"Discussion vs. Recitation" and "Mathematical Discussion": What might these look like

in middle school mathematics classrooms and what difference could they make?

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Introduction

Many people in the mathematics education community are using the word "discussion" but are only focusing on how this might relate to the content that students are learning. By making 'mathematical understandings' primary in our focus on mathematical discussions, we are not taking advantage of what a distinction between recitation and discussion can offer to us as it relates to the beliefs students may be forming about what it means to know and do mathematics. This is a very difficult shift for people to make because conventionally schooling has emphasized content and only content. Here my *primary* interest is not with "mathematical understanding" (although I do address that briefly) so much as different levels of engagement and ownership students are allowed within mathematics classrooms. My purpose in describing the two middle school mathematics classrooms in this paper is to shift the focus of the conversation about discussion in mathematics classrooms toward looking closely at patterns of authority and epistemological stances, often a less central topic in mathematics education literature.

In a study I conducted on the discourse in two "reform-oriented" middle school mathematics classrooms , one of my findings indicated that there were differences in the ways the teachers engaged students in mathematical ideas. Some of these differences were related to the way each teacher positioned him/herself with respect to the external mathematical authority and the epistemological stance s/he seemed to advocate. Teaching in the ways that the *Standards* advocate calls for a transformation in the way teachers orchestrate the discourse of their classrooms. In fact, a shift away from the "transmission model of communication" indicates a different role for the teacher in the classroom, among other changes. Here I focus on illustrating some of the shades of "discussion" and "recitation" within the context of these two mathematics classrooms, illuminating these differences to show that the discourse of a classroom can advocate different patterns of authority and different epistemological perspectives. I also use the idea of a "mathematical discussion" to examine more closely the mathematics in the classroom transcripts. I show that a "mathematical discussion," as defined by Pirie and Schwarzenberger , could be happening in both classrooms. However, a "discussion" as opposed to "recitation" only seems to be occurring in one of the mathematics classrooms. The difference is in the manner in which student ideas are pursued, who is allowed to talk and when, the kinds of contributions the students are allowed to offer, etc. All of these aspects of classroom talk are pivotal in determining the teacher's and student's roles and in influencing what students believe it means to explore mathematical ideas.

This article is divided into four parts. In the first part, I offer pertinent literature and define the terms "recitation," "discussion," and "mathematical discussion." I then describe the context, data and analysis used for this paper. To illustrate what shades of "recitation" and "discussion" looked like in two "reform-oriented" middle school mathematics classrooms, I offer extended transcripts from each classroom. In the fourth and final part of this paper, I look across the two examples and point out differences that occurred, showing how the talk patterns in the classrooms are advocating different roles for students and different epistemological stances.

Discussion, Recitation, Mathematical Discussion

The authors of *Standards* document pointed out that inherent in the discourse of a classroom are related issues about what it means to "know" and "do" mathematics, as well as issues about power and authority. In many mathematics classrooms, the textbook and teacher are typically viewed as the authority. However, this document encouraged reasoning and evidence as the basis for what counted as legitimate mathematical activity, drawing the locus of authority away from the teacher and the textbook. The teacher's role was described as that of a facilitator, one who initiated and orchestrated student discourse to foster student learning. Some of the things a teacher was encouraged to do included:

- Decide what to pursue in depth from among the ideas that students bring up during a *discussion*;
- Monitor students' participation in *discussions* and decide when and how to encourage each student to participate (p. 35, italics added).

While "discussion" is only part of the discourse of a classroom, many of the same concerns about discourse (e.g., issues related to what it means to "know" and "do" mathematics) are also apparent in the patterns of interaction in the mathematics classroom. By choosing the word "discussion" this document seems to be advocating a particular type of interaction in mathematics classrooms. What this word means is not apparent; how a teacher might orchestrate these kinds of interaction is also not clear.

Discussion/Recitation

There is literature that addresses, more broadly, how a teacher might establish his/her role in the classroom. Barnes , for example, distinguished between two types of teacher-roles in the classroom: "Replying" and "Assessing" (p. 112). When Replying, the teacher takes the students where they are at, even though s/he may wish to broaden and shape the understanding; when a teacher Assesses, s/he holds the student up to external standards. Barnes contends that Replying sets up a more collaborative environment and gives the learner confidence in actively interpreting subject matter.

This distinction of Reply and Assess, Barnes recognizes, is similar to Simon and Boyer's Understand and Judge. These dimensions are defined by the following (p. 112):

Figure 1

Understanding vs. Judging

UNDERSTANDING ß -------VERSUS--------à JUDGING

-accepts idea -positive evaluation ('good')

-clarifies understanding -negative evaluation ('wrong')

-reflects or paraphrases ideas -counter proposals, suggestions

-expands on someone's ideas -implies judgements

(should, should never, you

always, everybody ought)

According to Barnes, both roles are essential parts of teaching because one role

values the student's ideas and his attempt to make sense of the world; the other puts his ideas up against the larger community's standards. He also claims that if a teacher stresses Assessment over Replying, the students may focus on "externally acceptable performances, rather than […] trying to relate new knowledge to old" (p. 111).

When a teacher takes on a particular part in the communication process, the students seem to tacitly understand that they are to play a particular role in response to these mutually constructed norms. Barnes (1976) recognizes this when he points out that teachers who Assess/Judge more often have students who "present;" teachers who Reply/Understand tend to find students "sharing." The difference is in the predominant form of communication students may employ. When students "present," they use "final draft" talk. Final draft is the style students use when they are showing the teacher that they have the "right" answer—it is "a formal completed presentation for a teacher's approval" (p. 108). In contrast, students who "share" use "exploratory talk" which is more improvised because students are rearranging thoughts. Barnes recognizes that exploratory talk and final draft talk are "essentially a distinction between different ways in which speech can function in rehearsing knowledge—in exploratory talk, the learner himself takes responsibility for the adequacy of his thinking; final draft looks toward external criteria and distant, unknown audiences" (p. 113).

