenging young children to pose and then solve problems embedded in realistic contexts.

Young students posing problems in realistic contexts

Although some studies have investigated the extent to which children as young as ten years of age design and generate problems (Lowrie, 1999; Lowrie & Whitland, 2000; Silver, Leung & Cai, 1995) most studies have focused on children in upper-primary or secondary school settings. In general, these studies have provided children either with specific strategies or mathematics content to manipulate as a way of scaffolding their problem posing. English (1997a), for example, encouraged children to appreciate key ideas and their relationships to problems that were teacher generated before considering situations that might arise from extending these ideas. Then, before posing new problems, the Grade 5 children were given a series of questions that were intended to provide the children with information that could be used for designing their problems. These questions provided the children with a set of ideas or examples that could be used when they began to pose problems. In other studies (Lowrie & Whitland, 2000; van den Heuvel-Panhuizen, Middleton & Streefland, 1995), children have been encouraged to verbalise their thinking and consider problems associated with a particular content area (e.g., percentages) before posing problems. Writing problems for friends to solve is another way of contextualising problem scenarios (Ellerton, 1986; Lowrie, 1998). Such approaches allow the problem poser to relate problems to particular contexts or individuals.

English (1997a) proposed that greater attention should be given to problem-posing processes by making greater use of problem situations set within realistic contexts. It seems to be the case that young children are more likely to be able to pose “appropriate” problems when they have a meaningful context in which to situate the problem. Providing opportunities for children to scaffold their ideas and understandings is one way of ensuring that the problem-posing contexts are meaningful. In the present investigation it is argued that providing opportunities for the participant to pose and solve problems in a realistic context is another way of ensuring that scaffolded support is established. Moreover, it could also be argued that problem-posing situations allow children to have some control over the curriculum content and the type of learning activities presented in the classroom. Furthermore, the tasks or the activities children construct may provide insights into their beliefs or
attitudes they have toward mathematics and the way in which mathematical knowledge is developed.

The present study adds to the research literature on problem posing in a number of ways. First, it investigates the problem-posing capabilities of a participant who is only six years of age. Most previous studies have focused on children aged ten or older. Second, the participant had not previously been exposed to teaching-learning situations that encouraged problem posing. As a result, the influence of a scaffolded teaching-learning program could be evaluated critically. Third, the participant appreciated that she was expected to solve the problem posed. Consequently, the study established strong links between problem posing and problem solving. Furthermore, the investigation examines the problem-solving processes employed by the case-study participant and consequently moves beyond studies that have primarily focused on end-product problem-posing experiences, that is problem solution.

**METHOD**

This investigation involved a case study of a Grade 1 child (6 years of age) who was supported across a number of problem-solving activities with an undergraduate student in her final year of an education degree. The student worked with the Grade 1 child for one hour per week for five weeks as part of a compulsory course within the degree. During these sessions, the child was encouraged to pose her own problems and solve these tasks with support from her “teacher” or “critical friend”. It is quite difficult for young children to design appropriate problems without a substantial amount of practice (Ellerton, 1986), specific instruction (Leung, 1996) or guided questioning (Lowrie, 1999). The rationale behind matching an undergraduate student with the child was to provide support with problem construction. Further, since the child was so young it was assumed that she may required assistance in constructing problems that were challenging but still “solvable” in the given time period. On occasions, the undergraduate student had to use her professional judgment with respect to the degree of input she had in the problem-posing process. It could be argued, however, that the same balancing act occurs in most teaching-learning situations.

The case-study data were analysed in relation to the participant’s problem posing prior to problem solving and her problem posing during problem solving. Such analysis is in line with Silver’s (1995) notion that problem posing can occur before, during and after problem solving. The author (who was responsible to the practice teaching component of the undergraduate student’s course) observed each session and conducted semi-structured and open-ended interviews during the five weeks. Field notes were collated during these sessions and digital camera stills were also taken. In the present study, the case-study participant’s problem-solving processes were identified and analysed as she engaged in solving an open-ended task she had generated herself.

