RE-SEARCH RELATIONSHIPS: A SYSTEMS APPROACH TO MATHEMATICS EDUCATION USING THE METAPHOR OF A SEARCH AS A PARADIGM FOR CLASSROOM TEACHING AND LEARNING

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Abstract

For the last seven years, a research project focused on one North American Grade 2 teacher's efforts to develop young children's early mathematical concepts has given rise to a new paradigm for teaching and learning. By creating a classroom environment that promotes (1) reflexive psycho-pedagogical relationships and, (2) a systems-theory approach to learning, the teacher's pedagogy and children's engagement with learning have been continually refined through the SEARCH metaphor (Geoghegan, 2002). Being less to do with "didactic teaching" and more to do with "self-regulated learning," the research project has sought to highlight children's capacity to confidently generate creative propositions as one of the significant constituent elements of effective mathematics teaching and learning. Incorporating post-modern and systems-theory perspectives, this paper will discuss and demonstrate how the reflexive nature of self-regulation engages children in creative and productive mathematical thinking from a young age.

Introduction

The National Council of Teachers of Mathematics asserts in their Principles and Standards for School Mathematics (2000) that the need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. "In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed" (p. 4). Educators who work with young children have a tremendous responsibility and play a major role in developing children's mathematical thinking especially in light of the fact that many children have decided by age eight whether or not they will ever be "good at doing mathematics." Early childhood is an absolutely critical period for establishing children's mathematical futures.

Embodied in perspectives of early childhood education is the basic guiding principle of an integrated competent child rather than a deficient or developing junior version of an adult (Edwards & Knight, 2000). Unfortunately, the hegemony of "the deficient-child" and/or "developing-junior adult" theories of human development have had a restraining and detrimental effect in how learning mathematics is best achieved. A clinical preoccupation with cognitive development driven primarily by Piagetian stage-based perspectives, has been by and large at the expense of the systemic and integrated aspects of physical, emotional and social development that impact upon learning, especially in young children developing mathematical ideas. Radical-constructivist and social-constructivist theories have been influential in the last few decades in informing the epistemological dimensions of knowledge construction but the advancement of definable and defensible agendas for significant reform in mathematics teaching and learning has not followed suit. In keeping with Freudenthal's suggestion that humans should learn mathematics as an open rather than closed system, this paper describes psycho-pedagogical dimensions reflecting post-modern perspectives that emphasize "relationship" as a central tenet for young children's developing mathematical thinking. The emergence of effective mathematical thinking will be shown as a reflexive and systemic "relationship" that emphasises more than just cognitive development and by incorporating all areas of human growth and development generates more creative thinking and deeper mathematical learning.

The logic of "relationship" as derived from perspectives of complexity theory and systems theory emphasizes notions of change as a process that is holistic rather than piecemeal. Patterns of "organization and emergence" (Fleener, 2002) are central to the synergistic organizational dynamics of an evolving system, such as a classroom or a growing, learning individual. The whole is greater than the sum of its parts.

The proactivity of recognising the mathematics classroom as a dynamic and evolving system based upon "relationships" (cf. McClain and Cobb, 2001) can be extended metaphorically as a "SEARCH" – i.e. what learners and teachers do when searching for meaning; the learner searches to make meaning (qua construct knowledge) and the teacher searches to understand the learner's meaning-making processes. Learning and teaching become a reflexive phenomenon based in "relationship," where teacher and student search together as co-constructors to make sense in and of the meaning-making process. In this paper, I will discuss children's mathematical development in a classroom applying the "SEARCH" metaphor and then translate the "SEARCH" metaphor as a teaching heuristic presented by an acronym based in the "relationship" between SE = Social Emancipation, AR = Active Referencing, and CH = Creative Heuristics.

Modernist and post-modernist views

The modernist perspective of teaching and learning is regarded as a significant (if not "the") force in mathematics classrooms around the world. However, mathematics educators working with young children are increasingly challenging pedagogy and curriculum decision-making based upon modernist views of objectivity, absolutism, linearity and predictability. Modernist tendencies are being critiqued as having outlived their usefulness in 21st-century educational contexts. As the new millennium emerges, early childhood mathematics educators (in particular) are looking past modernist epistemology and turning more to post-modern emphases that represent learning as a process of systemic "relationships."