The Assessment role of teaching is aligned with the domination of recitation in the classroom. In recitation:

> the *teacher* follows a pre-scripted checklist of questions, information and concepts, sticks closely to a preplanned list of test questions rather than responding to the substance of what students say […] *students*typically give short, frequently tentative answers […] topic shifts can be abrupt as the teacher moves down a checklist of important points, as it were, making sure students remember them […] *discourse* is frequently choppy and lacks coherence .

In contrast, discussion is defined by its tightly interwoven comments and responses (Nystrand, 1995). Teachers make space for student ideas and the discourse is more balanced, although the teacher's voice is still crucial. Dialogic instruction involves fewer teacher questions and more conversational turns as teachers and students contribute ideas, make points, and ask questions. It is "less prescriptive since the actual conduct, direction, and scope of the discussion depend on what students as well as teachers contribute and especially on their interaction" (Nystrand, 1995, p. 17). Knowledge tends to unfold as the discussion proceeds; both personal and school knowledge are recognized and drawn upon. Student ownership is important.

When the organization of instruction is examined, we can find pedagogical "contracts" which define roles in the classroom. These instructional arrangements, Gutierrez shows, determine the "discourse patterns, rules of participation, and the nature of classroom interaction" (p. 15). In recitation, the Initiate-Respond-Evaluate exchange so common in classrooms has several key provisions: *knowledge* is rigid, impartial, autonomous; the *source of knowledge* is the textbook and the teacher—it is given; *the teacher's role* is to direct answers, introduce all topics, ascertain what is worth knowing; the epistemic *role of students* is restricted to remembering information from others, not figuring things out or initiating any knew knowledge (Nystrand, 1995, p. 16).

Nystrand (1995) associates the epistemological stance of "objectivism" with "recitation" because knowledge is "given." The focus is on "transmission" of knowledge from those who "know." This perspective is not in conflict with the Platonist view of mathematics because of its focus on mathematics as "a monolith, a static immutable product" p. 10). In a Platonist view, mathematics is "a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning" (Ernest, 1988, p. 10). Mathematics is something that exists and is there to be discovered. It is not a human creation. Although researchers have been inconsistent in showing the relationship between teacher's declared beliefs about the nature of mathematics and the teacher's instructional practice (Thompson, 1992), Gutierrez seems to indicate that they are very much connected in her claims about the key provisions associated with recitation. Not only does she describe how it portrays knowledge, but she also addresses how it defines the teacher's and student's roles in the classroom. The dominant view of how mathematics should be taught that would naturally follow this Platonist view of mathematics is "*content-focused with an emphasis on conceptual understanding*" . This view is characterized as instruction that "makes mathematical content the focus of classroom activity while emphasizing students' understanding of ideas and processes… it emphasizes students' understanding of the logical relations among various mathematical ideas and the concepts and logic underlying mathematical procedures" .

The discourse "contract" of <u>discussion</u> involves: on-the-spot grappling with ideas and problems which were not determined by the teacher; *knowledge* may come from students (and from their experiences) and is generated, and co-constructed among participants in the discussion; *students* figure things out; the *teacher's role* is to facilitate, guide, explore, anticipate, and analyze student responses. The epistemological stance associated with "discussion" is dialogism because "knowledge emerges from the interaction of voices" (Nystrand, 1995, p. 19) and the source of valued knowledge includes students' interpretation and personal experience. In this case, the focus is on the "transformation of understandings." "Dialogism" seems to be more aligned with a "problem solving view" of mathematics because of its emphasis on mathematics as a human creation and invention and as a process of creating (not as a finished product). In this perspective, mathematics is viewed as:

> [a] continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus mathematics is a process of enquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (Ernest, 1988, p. 10).

The view of how mathematics should be taught associated with a "problem solving" conception of mathematics is the "*learner-focused*" view (Kuhs and Ball, 1986). Students are actively involved in doing mathematics and are held liable for assessing the acceptability of their own ideas. The teacher is viewed as a "facilitator and stimulator of student learning, posing interesting questions and situations for investigation, challenging students to think, and helping them uncover inadequacies in their own thinking" (Kuhs and Ball, as reported in Thompson, 1992, p. 136).

How does the pedagogical format affect what students can learn? Nystrand (1995) claims that in <u>recitation</u> students are "merely doing school" (p. 17), going through the motions of trite and procedural interactions. This seems to be similar to Edwards and Mercer's (1987) "ritual understanding": students learn the rituals of what they are supposed to do and say rather than taking away the reasons for what they are doing. <u>Recitation</u> centers more on what Lotman calls "univocal" which focuses on the "accurate transmission of information." In contrast, <u>discussion</u> treats utterances as "thinking devices," allowing students to engage in "exploratory talk." Barnes, too, contends that what is learned is different in each mode, making the difference between rote learning and understanding.

Many authors point out that most classrooms operate somewhere between a recitation and a discussion. Also, as Cazden contends, one lesson can move from recitation to discussion within a matter of moments and sometimes the activity that is being engaged in determines the format of the lesson. For example, practicing times tables would require a different format than exploring the concept of multiplication in terms of repeated sets, area, etc.

These two case studies that follow will offer illustrations of some of the shades of gray that exist between discussion and recitation.

Mathematical Discussion

More specific to mathematics education, Pirie and Schwarzenberger outline what they believe to be the components of a *mathematical* discussion, focusing mainly on its use in small group interactions. They define this as:

It is purposeful talk

> i.e., there are well-defined goals even if not every participant is aware of them. These goals may have been set by the group or by the teacher but they are, implicitly or explicitly, accepted by the group as a whole.

on a mathematical subject

> i.e., either the goals themselves, or a subsidiary goal which emerges during the course of the talking, are expressed in terms of mathematical content or process.

in which there are genuine pupil contributions

i.e., input from at least some of the pupils which assists the talk or thinking to

> move forwards. We are attempting here to distinguish between the introduction of new elements to the discussion and mere passive response, such as factual answers, to teachers' questions.

and interaction

i.e., indications that the movement within the talk has been picked up by other

> participants. This may be evidenced by changes of attitude within the group, by linguistic clues of mental acknowledgement, or by physical reactions which show that critical listening has taken place, but not by mere instrumental reaction to being told what to do by the teacher or by another pupil (p. 461, italics in original).

