MCI02517

DEVELOPING INFORMAL WRITTEN COMPUTATION

Alistair J. McIntosh

University of Tasmania

<alistair.mcintosh@utas.edu.au>

Paper presented at the Annual Conference of the Australian Association for Research in Education, University of Queensland, Brisbane, December 2002

Abstract

Recent Federal and state mathematics curriculum documents have shifted from a requirement that all students should learn the standard algorithms for the four operations of addition, subtraction, multiplication and division to stating that all students should develop strategies for calculating mentally and should then acquire secure and understood, but not necessarily standard, written computation methods. While the move towards a greater emphasis on mental computation through the development of strategies is generally well developed in Australia, the implications of 'not necessarily standard written computation methods' has received very little attention. This paper describes preliminary work from a research project designed to explore the interface between mental computation and non-standard written methods. In particular the impact on teachers and students in grades 2 to 4 of at least delaying the introduction of formal written algorithms for addition and subtraction and the development of practical ways of building secure but informal written methods on children's mental computation strategies are described.

Introduction

Since the early 1980s, coinciding perhaps with the publication of the Cockcroft Report (DES 1982), increasing attention has been paid to mental computation in many countries, particularly in the early years of schooling. Three main reasons have been advanced for this increasing emphasis:

- that most calculations are done 'in the head' (Wandt & Brown, 1957; Northcote & McIntosh, 1999);
- that mental computation develops sound number sense (McIntosh, 1990);
- that mental computation promotes success in later work (DES, 1982).

In Australia this emphasis on mental computation has been endorsed by the National Statement on School Mathematics (Australian Education Council, 1991), the National Profiles (Curriculum Corporation, 1994), and in many state and territory curriculum documents.

The official position regarding mental computation is quite clear. While importance is still attached to swift recall of basic addition and multiplication facts, the recommendations go far beyond this; emphasis is to be placed on increasing the range of computations approached mentally at least to two-digit numbers, on the recognition that there are often several strategies for calculating mentally rather than (as is usually the case for written algorithms) the learning of one correct method, and on encouraging children to articulate and discuss their own and other children's mental solution strategies for particular calculations. For example, the *National Statement on School Mathematics* (Australian Education Council, 1991, p. 109) states that:

> Strategies associated with mental computation should be developed explicitly throughout the schooling years, and should not be restricted to recall of basic facts...students should be encouraged to develop personal mental computation strategies.'

This move to greater emphasis on mental computation has met comparatively little opposition.

There is a related change in approach to written computation advocated and described, though less clearly, in official documents. For example the *National Statement on School Mathematics* (Australian Education Council, 1991, p. 109) states that:

> 'The development of flexible computational skills can be inhibited by emphasising the practice of standard paper-and-pencil methods to the exclusion of all other methods. It is far more realistic to use a combination of mental and informal written methods most of the time, with paper-and-pencil recording seen as providing memory support...Consequently less emphasis should be given to standard paper-and-pencil algorithms and, to the extent that they continue to be taught, they should be taught at later stages in schooling.'

This change in policy is much less well known within Australia, and there is little evidence of it being adopted in schools, partly at least because very little guidance is offered to teachers: indeed there has been remarkably little research reported in this area and little experience of the actual benefits and associated difficulties. In particular the interface between 'personal mental computation strategies' and 'informal written methods' is largely uncharted.

Some researchers in the UK, the USA and the Netherlands (Kamii & Dominick, 1989; Hiebert et al, 1997; Carroll& Porter, 1998, Thompson, 1999) have made valuable contributions in this field, but to date there has been little research undertaken to assist teachers in moving forward in this area.

The current project

The current project is a DEST-funded Strategic Numeracy Research and Development Project following a joint proposal from the Department of Education (Tasmania), the Catholic Education Office (Tasmania) and AIST (Tasmania) in conjunction with the University of Tasmania. Thirty-three teachers of Grades 2, 3 and 4 in nine primary schools are involved, covering all three sectors, all areas of Tasmania, and including students from a wide range of socio-economic backgrounds. All schools agreed to modify their teaching of computation during the two-year life of the project.

The objectives of the project were two-fold: first, to investigate the process and the effects on students and teachers of developing informal written computation processes through grades 2 to 4, and, second, to determine which classroom strategies and approaches for the encouragement of informal written computation are most effective in developing students' number sense and computational ability and to analyse critical features of these approaches. In view of the short time span of the project (effectively only eighteen months in the classrooms), it was decided to concentrate on addition and subtraction only.