Perspectives such as "systems theory" (Bertalanffy, 1968; Capra, 1996), "constructivism" (Glasersfeld, 1995; Cobb & Bauersfeld, 1995), "ecological systems theory" (Brofenbrenner, 1992), "autopoietic systems" (Luhmann, 1995), "problem-centered learning" (Murray, Olivier & Human, 1998), "classroom socio-mathematical norms" (McClain & Cobb, 2001) and "SEARCH " (Geoghegan, Reynolds & Lillard, 1997; Geoghegan, 2002) are cohering around the world in a synergy that recognises systemic "relationships" and the interconnected, emergent and reflexive nature of learning and teaching mathematics.

Systemic relationships

With scientific developments such as Heisenberg's "uncertainty principle" during the 20th century, quantum physics showed that the ideal of modernism, namely "*scientific objectivity*," no longer holds (Wheatley, 1994). New scientific thinking in the post-modern 21st century is portraying the world as a system that undergoes change as a

result of interacting with itself, its environment or other systems, where nothing is independent of "relationships" and everything is in a constant flux of dynamic "structural plasticity" (Maturana & Varela, 1980). In post-modern terms, our world exists as "relationships," as a world of process, not things. The world of "relationships" is in a spontaneous state of becoming and self-organization. These are the basic principles of systems theory and infer that the future will be unpredictable and everything in the world will be open and susceptible to processes of change. Reality is itself in process and the ultimate principle will be a "relationship" with experience; but not experience removed or separated from ourselves as Plato or Descartes would have us believe but rather, experience as the reality of our being (Whitehead, 1978, cited in Doll 1993). The implications of promoting a systems-theory perspective of learning mathematics are enormous.
The idea of systemic "relationship" has become the hallmark of economic, political, social and scientific progress. In order to keep abreast with these new international paradigms educators also need to be responsive with new conceptions of knowledge, teaching and learning.

In mathematics education an imperative based upon a logic of systemic "relationship" is emerging as foundational to the idea of the post-modern classroom whereby teachers and students, and teaching and learning, are cast in a new concept of curriculum. The mathematics classroom is being portrayed as a multi-faceted self-organising system reminiscent of Ilya Prigogine's dissipative structure wherein order is not imposed but created from within and Fritjof Capra's notion of an integrated whole arising from the "relationships" between its diverse parts. Working with a systems approach to mathematics education requires teachers to operate in a very different manner. Rather than being bound by pre-determined outcomes and imposed hierarchical curriculum decision-making, nonlinear and self-organising models that encourage the teacher to spontaneously experiment and play with them and observe what happens will be required. Open, dynamic, creative, and adaptive processes within the classroom will form the framework for exploration so that the critical points and homeostasis of the classroom's system are recognised and harnessed. If we view the classroom as an unfolding, evolving and open system determining its own dynamics and direction, and through dynamic interconnectedness determining its own meaning, then everything is free to adapt and open to change; the "relationship" between teaching and learning mathematics takes on a completely different perspective (Geoghegan, 2002).

Wheatley (1994) suggests that

> *we are beginning to recognize [classrooms] as systems, construing them as "learning organizations" and crediting them with some type of self-renewing [coherent, evolving and interactive] capacity (p. 13) . . . we talk of quantum interconnectedness, of a deep order that we are only beginning to sense . . . a constant weaving of relationships, of energies that merge and change, of constant ripples that occur in a seamless fabric [of learning]. (p. 20)*

> *This is such a remarkably different approach to analysis, this sensing into the movement and shape of a system, this desire to be in harmony with it. The more we develop a sensitivity to systems, the more we redefine our role in*

managing the system. The intent is not to find the one variable or set of variables that will allow us to assert control. This has always been an illusion anyway. Rather, the intent becomes one of understanding movement based on a deep respect for the web of activity and relationships that comprise the system. (Wheatley, 1994, pp. 110-111)

The SEARCH begins

The articulation of a new awareness and sensitivity to the "relationships" between learning and teaching, and teacher and student, might prove helpful in order to implement mathematics reform agendas that are beckoning a new approach to education in the 21st century. Traditional approaches steeped in the modernist paradigm are no longer adequate. The heuristic "SEARCH" provides a basis for teachers to make a paradigm shift that emphasises "relationship" as a central tenet of the learning experience.