In general, students participate in mathematical discussions by "making conjectures, presenting explanations, constructing arguments, etc. about mathematical objects, with mathematical content, and towards a mathematical point" .

Pirie and Schwarzenberger use this definition to locate mathematical discussions and to make inferences about the mathematical understanding that the students are displaying in the talk. To do this, they distinguished between to different kinds of statements: reflective and operational. Reflective statements describe concepts and the connections between them and are linked to relational understanding; operational statements focus more on actions and are related to instrumental understanding. The authors claim that a further distinction is needed between the language being used and the statements being made. "Reflective statements can be made in ordinary language and conversely new mathematical language can enable operational statements to be made in new and powerful ways" (p.

466). They describe this distinction as important because people may think a student is not understanding something relationally if they are speaking in ordinary language.

Lo, Wheatley & Smith recognize this definition in their work, but offer their own definition in focusing on mathematics *class* discussions. Their main goals were to "characterize the potential learning opportunities and to infer student constructions of mathematical meaning in class discussions" (1991, p. 1) and "to understand the role of class discussion in the construction of mathematics relationships" (1991, p. 32). In another article they discussed one student’s "participation, beliefs, and arithmetic development in mathematics class discussions throughout the school year" (p. 45). However, they reported the student’s beliefs about effective sharing and disagreement. They did not address how using a discussion format may have influenced his views about what it means to know and do mathematics. Their conclusions (1991) indicated that class discussion is a rich environment for mathematics learning because "students had a more elaborated and integrated thinking schema and greater awareness of their own thought processes…were able to make sense and communicate their mathematical ideas more effectively" (pp. 20-21).

While this paper draws from these definitions to discuss aspects of the mathematics that appear different in the two classrooms, my focus is not on student understandings per se. Instead, I use these definitions to show that in both classrooms all of the components of a ‘mathematical discussion’ seem to exist. However, I will show that in one classroom many of these aspects appear to be part of the teacher’s role, not the students.

Background and Context of the Study

Data Sources and Analysis

This paper draws from a larger database that I acquired during almost three years of work in two 8^{th} grade mathematics classrooms. The two teachers, Josh and Karla, whose classrooms I worked in were involved in piloting the NSF-funded Connected Mathematics Project curriculum. One of each teacher’s classes was the focus of this study: Josh’s first hour class and Karla’s second hour. As a participant observer in the classroom, I observed 2-5 times per week in each classroom and took detailed field notes on my laptop. In addition, I video- and audio- taped in the two classrooms during a few of the algebra units used in the two classrooms, including *Moving Straight Ahead* , *Thinking with Mathematical Models* and *Say It with Symbols* .

I interviewed most of the students in both classrooms 4-6 times during the final school year, posing algebraic tasks to students in small groups and asking them about their classroom experiences. The teachers also were interviewed four times during my final year in their classrooms. These interviews focused on their backgrounds, their perspectives on their own role in their classrooms and how they thought about establishing and maintaining the norms (e.g., the responsibilities, rights, roles, and expectations) in their classrooms.

The transcripts I use here came from March, 1998, when students were working in the last algebra unit, *Say It with Symbols*. Because the two teachers paced themselves differently, I have very few lessons captured where both teachers were focusing on the same pages in the same unit. The excerpts I have chosen to illustrate students’ mathematical engagement in the two classrooms are representative of the type of interactions that took place whenever the classes was "exploring" something in each class. Even though both teachers used the same textbook, the explorations that took place in each classroom were different. At the very least, the amount of time each teacher took to explore mathematical ideas was different. There were other aspects of the interactions, however, that felt different to me. Noticing this led me to look more closely at what was happening in the discourse of the classroom, focusing on the distinctions defined earlier (i.e., recitation, discussion and mathematical discussion).

Examples from the Classrooms

The Mathematical Background for the Observations

Prior to these classroom excerpts, students have worked with the following mathematical ideas: 1.) order of operations, 2.) equivalent expressions (using an area model, which introduced the terms "factored form" and "expanded form"), 3.) algebraic properties (i.e., distributive, commutative), and 4.) writing quadratic expressions. During the following observations, students had just finished solving linear equations and were beginning to solve quadratic equations. Problem 4,4, the topic of the class period, is given below in Figure 2. While Karla's class actually spent over two and a half class periods on this topic, I have chosen one "brief" segment of transcript to illustrate the exploration that took place. In Josh's classroom, these same topics were the focus of one class period.

Figure 2

4.4 Solving Quadratic Equations

Different ways of thinking about a problem can lead to different methods for solving it. For example, finding the *x*-intercepts of the graph of $y = x^2 + 5x$ to find *x* values is the same as solving the equation $x^2 + 5x = 0$.

In earlier units, you solved quadratic equations by using tables and graphs. For example, to solve $x^2 + 5x = 0$, you can trace a graph of $y = x^2 + 5x$ to find the *x* values for which $y = 0$. Or you can make a table of values and look for the *x* values that correspond to a *y* value of 0.

<graph of parabola is given;

x-intercepts are labeled as ordered pairs>

x	***y***
-7	14
-6	6
-5	0
-4	-4
-3	-6
-2	-6
-1	-4

0	0
1	6
2	14
3	24

The solutions to $x^2 + 5x = 0$ are called the **roots** of the equation $y = x^2 + 5x$. A quadratic equation must have zero, one or two roots. If r is a root of an equation, then the point (r, 0) is an x-intercept of the graph.

Algebra provides important tools that can help you solve quadratic equations such as $x^2 + 5x = 0$ without using tables or graphs. This problem illustrates a symbolic method for finding roots of some quadratic equations (from *Say it with Symbols,* p. 57).

Karla's Classroom Example

Prior to the transcript below, Karla had students using the tables and graphs on the graphing calculator to find where $y = 0$ (or the x-intercepts or solutions) for some given quadratic equations. They went through a couple of these as a class. Students shared how they found the solutions using the graph and/or table, depending on which representation they found most helpful. During this discussion, Mark told Karla that he could find the x-intercepts using just the expanded form of the equation $y = x^2 + 5x$.

Exploring Solutions to Quadratics Using the Expanded Form of an Equation

Karla: […] Mark has now thrown something out on table- thinking about, is there a way to do this without the table and graph since the point of this unit, Max, is symbols. Mark, your idea was?