Each school nominated two Key Teachers who have attended six-half day sessions each year, and have acted as catalysts for maintaining the momentum of the project in their schools while also providing invaluable feedback on successes and concerns. Indeed their comments and discussions have materially affected important aspects of the project.

All teachers in the project attended three half-day sessions each year at which new key ideas were presented and discussed and shaped into practical proposals for the classroom.

Data has been gathered at several points in the project: teacher and student surveys, school recommendations based on teachers' joint experience in the project, tests of children's mental computation ability and video-taped interviews with individual teachers and children.

Stages of the project

Table 1 shows the four stages of the project.

Table 1

Four stages of the Developing Computation project

Stage	Duration	Focus
1	June - Dec 2001	Teachers familiarise themselves with the development of mental computation through strategy approach in their classes. No strictures regarding teaching of formal written algorithms
2	Jan - May 2002	Development of mental computation through strategy approach with new classes. No teaching of formal written algorithms to these classes.
3	May - Oct 2002	Exploration and encouragement of informal written algorithms. Development of mental computation through strategy approach continues. No teaching of formal written algorithms
4	Oct - Dec 2002	Teachers make their own decisions regarding teaching of mental computation, the encouragement of informal written methods, and the teaching of formal written algorithms

Stages one and two, which occupied well over half the time of the project, concentrated only on mental computation: while this appeared to leave relatively little time for the central purpose of the project, which was to look at the development of 'informal written computation', this long and deliberate introduction was critical, both for children and teachers. For children, since the purpose of the project was to observe the process of working on informal written computation with children who have developed strategies for calculating mentally and have a certain level of competence, it was clearly necessary first to concentrate on developing children's mental computation ability.

For teachers the case was similar. All teachers in the project had previously adopted a traditional approach to the development of written computation, in which the connection with mental computation was almost entirely restricted to the children's use of basic facts (single digit computation) in computing the partial sums of a multi-digit addition or subtraction (although several of them had already begun to introduce mental computation activities based on strategies and discussion). This project asked them to adopt an approach that was radically different from the traditional approach to computation in at least three fundamental aspects. First, at least as much attention was to be given to children's mental computation of two digit numbers as to single digit computation. Second, the approach was to emphasise not memorisation of individual facts, but the children's invented or informal strategies for computing the calculations mentally, and their shared oral explanations of their strategies. Third, when numbers became larger than could be dealt with mentally, instead of learning from the teacher one standard computation method which is both alien to children's natural computation processes and unconnected with their mental methods (Plunkett, 1979), children would build on the mental strategies they were using and develop ways of extending these by the use of pencil and paper. Stage one allowed teachers to explore this new approach to mental computation with their existing class in a relaxed way, discussing and appraising the effects with colleagues from their own and other schools. Stage two, starting at the beginning of a new school year, allowed them several months with a fresh class, developing these children's mental computation with an increased confidence and sense of direction. Thus the teachers built up their own confidence both in themselves and in the project in an atmosphere that, while quite new and demanding, was not perceived as threatening.

The reactions of the teachers to stages 1 and 2 were extremely positive. They remarked on the much greater interest and involvement of the children when asked to explain their own methods and an increased competence in discussing and explaining orally. Indeed when one teacher expressed some doubt about the possible reactions of parents, two teachers from different schools remarked that parents had come up to them asking what they had been doing in maths, as their children had suddenly begun to talk freely about numbers and regularly asked them to give them calculations to do mentally. Stages one and two of the project were therefore perceived by teachers as positive and rewarding.

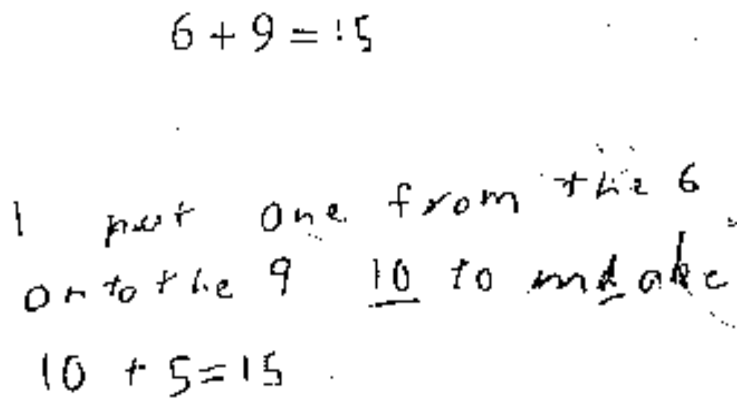
Stage three however raised quite new and potentially threatening aspects for teachers (and indeed was the reason why several schools who were approached originally to join the project, declined to do so). First, it asked teachers to explore 'informal written computation' (a term which it would be fair to say would have been quite unfamiliar to most before the project), and to withhold the teaching of formal written algorithms, a practice which most, if not all, teachers would have considered central to their mathematics programme.