**Profile of the case-study participant**

Jane’s regular classroom teacher considered Jane to be a quite capable student with a sound understanding of most foundation mathematics concepts. Although Jane’s spatial reasoning skills were quite well established, her understanding of most other mathematics content was comparable to many other members of her class. The regular classroom teacher indicated that Jane’s overall mathematical performance could be classified with “those students who were in the upper third quartile of the class.”
Jane’s critical friend [the undergraduate teacher Sally] reported that she was an articulate student who possessed a “bubbly personality” and who was quite at ease when discussing the manner in which she went about completing tasks. At the end of the second session, Sally commented that Jane was:

…an enthusiastic student who was always willing to do her very best. She was prepared to talk about the processes she used to solve problems and was happy to elaborate on responses when I needed more information. Her spatial concepts seemed more sophisticated than her measurement or number concepts. She was willing to add and subtract various combinations of numbers but did not always do so accurately. Her mathematical visions were well ahead of competencies. In other words, her ideas were advanced but her ability to sequence and then complete the tasks correctly didn’t always eventuate. She was able to relate many mathematical ideas to real-life experiences…her father was an architect and she seemed to have a good understanding of the usefulness of mathematics in society. (Sally’s journal)

Researchers, who have conducted problem-posing studies with young children, have argued that the most capable students are more likely to be able to generate rich scenarios (Ellerton, 1986; English, 1997a; Lowrie, 1999). Jane was selected as an appropriate case-study participant because was mathematical capable, she was able to verbalise her thinking and she could relate mathematical concepts to realistic contexts.

The problem-posing sessions

For each of the three problem-posing sessions, the undergraduate teacher was responsible for assisting the Grade 1 child to pose problems that both were open-ended in nature and appropriately structured. A topic or scenario was negotiated with a problem or series of problems generated from the theme. Jane decided that her problem would involve building a model of the playground. Once the collaborative team was happy with the task the student teacher attempted to establish whether the Grade 1 child was able to determine the type of understandings and strategies that would be required to complete the task. Importantly, Jane knew that she was able to seek assistance from her “teacher” to solve the task. This added another dimension to the study because the child was not inhibited by their inability to complete computations or solve multi-step problems. Thus, she was challenged to consider the types of strategies and methods that she would need to use in order to complete the task without being restricted by a lack of content-specific knowledge.

A series of questions were posed by the undergraduate student to elicit this information before the participant began the task and included: What might we collect to build this?; Where will we get these materials?; What is going to be the hardest thing to do?; Will we need to do some mathematics?; when do you think you will need help from me?; and Where should we begin? The following section describes the problem posed by Jane and investigates in more detail the way in which she responded to these questions before attempting to solve her problem.

RESULTS

Problem posing before problem solving

Jane decided that she wanted to construct a model of the school playground to fulfil the requirement of generating a problem that required mathematical thinking. Jane was asked to indicate what type of mathematics problems she would encounter before actually beginning to construct her model.

Interviewer: What sort of Maths problems will you need to work out?

Jane: I will have to measure, do pluses [addition] and minuses [subtraction] and make sure I get the size of things right.
Interviewer: Anything else that will make you think?

Jane: I have to put things in the right place. I think I will need help to measure everything.

Interviewer: Well done, positioning objects in the correct place is certainly mathematical. Is there anything that will be quite difficult to do?

Jane: It will be hard to fit all of the playground on to a model I can pick up…and because we wouldn’t have enough time to finish it if it was too big.

Interestingly, Jane recognised that it would be too difficult to make a model of the entire playground “because we wouldn’t have enough time”. The undergraduate student (Sally) working with Jane helped her modify the task to include a section of the playground that contained a boundary fence, the sandpit and some large trees.

Jane thought that the best way to begin constructing the model was to position all the important objects on a large piece of paper (which was later replaced by a piece of thin wood). With respect to positioning these objects, Jane wanted to “step out” how far each landmark was from the other.

Interviewer: How will you decide where to put the trees?

Jane: You have to measure how far they are away from the other ones.

Interviewer: And how will you measure the distances?

Jane: By going to one tree and stepping out how far it is to the next tree.

Interviewer: Then what do you do?

Jane: You put the first tree on the board then you have to put it [the second tree] where it looks right.