Mark: That the 5*x* is, you make that, if you make that just the opposite, then that could be one of you- your *y*–intercepts. If it was negative 5*x* then that would be five.

Karla: So, you're wondering if maybe there's some way of looking from the equation, you could have gotten this [the solution]. [writing on the board] Dale?

Dale: In the *x*-two, since there's like no number [meaning the coefficient of the term was one], you can kinda assume it's zero, zero.

Karla: [writing on the board] Since there's no number, you're assuming then that [one of the *x*-intercepts] might be zero?

Dale: Yeah [inaudible] equation *x*-two that's [inaudible]

Karla: Can you say that again?

Dale: When it's like y equals x-two, just that equation, that's the only spot where it went [inaudible]

[Dale is adding the fact that there needs to be another x-intercept (in addition to the one Mark just gave). He knows this from his experience of working with the graph of $y = x^2$].

Mark: It would have to be [overlapping-inaudible]

Karla: Oh, yeah. We graphed this one a lot, just the basic old parabola [i.e., $y = x^2$]. When it was just that x-squared, it did cross at zero, zero. That's the one I made you graph by hand before we got the graphing calculators out again. Hhmm [pause] Let's try some more and see if Dale and Mark are on to something here.

Without Karla eliciting the suggestion, Mark offered that he had a way to find the solutions without using the graphing calculator. In doing so, Mark stated that one of the "y-intercepts" would be the opposite of "5*x*." Technically, he should have used the term "x-intercepts" and the solution should have been the opposite of 5, not 5*x*. However, since students were in initial stages of trying to make sense of solving quadratics, Karla may not have thought it was an appropriate time to be concerned with the language because she was trying to understand Mark's ideas. Karla recast what Mark said, making the statement more general—what he was actually doing was trying to figure out a way to get the answer from the expanded form of the equation. Dale offered another, related, idea. He remembered manipulating the equation $y = x^2$ and that the solution had been (0,0). He assumed that the solution (0,0) in this equation was related to the fact that there was an x-squared with "no number" in the equation. Karla reformulated what Dale said and then ended with a drawn out "Hhmm… Let's try some more and see if Dale and Mark are on to something here." Essentially, she was inviting students to explore possibilities. Charity became excited and exclaimed:

<u>(Continued)—Exploring Quadratics of the Form (x^2 + bx)</u>

Charity: They are! Ms. Delmont-

Karla: What?

Charity: They are.

Karla: Why are they on to something?

Charity: 'Cuz I just tried it, um, the same thing but with 3*x* and it's the same thing.

Karla: So, let's go to our y equals [on the graphing calculator] and let's try Charity's [$x^2 + 3x$]. First, would we agree this is going to be a parabola?

Mark: Yeah.

Karla: [comment about tripping over the cord for the overhead projector] Would you agree that this is going to be a parabola?

Max: Yeah.

Karla: Um, tells me it's quadratic. What's telling me it's gonna be quadratic?

Max: Because it looks the same as the [inaudible] except you changed the numbers.

Karla: It's got the *x*-squared, it looks the same; it's got the three there instead. With that in mind, Mark and Dale, tell me where you think it's gonna cross before I look at it.

Mark: Negative 4*x* and zero, zero. [Mark continued by saying that he had gone on to another equation and then answered the original question]… zero, zero, and zero, negative 3.

Ariel: Ms. Delmont [inaudible] zero, zero [inaudible] squared

Karla: That's what Dale's suggesting. We've got just a plain old *x*-squared, Dale's suggesting it's gonna cross at zero, zero. And Mark's saying it's gonna do the opposite of this one [points to the coefficient "b" in the equation]. So we're guessing, zero, zero and negative three, zero? And, Mark, you're also trying this one?

[Mark told her he had already done that one and moved on to the form x^2 – b*x*-- Karla continued with the x^2 + b*x* form and had students check their predictions on the graph and table of their graphing calculators. Students confirmed that they were correct.]

Karla told the class that they were going to start with Charity's example and asked Mark and Dale to tell her where it was going to cross before they actually put the equation into the graphing calculator. Mark appeared to give the wrong answer, but then clarified by indicating that he had already moved on to another problem of that same form. Mark wanted to move the conversation to the form x^2 - bx, but Karla appeared to ignore Mark's comment as she went back to the first form (i.e., x^2 + b*x*). Soon Charity indicated that, like Mark, she had begun to explore the next form of quadratics. This became the next topic of discussion:

<u>(Continued)—Exploring Quadratics of the Form (x^2 – bx).</u>

Mark: If you're adding a negative *x*-

Charity: If you add a negative *x*, *x* will be positive

Mark: Ms. Delmont-

[other students talking, too-inaudible]

Karla: Tell me about negatives, Charity?

Charity: If you add-

[Karla interrupted and asked Kevin to be on task]

Charity: If it's a negative *x*, it's gonna be a positive.

Karla: Give me an example so I know what you're-

Charity: Um, *x* – um, *x*-two plus negative 3*x*.

Karla: And, where is this one gonna cross?

Charity: Um, when *y* is at zero, it would be at 3, cross at positive 3.

Karla: At positive three?

Charity: Yes.

Karla: What about this *x*-squared idea that Dale gave us? Is it still at zero, zero?

Charity/Mark: Yeah/Mmhmm.

Karla: Let's see if it does. Charity's probably right. I just inserted a negative and left everything else the same [in her equation on the overhead graphing calculator]. It's still a parabola?

Fs: Yeah

Max: You can't see until you graph it. Oh, [inaudible—laughing]

Karla**:** Charity thinks it's gonna cross at zero, zero and positive three.

Barbie: It does!

Karla: Looks like it. Check it on my table just to be sure.

[Students are talking while Karla changes her graphing calculator screen to show the table of values for the equation.]

Karla: Hhhmm, it does.

Charity began this segment of discussion by telling Karla that "if it's a negative *x*, it's gonna be positive." Karla asked her to ground her claim in an example, so that she would understand what Charity meant. Karla returned to Dale's idea and asked if zero, zero was a solution, too. She used the overhead graphing calculator and checked the solution on both the graph and the table. Mark then moved the conversation to another case:

(Continued)—Exploring Quadratics of the Form ($-x^2 + bx$).