Surprisingly, the holding back from teaching formal written algorithms did not raise anything like the anticipated opposition. It would appear that teachers were satisfied with the tangible improvements in competence and attitude resulting from the greater concentration on mental computation, and were in many cases very aware from their experience that the traditional approach to teaching standard written algorithms, even with an emphasis on understanding and the use of materials such as MAB, often caused great difficulties for children. However the encouragement of 'informal written methods', as advocated in the curriculum documents, constituted a quite foreign and unexplored area of classroom practice.

Early in the project, teachers in all the project schools were asked to give to their classes some calculations which were at the edge of the children's ability to calculate mentally and were asked to invite their children to 'use paper and pencil in any way they found helpful'. The purposes of this were to ascertain the range of ways children interpreted this activity, to gauge the extent to which formal written algorithms had already been met by children, to examine the range of informal written methods used by children before any classroom discussion or deliberate teaching of such methods, and to make some initial hypotheses as to how teachers could build on children's informal written work. The remainder of this paper describes the range of responses by children, and describes the resulting course of action proposed to the teachers.

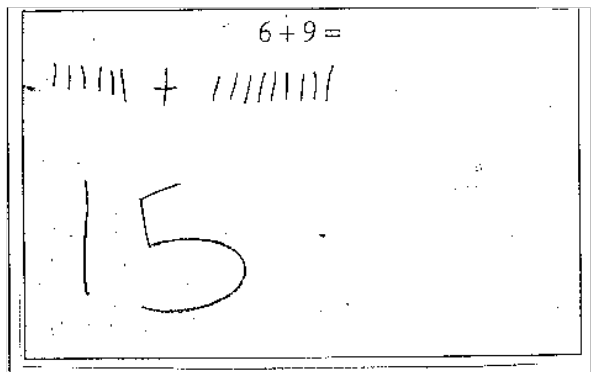
Children's use of paper for calculating

The children's responses could be put in four categories:



1. *Written explanations of the way in which the child had in fact performed the calculation mentally*. Figure 1 gives one example.

Figure 1:A written explanation of a mental computation strategy



2. *Marks representing individual units, which were then counted*. Simple examples are given in Figures 2 and 3 (Figure 3 showing a significant step forward from Figure 2).

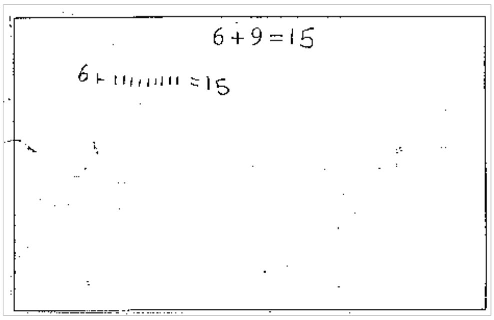


Figure 2: Marks representing all individual units to be counted

Figure 3: Recognition that there is no need to represent the 'six' by individual units

The most ambitious use of marks to represent individual units was exhibited by a child calculating 53 x 24. The child drew 53 circles covering the page, and had then started to mark 24 lines round the inside of about half the circles before abandoning the attempt.

2. *More sophisticated informal use of paper to aid in calculation*. These were relatively uncommon, but varied and idiosyncratic and often involved partial calculations with lines showing the connections. One example, the use of a blank number line (which had earlier been modelled by the teacher) is given in Figure 4. Note that the source of the error can be easily located.