The SEARCH heuristic has been formulated upon the reflexive complementarities of three psycho-pedagogical dimensions, namely, Social Emancipation (SE), Active Referencing (AR), and Creative Heuristics (CH). SEARCH is given form by the notion that "learning is not a destination, nor simply a journey but rather, a never-ending search." Knowledge is not derived from an objective reality but rather, is a personal and intersubjectively connected sense of meaning making. The notion of searching for meaning is made more multi-dimensional by relating not only to a child's search to make sense of mathematical ideas but also through the teacher's role of searching to make sense of each child's cognitive, social, emotional, and physical needs in order to make developmentally appropriate decisions about what to include in the daily curriculum.

As such, the search for mathematical meaning is portrayed as a complex reflexive "relationship" founded upon sociocultural and sociomathematical norms (McClain & Cobb, 2001). The SEARCH heuristic finds agency within the new generation of mathematicians and scientists who seek to explain learning and knowlegde, as "… a shift from quantity to quality… characteristic of systems thinking" (Capra, 1996, p. 135). Research has shown that negotiation of sociocultural and sociomathematical norms is an ongoing process and that students' participation in the negotiation process crucial. The execution of each mathematical experience is contingent upon children's emerging appreciation of their teacher's and their own roles during mathematics lessons. As classroom norms are negotiated new "relationships" are established and the class co-evolves into new roles with new expectations. As the roles evolve, new norms emerge and so the cyclical and reflexively emergent nature of learning manifests as a process of continuous self-regulation and self-organization.

The SE in SEARCH

In a SEARCH classroom, a problem-centered approach to learning imbued by constructivist perspectives is fundamental. Learning experiences are characterised by notions of partnership, personal agency, individuality, respect for each other, freedom, democracy, child-centeredness, active engagement, developmentally appropriate discourse, and the opportunity to function creatively. These perspectives interweave in "relationship" with each other together to create a learning environment that generates and interconnects a diversity of mathematical development.

Children's analysis and discussion of mathematics that takes place in the SEARCH classroom reflects what it means to "know" and to "do" mathematics and thus what it means to learn mathematics. Sustaining the inter-relational psycho-pedagogical dimensions of mathematical learning is based on (1) replacing competition with cooperation, (2) fostering collaborative cooperation for reaching solutions to social as well as academic problems, (3) respecting each individual's attempt to have an opinion, (4) rejecting coercive authority, and (5) promoting personal agency.

Deriving from principles of "relationships," mathematical learning based on respect (by teacher and students) for student intellectual, social, physical, and emotional development is akin to the self-regulating and self-creative basis of complex adaptive systems. Such an elaboration of the classroom environment resonates with what is portrayed in the writings of Dewey in that the classroom is a place "where the inhabitants - students and teachers alike – [are] invited to find personal fulfilment and social well-being in their daily activity, a place where the ultimate test of knowledge [is] to be its usefulness but where the useful [is] to include the aesthetic, the contemplative and what some would call the spiritual aspects of human experience" (Jackson, 1990, p. xxxvi).

One consistent and enduring dimension in a SEARCH mathematics classroom is a child-centered approach. A child-centered approach is characterized by the encouragement of democratic, caring, ethical, respectful, self-directing, self-organizing, and socially-relational principles. Control and freedom in the classroom is constituted by a democratic outlook akin to Dewey's reform agenda that advocated self-control as the aim of the educative process. Children's freedom to search together to establish autonomy and independence are major facets of learning mathematics. Such is the focus of the SEARCH classroom. The social and emancipatory dimensions of constructing mathematics knowledge is precipitated by a community engendered with a freedom based upon independence as well as inter-dependence. From this perspective, each child's social emancipation is permeated by self-control and respect for others to become fundamental in fostering personal "relationships" with others in the SEARCH classroom.