Mark: If you have like a negative *x*-squared plus just a positive 4*x*, it's gonna be a positive 4 and zero? [Mark is asking this question because he entered the expression $-x^2 + 4x$ into his graphing calculator and the solutions he found in the table did not result in positive 4.]

Max: Then why is it a zero with the *x*-squared?

Mark: Plus 5*x*, it'll still be a positive 5*x*

Karla: It just doesn't do the opposite?

Mark: Mm-mm.

Karla: Let's look at that one.

Max: Could you like write down what she just said?

Charity: It's [the graph] gonna be an up- It's gonna be an upside down, like thingymajigger.

Ms: Negative [inaudible]

Karla: What do you mean, an upside down thingamajigger?

Charity: Upside down [motions with her finger an upside down U]-

Barbie: Parabola?

Charity: Yeah.

[others talking, inaudible]

Mark: A hill thing.

Karla: How do you know it's gonna be a hill thing?

Barbie: Because-

Mark: [inaudible]

Barbie: Yeah

Charity: [laughs]

Karla: Yep, like this hill thing [the graph of the equation appears on the overhead screen].

Fs: Ooo.

Karla: Oh.

Charity: Yep.

Karla: That did something different, though.

Fs: It crosses at zero, zero and-

Mark: Five, zero.

Charity: It crosses at zero-

Kevin: The table, look at the table.

Karla: Okay.

Kevin: It’s way different.

Karla: Tell me about the table

Kevin: It’s like way different

Fs: Yep

Darla: So if it’s a negative *x*-squared, then [inaudible]

Mark asked Karla if the solution was going to be positive this time instead of the opposite like they had found in the other cases. Karla did not offer an explanation. Instead, she proposed a question that made Mark’s claim more general; that is, instead of thinking about the solutions only in terms of whether they were positive or negative, she pushed them to think about the solution as no longer being the "opposite" of the coefficient. She told students to "look at that one." Charity noticed right away that the graph was different—it was now an "upside down thingymajigger." Karla asked her to clarify what she meant. Charity used her fingers to motion the shape and Barbie offered the word "parabola," which was confirmed by Charity. Mark re-named the shape as "a hill thing" and Karla repeated this phrase in her next question. This word appeared in past class observations to indicate one type of quadratics (i.e., those with negative *x*-squared terms). This may have been the reason Karla appropriated this instead of "parabola," or she may not have heard the female student offer "parabola." Karla then turned the conversation to organizing all the cases that they had presented:

<u>(Continued)—Organizing the Cases</u>

Karla: I think I gotta figure all this out. Let me write down what you guys found. [overhead screen is put up so she can write on the board] It’s kinda all over the place, let’s try to put it all together, what we’ve got. Let’s see, how about we look at the ones that had positive *x*-squared plus a positive *x*-

Fs: Zero [overlapping voices]

Karla: -and find out what we’ve got

Charity: Zero, zero and the negative

Karla: An *x*-squared and 3*x*, *x*-squared and 4*x*, *x*-squared and 5*x* [writing on board-long pause]. I think those are the ones that we looked at, right?

Ms: Yeah

Karla: And when those three were equal to zero [writing-sets all three expressions equal to zero], the *x*-intercepts- we always had a zero, zero [writing these as ordered pairs]. Why did we think we were always going to have a zero, zero?

Dale: Because of the *x*-squared.

Karla: Because of the *x*-squared? [writing] And we already knew the *x*-squared was gonna cross right there [writes intercept for each equation-long pause]? And then Mark, you started us out thinking about this one [the examples where the equation was of the form x^2 + b*x*]. How did you get those negative 3's, negative 4's and negative 5's?

[Mark was working on his graphing calculator and appeared to be not sure which case Karla was asking him to talk about as he reported on a different case than the one about which she asked. She briefly restated what they were doing and which case they were currently reporting.]

Karla: What did you say would happen?

Mark: The, one of the y-intercepts, or the *x*-intercepts, would be at the opposite of, like if it was a positive, like *x*, positive number and then *x*, it'll be at negative that number *x*.

Karla: [writing on board-long pause] Opposite of that coefficient of *x*?

Mark: Yeah

Max: What's a coefficient?

Barbie: [Defines this term for him-inaudible]

Karla: Then Charity started looking at this.

Barbie: [inaudible]

Karla: Another group we might say, was *x* squared plus a negative coefficient in front of *x*? And what did we find out there for the *x*-intercepts? [writing on the board-long pause] Am I going too fast?

Ss: No [inaudible]

Karla: I'm just- you guys came up with some interesting things that kept happening, but I had it all over the board and I needed to pull it all together somehow to see what we found. Charity started looking at what if we didn't add a positive *x* number for *x* squared, what if we had a negative *x* number? And, Charity, what did you find was happening to the *x*-intercepts that way?

Charity: Um, well, it's gonna be zero, zero and then, um, it's gonna be- positive, like, it's not gonna be negative numbers, it's gonna be positive.

Karla: So, kinda like Mark's thing over here, too?

Charity: Yeah

Karla: It's gonna be the opposite of that? This again was because of the *x*-squared. [writes (0,0) on the board-pause] And then we found this. [writing on board] But then the weird thing that happened was when we threw a negative in front of the *x*-squared.

Max: It turned it over

> Karla: Yeah. [student whistles-long pause] We only looked at one of those. [writes $-x^2 + 5x = 0$ on the board-long pause]. Max said it flipped over when the negative *x*-squared was there, the parabola became a hill.

Fs: Oh.

> Karla: So, the ideas that Charity and Mark were bringing up with this opposite coefficient stuff fell apart right here. It wasn't the opposite of this coefficient, but the same.

>> Charity: Well, so that means the $5x$ would have to change, too, because normally that, normally, in your intercepts it would be negative five, zero

> Karla: That's how it's been working out, you're right. Maybe it was just that one, what if it was like $-x$-squared and $6x$?

Fs: Well, we could try it.

Karla: Try it and see?