Not surprisingly, she did not know how to transfer this information accurately onto the paper but did not see this as a problem because “you have to put it where it looks right.” Jane did not have an understanding of scale (after all she was in Grade 1) but appreciated that her model needed to represent the 3D world because “that tree will have to be the biggest thing in my model.” She also realised that the tree had to be taller than the length of the sandpit.

Despite the fact that Jane was not able to develop a scale she felt that it was important to measure everything. Her greatest concern was working out how to measure the large tree. Initially, she thought that her “teacher” could stand on a ladder with a large tape and she could read the measurement on the ground. Before they had a chance to “act out” her proposal she conceded that the school did not have a ladder that tall. She then hypothesised that they could measure the shadow of the tree in order to ascertain the height of the tree. She was not convinced that this was valid but felt that it must have been a good idea based on the Sally’s reaction. She commented, “I will have to make sure the fence is high but not as high as the trees.” Jane was actually creating new problem-posing situations to solve within the context of the original problem she had designed. The open-ended nature of the task provided her with the opportunity to set personal outcomes as the problem evolved.

Problem posing during problem solving

As Jane begun constructing her model, the context in which she posed problems changed. She raised questions in relation to particular problems encountered as she completed particular tasks (for example, completing the fence). The relationship between these design elements also provided challenges. The following transcript illustrated an incident where Jane realised that the actual construction of one part of her design affected other aspects of the model.
Jane: I will have to make really tall trees now because they [the trees] will have to be a long way over the fence.

Sally: How much taller do you think the trees will need to be?

Jane: Lots taller.

Sally: Well, could the trees be two or three times taller than the fence?

Jane: I don’t know. It’s easy to tell with the sandpit because it’s not too long. I could get some string and get you to cut it off at the top of the fence. We could then see if it was longer [than the sandpit].

Jane was able to make sense of initial understandings associated with proportion and ratio by identifying a number of informal-measurement approaches to “satisfy” her personal problem-posing scenarios. Although she was not able to verify her calculations in a formal manner she did engage some quite rich learning experiences.

There were instances when Jane expressed a view that the problem was too difficult to solve. On one occasion she commented that “the fence looks real but the trees don’t…the tree is too tall to measure and I don’t like real big numbers anyway.” On another occasion she indicated that “I will have to make sure that the sandpit is not too long or the trees won’t fit…maybe I should do the trees [place them into position] first.” Being challenged to pose questions and identify difficult scenarios during the construction phase of the project meant that Jane used a variety of approaches and went beyond trial-and-error strategies in order to complete the task.

CONCLUSION

The case-study participant described in this study was able to design her own open-ended problems and then collaboratively solve the task over a three-week period. She was able to identify mathematical concepts associated with the task and highlight areas where assistance may be required. English (1997a) argued that both number sense and novel problem-solving skills play an important role in problem generation. Although Jane’s number sense could have been a restrictive factor in her ability to complete the activity in a manner that she desired, her problem-solving skills were able to compensate for any limitations in content knowledge. In fact, some of the approaches she used to construct her model were considered both creative and sophisticated for a Grade 1 student.

Silver (1994) maintained that problem posing not only provided an opportunity to view students’ understanding of mathematics but also provided an appreciation of the content and character of their school mathematics experiences. This activity certainly did not reflect the type of experiences Jane was exposed to in a school context. This was the first time she had been engaged in this type of problem-posing environment. Nevertheless, Jane may have been exposed to model building due to the fact that her father was an architect who predominantly worked from home. With the support of a critical friend she was able to engage in mathematics at a much more sophisticated level than she had previously encountered at school. The open-ended nature of the task allowed her to modify the problem with flexibility so that the content knowledge and skills she possessed could be used most effectively. The fact that she had control over the learning ensured that she remained enthusiastic and motivated even when she faced difficulties.

The structure of the problems Jane posed before creating her model were predominately “what-if” scenarios or prediction-based questions about difficulties that may arise during model construction. As she began working on the task, Jane’s questions were quite context specific—usually posed to support her limited content
knowledge or consolidate the types of problem-approaches used to measure informally. Finally, the design and make nature of this investigation allowed the participant to pose problems in a realistic context that supported personal problem-solving strengths and interests.

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References