The AR in SEARCH

In the SEARCH classroom, children's mathematical thinking revolves around efforts to let them negotiate meaning for themselves and amongst themselves, to build upon their previous knowledge, and to be viewed individually as unique learners with unique and interesting ideas. There is a distinct effort by the teacher not to tell children the answer. The aim is to provide a learning environment in which children are unencumbered (by pre-conceived and pre-determined thinking) in their search to construct mathematical meaning.

The emergent nature of "relationships" is also reflected in actively engaging children in their mathematical experiences. The learning environment is strongly activity-based, hands-on, participative, engaging, motivating and interactive. A sociomathematical norm constituted through an expectation that mathematics lessons should be an active use of time, space and thinking to develop mathematical ideas should prevail. Not only is cognitive growth addressed, but also social, emotional, and physical growth. The teacher's role is not to "teach" children but to facilitate experiences that provide active engagement in the construction of mathematical ideas. Learning based upon active engagement develops around a working "relationship" and alliance with the principles of partnership and friendship, not just in an inter-active but also an intra-active sense. Children are expected to utilize all their senses and capacities inter- and intra-actively, in "relationship," in a holistic fashion to think mathematically and to share their ideas.

In light of constructivist perspectives that portray knowing as active, and that new knowledge is based on previously constructed knowledge, the SEARCH classroom rejects structuring mathematics lessons around transmission of instrumental and procedurally rule-oriented thinking. Instead, inter-dependent learning wherein the construction of mathematical knowledge harmonizes in a reflexive "relationship" between group interaction and individual sense making is established. This reflexive view of learning implies that knowledge construction is not purely a personal experience. Rather, learning is an active participation in a dynamically volatile environment wherein effective engagement involves self-directed harnessing of contextual variables and not merely a blindly faithful submission to imposed directives determined by the teacher.

Students need to be encouraged to resolve perturbations with a sense of confidence that the search for solutions is "do-able." Thus student's mathematical sense making is portrayed not only as an active and confident foray into new mathematical conceptualizations but also, and importantly, an expedition supported by conceptualizations which already are constructed as "sensible" and "meaningful." Meaning making and having confidence go hand in hand, each building upon and generating the other. From such a perspective, SEARCH highlights "Active Referencing" as a constructive inter- and intra-activity process which actively (rather than passively) engages previous meaning as reference points to assist in making sense of new concepts.

The CH in SEARCH

In his search to explain human development Piaget's guiding question was, "How does knowledge grow?" In his wisdom, he eventually remarked, "The essential functions of intelligence consist in understanding and in inventing. . . . It increasingly appears, in fact, that these two functions are inseparable" (1971, pp. 27-28). His book "To Understand is to Invent" (1973) is testimony to his revelations that invention is knowledge construction. Children consistently like to be inventive and creative. David Elkind amongst many, argues that there are no limits to a young child's curiosity and imagination. Our job as teachers is to encourage and support the many different ways of thinking that children bring to the learning context. Teachers who orchestrate opportunities for children's creative endeavour during mathematics lessons provide the ideal environment for learning. Such an imperative is based on the assumption that through creating, children invent mathematical meaning.

Considerable time in mathematics lessons should be dedicated to children inventing new ways and devising uniquely creative ways for discussing, validating, expressing, and describing their mathematical conceptualizations. As David Elkind says, the child is most happy to share his or her thoughts if we as teachers show interest in their ideas. We get insight into the child's thinking. If we communicate that we are interested, not in terms of right or wrong, but in a genuine attempt to comprehend what the child is proposing, we are able to promote the child's sense of self-confidence and security in expressing their own and often unique mathematical ideas.

In the SEARCH classroom, children's personal mathematical sense making revolves around an environment that is conducive to spontaneous and creative discussion. Establishing such an environment engenders a patience and tolerance for children's freedom and willingness to speak openly. In order to foster children's capacity to express unique ideas teachers need to allow children a measure of freedom to roam the room with autonomous flexibility and interact with their peers in informal ways. The children's freedom is based upon a sociocultural norm that during group time, as long as they respect each other's attempts to complete the projects assigned, they are free to sit, cluster, or gather to work wherever they wish.