[…]

Karla may have felt there were too many strategies being offered and decided to try to organize all of the suggestions. In doing so, she had students go through the type of problems they had discussed, the examples they had used, and the solutions they had found. Each time, Karla offered her own "paraphrastic interpretation" of what students offered, controlling the common knowledge of the classroom. This continuing conversation lasted for more than two class periods. During that span of time, Karla and her students offered other forms of quadratics that included $ax^2 + bx$ and $ax^2 + bx + c$ (where a, b, and c were all nonzero integers). In following the textbook's lead and keeping in line with what students would need to know in high school the following year, Karla tried to return to finding the solutions from the factored form. However, students continued to explore the solutions through using the expanded form of the equations by offering various forms of the equation and examining the tables and graphs on the graphing calculator.

Josh's Classroom Example

The transcript from Josh's class begins shortly after the bell rang. Like Karla, Josh's mathematical goal was to get students to understand how to use the factored form of the equation to solve a quadratic function.

Solving Quadratic Equations

> Josh: […beginning of class period…Josh is reviewing what they have been doing in this unit up to this point.] Those were all linear equations, solving linear equations; taking two linear equations, setting them equal to each other and finding the value for *x* in which they're the same, right? We're doing this all symbolically. You've always had graphing calculators you can fall back on to help you make sense of things. Um, but there's a lot more than just linear. There's also, among other things, quadratics. If you look at Problem 4.4, let's

take a look at it. Would you read, please, where it says, "Solving Quadratic Equations"? It's on page 57. Would you read that please, Sandy, would you read that please?

Sandy: [reads aloud—see problem that was given earlier]

Josh: Okay let's stop there for a second. (I-R-E) This--How do you know, first of all how do you know it's a quadratic equation?

Ss: *X*-squared [multiple responses of this]

Josh: An *x*-squared. If I graphed it, I'm probably gonna get some- a parabola here. It's gonna go up and come down or come down and go back up. In the *Moving Straight Ahead* unit, we said that a *y*-intercept was where *x* was zero? Right?

Ss: Mm-hmm

Josh: Where are the *x*-intercepts for this? So, in other words if I set this equal to zero, if *y* – if you want to put *y* over on this side because it's a *y*, it doesn't make any difference [referring to writing the equation either as *y* = the equation or the equation = *y*—pointing out that the order one writes it in doesn't affect the equation]. What are your *x*- intercepts? There might be one, there might be two or there might be zero. Use your graphing calculators. (I-R-E) Where are the *x*-intercepts? In other words, where is the parabola going to cross the *x*-axis?

Alicia: It's in the book.

Josh: Pardon?

Robert: 0 and negative 5.

Josh: You looked that far ahead already [to Alicia]. Robert?

Robert: 0 and negative 5.

Josh: (I-R-E) How did you know that Robert?

Robert: Cuz I looked at- first at zero and they're both zero and then [inaudible] at negative 5, it's zero.

Josh: Okay, so, how many entered the equation into their graphing calculators and tabled it and looked to see what *x* was when *y* was zero? Now, if you didn't do that, obviously it's right in your book right there. (I-R-E) If you look at the table in your book, where are the two *x*-intercepts again?

Ss: Zero and negative 5

[Using the graphing calculators, they look at the graph and table and go through what the minimum, maximum and line of symmetry are—Josh tells them that they know much more than they think. He also has them read information from the book that states that the *x*-intercept is the same thing as finding the "root" or the solution to the quadratic equations. Josh reminds them that the focus of this book is to do things with symbols and changes the topic to this subject at this point. Approximately 15 minutes of talk are omitted to get to the point where they are solving quadratics.]

To focus students on solving quadratics symbolically, Josh asked Sandy to read from the textbook. Alicia seemed to understand that the textbook was a place to find answers to Josh's questions as she told him that the answer was in the book. Josh first took note of Alicia's remark and then called on Robert to respond and offered a positive evaluation (i.e., "okay") of his reply. Josh drew from the textbook and directed student attention to the table that was given there. He moved the talk to focus on what he had defined the classroom activity to be earlier: solving quadratics symbolically.

<u>(Continued): Solving Quadratics Symbolically—Writing Expanded Form in Factored Form</u>

Josh: Okay, symbolic. Let's take a look at Problem 4.4. If you look on page 58, we're gonna look at another quadratic. It asks you look at-*x*-squared plus 3*x* is in expanded form. Would you write that in factored form [puts up overhead screen]. In fact everyone should be able to do that by now.

[Quiet overlapping talk while students work on this]

Josh: If I was gonna draw a rectangle with the area of *x*-squared plus 3*x*, how can you do that [referring back to the area model they used to factor quadratics in the *Frogs* unit]?

[Ss talking quietly]

Josh: This is in expanded form? I'd like to have an equivalent expression in factored form. So, what can you factor out of both terms? (I-R-E) Who has [inaudible]. Alright, Jennifer, what did you do?

Jennifer: I, I, um, I know that *x* times *x* is *x* squared.

Josh: Okay.

Jennifer: And 3 times *x* is 3*x*, so put *x* plus 3 in parenthesis.

Josh: How many have *x* times the quantity *x* plus 3? [writes on the board] So, this is in factored form? (I-R-E) Could I draw an *x* by *x* plus 3 rectangle?

Ss: Yeah.

Josh indicated that the textbook was defining the focus of their classroom activity when he asked students to look at page 58. He reminded them this equation was in "expanded form" and requested that they write it in "factored form," telling students that this was something they "should be able to do by now." He called on Jennifer to describe what she had done and offered a positive evaluation to both of her responses. He drew the rectangle and again reminded the class that the dimensions of the rectangle gave them the factored form of the equation. The excerpt continued with Josh referring to the book again to define what "else we are supposed to do":

<u>(Continued): Solving Quadratic Equations—Finding the Solutions</u>

Josh: [Draws it on the board] Something like that? So I've got an *x*-squared piece and the 3*x* piece in expanded form? Okay? That's no big deal. What else are we supposed to do? It says, [reading from book] "find all possible solutions to the equation x^2 plus 3*x* = 0." (I-R-E) Translate that for us, what are they asking for?

Sandy: The solutions.