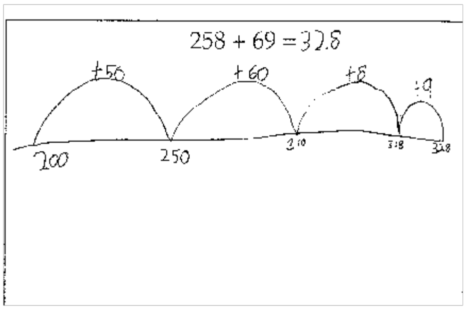


Figure 4: Use of a blank number line to aid (or explain) calculation

3. *Attempted use of the formal written algorithm*. Clearly many children had been exposed to these. What was amazing was the extent of the misunderstandings revealed, which confirmed the need for exploring alternatives. One example is given in Figure 5. This is by no means the most confused, though it reveals confusion between addition and subtraction algorithms, and provides an answer which is clearly too small. What is revealing is that the child has written the correct answer (it is not unreasonable to suppose that this was calculated mentally) but has rejected this in favour of the 'officially correct' way to calculate.

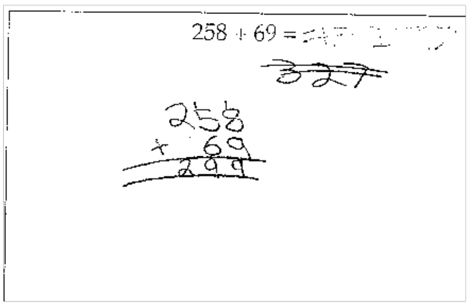


Figure 5: An attempted use of the formal written algorithm

From informal to formal written methods: a proposed pathway

The second and fourth categories were clearly not useful as stepping-stones from mental to informal written calculation method, while the third would involve imposing particular recording forms on children, which would run counter to the intention to build on children's own mental methods. The first category, written explanations of mental methods, while much the most common response by children, seemed at first not relevant. However further consideration suggested that it could be an excellent first stage in the process and one which could be used as a bridge. Its strengths as a springboard are that it relates directly to mental methods used by individual children, and it is by its nature fully understood by the child. The challenge was to find ways of extending it, that is, to move it from representing explanation of a written method to becoming a method for calculating on paper. Table 2 shows the six-step process that was proposed by the author and adopted by teachers.

Table 2

Six stage process for moving from mental to written computation

Stage	Process
1	Strengthen children's mental computation with two digit numbers.
2	Encourage children to explain their methods using pencil and paper.
3	Compare, discuss and so refine their written explanations.
4	Strengthen this method with further calculations of similar difficulty
5	Extend its use by adapting it to calculate more difficult examples
6	Consolidate it as an 'understood, secure written method'.

Step three is a critical stage. As an example Figures 6 and 7 show two examples of explanations of computation methods for the same problem by different children.

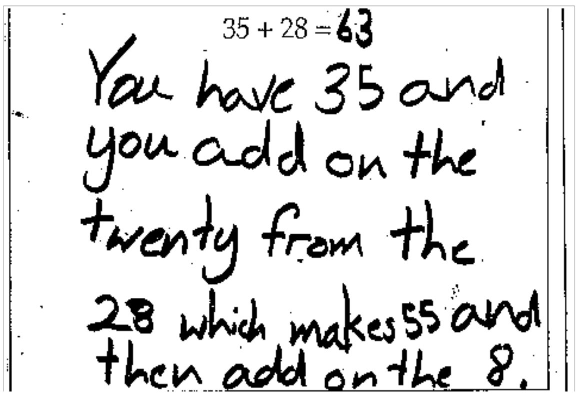


Figure 6:

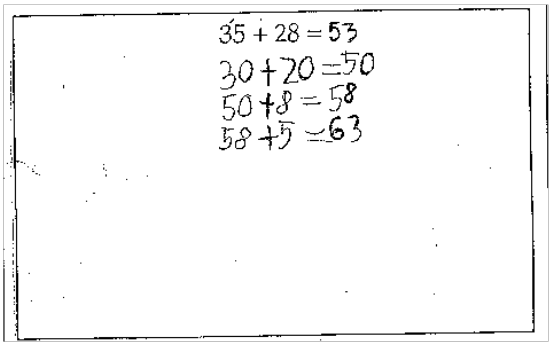


Figure 7:

While both are clear and correct, the form of recording used in Figure 7 is more concise and capable of being extended than that of Figure 6. Thus children were encouraged to work on their explanations with the help of the teachers, using three criteria for assessing their value based on those proposed by Campbell, Rowan & Suarez (1998): *efficiency* (Is it a concise and effective method?), *validity* (Is it mathematically sound?), and *generalisability* (Could it be used on a wide range of computations, especially with larger numbers?).