The SEARCH classroom encourages a dialogic community wherein each child has a voice and yet respects the opinions of others. Each child values and respects the contribution made to the classroom by other children. All children are encouraged to do their own thinking and to express their own ideas. It is through the public exposure of their ideas that each child reflects and reconsiders the value and viability of his or her thinking. Classroom lessons become a time for open and candid discussion about mathematical propositions. Children present their propositions for public scrutiny without fear of shame or discredit. Within a forum of open and creative discussion, children's ideas are revised, re-examined, reconstituted and reformulated after hearing different points of view. A climate of creative endeavour overcomes many of the roadblocks that prevent children from making breakthroughs in their mathematical thinking. Instead of floundering in a sea of misconceptions and anxiety, each child engages "play-fully" with mathematical ideas, constantly comparing and contrasting their own thinking with their peer's.

Every attempt for children to express creative mathematical propositions should be encouraged. Unique and inventive ways of "doing" mathematics emerge every day. Such a fluid and inventive learning environment might be considered a recipe for chaos to the unsuspecting. The important issue here is that the model of classroom engagement that fosters self-regulation is not just operationalised during mathematics lessons but implemented for the entire school day. As Jantsch (1980) suggested, as a system changes, it does so by referring to itself. Self-reference is what facilitates orderly change in turbulent environments. "The natural dynamics of simple dissipative structures teach the optimistic principle of which we tend to despair in the human world: the more freedom in self-organization, the more order" (p.40). In other words, the control of the class endures in systemic order because the "relationship" between learning, freedom and autonomy is self-regulating. The driving force for learning, according to the emerging views of New Science, is to be found not in controlled or imposed structures, but in life's inherent tendency to "create novelty, in the spontaneous emergence of increasing complexity and order" (Capra, 1996, p. 228).

Whitehead (1978, cited in Doll 1993) believed that the "ultimate principle" of reality itself was a relationship of becoming and perishing – a relationship of experience. He contended, in contrast with a Newtonian view of an ultimately atomistic and mechanical reality, that reality was a set of relations. In consonance with Dewey and Piaget, Whitehead thought that the pupil's mind was "a growing organism" and that "the only avenue towards wisdom is by freedom in the presence of knowledge" (p.30). For Whitehead, "ideas give power to create, to bring into actual existence an infinitude of possibilities. . . . For this reason it is not only good that we, as teacher and students, throw 'ideas into every combination possible'; it is essential we do so. For in this 'throwness,' meaning, experience, and reality are created" (Doll, 1993, p. 145).

Conclusion

The implications for teaching in a classroom characterized by open, dynamic, creative, and adaptive "relationships" carry a very different imperative when contrasted against current modernist approaches to mathematics education determined by teacher domination and control. A different system is required.

Organizations and business systems outside the education sector are increasingly paying heed to the properties of self-organization and self-renewal. Some theorists have termed these "adaptive organizations," where the task or problem to be solved determines the organizational form (Dumaine, 1991). Such organizations are depicted as avoiding rigidity or permanent structures and instead developing a capacity to respond with greater flexibility to external and internal change. Teams, action, knowledge, expectations, and norms emerge in response to needs. When a need changes, so does the organizational structure.

It is acknowledged that a self-regulating systemic classroom requires a high degree of patience and a considerable tolerance for chaos. Establishing "relationships" and operating close to the border of chaos requires considerable managerial fluidity - rather than managerial rigidity. Teaching this way requires adaptability to operate with constantly emerging divergent ideas – to allow ideas to "spiral off into the unknown, as in the beautiful iterations of complex numbers found on the edge of the Mandelbrot sets" (Doll, 1993, p. 126).

Should classroom teachers continue to deliver mathematics education using traditional methods, "relationships" between students and mathematics will certainly be formed, and following current international trends, the "relationships" will not be positive ones. In light of the New Scientific and post-modernist paradigms, teaching and learning mathematics must enter the 21st century with a new spirit. The SEARCH for good mathematics education might take us forward.

As Dewey says:

One can think reflectively only when one is willing to endure suspense and to undergo the trouble of searching. (cited in Pollard, 2002, p. 4)

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