Josh: The roots or the *x*-intercepts. In other words, I'm setting *y* equal to zero. What values of *x*, value or values of *x* give me a *y*-value of zero? Who can answer this for me? I don't care how [meaning they could use the graphing calculator or not], I'd just like an answer. What are the *x*-intercepts, what are the-uh, roots? Are there any? There might not be. You might, sometimes you get an equation where the parabola looks something like this and it never does cross the *x*-axis. PJ has an answer; Abram has an answer; Chuck has an answer. [naming all the people who have their hands raised] [long pause] How are you finding- Matt, you know how to find the *x*-intercepts?

Matt: Umm.

Josh: What are they? Alicia, Jennifer, Robert have an answer. (I-R-E) Carl, Jaylen, how? Jaylen, what are the *x*-intercepts?

Jaylen: Zero and negative three.

Josh: (I-R-E) How'd you get that, Jaylen?

Jaylen: I put it in the graphing calculator and found the zeros.

Josh: Okay, make sure you all can do that anyhow. (I-R-E) What equation did you enter into your graphing calculator?

Jaylen: I put *x* times *x* plus 3.

Josh: Which one?

Jaylen: [Inaudible—Josh pulled down the overhead screen]

Josh: Okay, so, you did the one that was in factored form?

Jaylen: Yeah.

Josh: Doesn't make any difference. Let's clear this [the equation that was already in his graphing calculator] out. And, if you don't mind, I'll just um, do the first one. So *x*-squared plus 3*x*? Oops, try again.

Ms: You pushed the [inaudible].

Josh: Yeah, I know. *X*-squared plus three *x*. (I-R-E) Okay, so, you went to the table?

Jaylen: Yeah.

Josh: Okay, oh, geez. I used this one. (I-R-E) So, let's see, I'm looking for, what are you looking for, Jaylen?

Jaylen: The *x*-intercept is zero.

Josh: Okay. (I-R-E) Agree?

Ss: Yep.

Josh: The *x*-intercept is what *x* is when *y* is zero? So there's one, but—

Ms: Negative [inaudible]

Josh: There's another one, too, right?

Chuck: Yep.

Josh: So, how many have the *x*-intercepts as being at –3 and 0? Okay, easy enough? [puts the overhead screen back up] What else were we supposed to do? We're gonna kinda walk ourselves through this problem. (I-R-E) What else were we supposed to do?

Jonah: It says [reading from the book] find the *x*-intercept where *y* equals *x*-squared plus 3*x*.

[Josh walks them through the remainder of the problem. About 2-3 minutes of transcript is deleted.]

Josh called on students to "translate" what "they" were asking, drawing students into interpreting the textbook's activity. Instead of picking up Sandy's choice of words (i.e., "solutions"), Josh continued to use "roots" and "*x*-intercepts" throughout this segment of transcript. He then outlined the process one would go through to do this—"setting *y* equal to zero, what values of *x*, value or values of *x* give you a *y*-value of zero?"

Even though Josh had stated earlier that they were going to use symbols now, he told students that he did not care how they answered his question, indicating that they could use a graphing calculator if they had wanted. Josh implied that he would have liked them to use symbols when he said "make sure you all can do that anyhow." This seemed to send the message that this was the "least" of what they should be doing. He again referred to the textbook and asked students what they were "supposed" to do. (Continued): Solving Quadratic Equations—Without Using the Graphing Calculator

Josh: We touched on this a little bit in the *Frogs* unit. See how good your memory is. Do you remember something?

Fs: [inaudible]

Josh: I know it was a long time ago, over a month. Alicia remembers something; Sandy remembers something; Chuck remembers something. Chuck, what do you remember?

Chuck: Uh, what you need to get to get zero at *y*, the three is zero and the *x* is zero

Josh: Hang on a second. (I-R-E) First of all, I know Jaylen said the *x*-intercepts are- what were they, negative 3 and 0? [writing on the board]

Chuck: Yeah.

Josh: Or negative three, zero, zero, zero, as far as points are concerned. (I-R-E) Chuck, what did you, say what you said again, what'd you say?

Chuck: Uh, what you need to get to zero [inaudible] you got the three and the *x*

Josh: I-I think I understand what you're trying to say, but I'm not sure anybody else does. "What you need to get to zero," okay? Do you [the rest of the class] understand what he's saying? (I-R-E) Jonah, do you understand what he's saying, "what you need to get to zero?"

Jonah: Not really.

Josh: Okay, I didn't think so. I don't think most people do, Chuck. Can you clear it up, what do you mean by, "what you need to get to zero"?

Chuck: The number you need to get to zero. Like 3 times what to get to zero. Three times zero is zero and negative three, zero is zero.

Josh: What are you looking at, the expanded form or the factored form?

Ms: I have no idea.

Chuck: I looked at the expanded form.

[Other students talking]

Josh: For both of them? [This question was directed at Chuck. Chuck nods yes.] Okay. (I-R-E) Mindy, what do you think?

Mindy: [inaudible] think about what number you take times to get to be zero [inaudible] whatever the three times what equals zero.

Josh: [inaudible] for expanded form, I didn't hear that-you said what?

Mindy: Like the number three times whatever [inaudible].

Josh: 'Kay. But, what if didn't know what the *x*-intercepts were. How could you solve one of these questions for zero? Now, symbolically we could solve- because of the linear problem, we could solve it, right? We could solve it symbolically?

Ms: Yeah.

Josh: How could you do that here? (I-R-E) You do an *x*-squared times zero?

Sandy: In factored form, um, in the parenthesis, *x* times zero, or *x* plus 0 is zero and then *x* plus negative 3 equals zero. So you need to use the negative or positive of that number.

Josh: It's a multiplication problem, isn't it? You're multiplying *x* times the quantity *x* plus three? Isn't any number times zero, zero?

Ss: [overlapping] Yeah

Josh: Just a little tiny bit about this. If you can get either of these terms equal to zero, since anything times zero is zero, wouldn't it be an *x* intercept? (I-R-E) How can you get that term [*x* + 3] equal to zero, PJ, what would you put in for *x* to get to equal zero?