After initially expressing some nervousness at the start of stage 3, teachers have been increasingly and without exception strongly supportive and enthusiastic, because of the perceived impact on children's confidence and enjoyment. Stage 3 onwards was seen to have strong links with 'conferencing' with which they were familiar from their language teaching, and so it was felt to be quite a natural and comfortable process.

Several schools held parents' sessions and reported little or no opposition, because parents appreciated the growth in confidence and competence of their own children.

Teachers comments and opinions to date

Towards the end of the project, teachers gave a composite response from each school to a series of questions in which they were asked to use their experiences of the project to give advice to other schools. The following is a summary of the responses from the nine schools.

Do you recommend teaching mental computation via a strategies approach? Why? From what grade?

Schools were unanimous in supporting this, citing children's increased confidence, greater understanding of number and place value, awareness of different ways of calculating, and acquisition of a range of strategies. Most schools saw Prep or Kindergarten as the time to start for most children.

Do you recommend teaching standard written algorithms? Why? From what grade?

Again schools were unanimous in supporting this, mainly on the basis of 'need-to-know'. However all agreed that their views had been considerably modified by the project. Whereas before most said that they would have advocated the introduction of standard written algorithms from Grade 1 or 2, they now suggested Grade 4 as more appropriate. They saw the increased understanding of numbers and place value acquired through concentrating on mental computation provided a much more secure basis for a later introduction of the written algorithms.

Do you recommend that children develop some informal written algorithms? Why? From what grade?

Schools were unanimous in supporting this. They stated that it helped children to develop understood ways of dealing with larger numbers, that seeing their thinking strategies in writing helped them to clarify their mental strategies, and that the presence of written work helped teachers to 'see' children's thinking and to work on misunderstandings. Most schools saw Prep or Grade 1 as the time to start for most children.

The purpose of the project was not to find or propose a specific solution, but to explore with teachers the consequences for themselves and their children of moving in this direction, and to provide some insights from their experiences that may help other schools as they face the same situation. The project sends out two clear messages. First, it confirms the recommendations quoted earlier, that the introduction of mental computation through a strategies approach is enormously valuable and empowering for children. Second, it makes clear that this change has wider implications for changes in our approach to teaching written computation, which we should not ignore.

References

Australian Education Council (1991). *A national statement on mathematics for Australian schools.* Melbourne: Curriculum Corporation.

Campbell, P.F. Rowan, T.E., & Suarez, A.R. (1998). What criteria for student-invented algorithms? In L.J.Morrow & M.J Kenney (Eds.) *The teaching and learning of algorithms in school mathematics: 1998 yearbook of the National Council of Teachers of Mathematics.* Reston,VA: NCTM.

Carroll, W.M., & Porter, D. (1998). Alternative algorithms for whole-number operations. In L.J.Morrow & M.J Kenney (Eds.) *The teaching and learning of algorithms in school mathematics: 1998 yearbook of the National Council of Teachers of Mathematics.* Reston,VA: NCTM.

Curriculum Corporation (1994). *Mathematics - A curriculum profile for Australian schools*. Melbourne: Curriculum Corporation.

DES (Department of Education and Science), (1982). *Mathematics counts* (Cockcroft Report). London: HMSO.

Hiebert, J et al. (1997). Making sense: Teaching and learning mathematics with understanding. Portsmouth, NH: Heinemann.

Kamii, C. & Dominick, A. (1997). To teach or not to teach algorithms. *Journal of Mathematical Behavior, 16*(1), 51-61.

McIntosh, A.J. (1990). Becoming numerate: developing number sense. In S. Willis (Ed.) *Being numerate: what counts?* Hawthorn, Victoria: Australian Council for Educational Research.

Northcote, N & McIntosh, A. (1999). What mathematics do adults really do in everyday life? *Australian Primary Mathematics Classroom, 4*(1), 19-21.

Plunkett, S. (1979). Decomposition and all that rot. *Mathematics in School*, *8*(3), 2-5.

Thompson, I. (1999). Written methods of calculation. In I. Thompson (Ed.) *Issues in teaching numeracy in primary schools*. Buckingham: Open University Press.

Wandt, E. & Brown, G. W. (1957). Non-Occupational Uses of Mathematics: Mental and Written - Approximate and Exact. *Arithmetic Teacher*, *4*(4), 151-154