PJ: Negative 3

Josh: Because negative 3 plus 3 is zero and then negative three times zero is zero. (I-R-E) How can I get that term [*x*] equal to zero?

Ss: Zero

Josh: Zero, (I-R-E) what's zero plus three?

Ss: Three

Josh: But what's zero times three?

Ss: Zero

Josh: Do you remember that?

Ss: Yeah

Josh offered a very open question: "Do you remember something?" Chuck may have offered a somewhat cryptic response as Josh asked him to "hold on a second." Josh revoiced Chuck's numbers into ordered pairs, making the answer more mathematically precise. He asked the rest of the class if they understood what Chuck was saying and three times restated, "what you need to get zero." Chuck had a difficult time clarifying. Josh tried to help him make his response more clear by asking him if he was looking at the expanded or factored form. Chuck said he was looking at the expanded form, which may have surprised Josh because he responded with the question, "for both of them?" Instead of asking Chuck to articulate his process further, Josh asked Mindy what she thought.

Josh may have been hoping that the process for solving linear had built a foundation for students to see how they could solve quadratics. He then scaffolded the process by going into a "tiny bit about this." He asked PJ what he could put in to make (*x*+3) equal to zero. PJ gave the correct response and Josh revoiced the response to show the process involved; that is, when you substituted –3 in for *x*, the result came out to be zero and –3 times zero gave them zero. Josh continued the process by asking students how to get the other term to be zero. Students talked him through this one as a class and when Josh asked, "Do you remember that?" students responded in the affirmative.

Earlier, I encouraged the reader to notice the pervasive I-R-E sequence in the segments of transcript as it is a defining characteristic of a "recitation." Throughout this 20-30 minutes of class, Josh utilized that sequence at least 20 times. Some of the questions Josh used in the "Initiate" slot were initially phrased in an "open" manner but became rephrased until they were "closed" (Barnes, 1969) in nature, funneling students toward the answer for which he was looking. The textbook is drawn from throughout the class period and not only defines the focus of the lesson, but it also defines the process the students will use to solve the quadratic equations. In this way, it is one source of knowledge from which the students may draw.

Looking Across the Examples

From the lens of "discussion" vs. "recitation": What does this lens say about what students in each classroom are learning?

From the more general perspective of "discussion" and "recitation," it appeared that students were engaging in discussion with their peers in Karla's classroom. Karla took on the role of summarizing and recording student ideas. She encouraged the class to check their answers on the graphing calculator, using the table and the graph for verification rather than her confirming answers. The activity and discourse boundaries in this excerpt are significantly relaxed as students talk both to Karla and to each other. Student responses also build on previous student responses. Karla does not keep her utterances and intervention to a minimum, but she does not explicitly assert which ideas are correct.

The topics that were discussed in Karla's classroom were often introduced by students. Each of the cases were offered first by students and then taken up by Karla to be discussed. The movement from one case to another did not always take place as quickly as some students would have liked. Mark advanced rapidly and continued to share cases that were ahead of the class discussion. At least two of Mark's contributions were not recognized by Karla while she worked with the class to explore and evaluate earlier ideas.

Students in Karla's classroom were finding many cases of quadratics and exploring them. They were hypothesizing the solutions and using two representations on their graphing calculators to validate their choices. They were working together to articulate what they were thinking. They were sometimes questioning each other, offering other ways to think about the mathematical ideas, and helping each other understand what they were saying. In line with Karla's role in "Replying" to students, students seem to be "Sharing" rather than "Presenting," especially before Karla said she needed to organize the ideas.

The use of this type of discussion in Karla's classroom was not unproblematic. In fact, there were many times when Karla had to remind students to be on task. She struggled not to let some students dominate the conversation. While many students were actively engaged in the conversation, there were always a few who were off task. This was apparent when students asked inappropriate questions (e.g.,, Kevin once asked, "Do you want to hear what I dreamed about last night?" and "Why don't we just talk in here like we did last year [during homeroom]?") or engaged in off-task behavior (e.g.,, making faces at other students across the room, drawing pictures unrelated to mathematics).

Another crucial aspect in the teacher's ability to lead discussions in the classroom is that of adequate and appropriate subject matter knowledge for teachers. Both Josh and Karla talked about how important their subject matter knowledge was in their teaching. In fact, Karla told me that she thought Josh's deeper subject matter knowledge allowed him to:

> […] see a lot more, I think. He knows more math than I do and if a kid said something, sometimes I wouldn't even get what the kid was saying and he would. He would understand it better or he would see where it could lead and so he could do that so much better than I could. I've learned a lot more math from him because I'd go to him with, "So and so said this and that was so stupid" and then he'd say, "No, this is what they're saying. This is where it's going." And so I think he's so much better at knowing those kinds of things—where everything goes, how everything connects and I learned tons from him that way (Interview K-4, 1/07/00).

In contrast, Josh's class seemed to be taking part in a recitation. The pervasive

I-R-E structure kept the classroom talk very centered on the process that Josh was advocating. Josh's role seemed to be "Assessing/Judging." This role became apparent in Josh's consistent evaluations of student responses, his counter proposals and suggestions and his use of "should." In response, students "Presented" final draft talk. Josh's role was to offer topics, evaluate responses, ask questions, make connections (e.g., in his statements about what they learned in this and other units), etc. Students seemed to understand that the textbook was the entity that defined the activity in the classroom and it also was a place in which to find answers. Student's role seemed to be one of answering questions and offering confirmation. They described how to do things and why. However, these were not offered until Josh requested them. In that sense, students seemed to have learned the "appropriate" way to respond. This was true when Josh used tag-forms; students offered affirmation whenever Josh posed a tag-question, showing that they knew that this was the appropriate thing to say.

The discourse boundaries were not as relaxed as Karla's because Josh functioned within the Initiate-Respond-Evaluate sequence most of the time. Josh controlled both the topic and who got to speak. At least three times he called out the names of students who had their hands raised. At the end of each of these turns, Josh was the person who selected who spoke, instead of students self-selecting as they sometimes did in Karla's classroom. Rather than drawing on students as a source of knowledge, Josh and the textbook were defined in this manner. In fact, later in the class period Chuck offered a generalization:

<u>(Continued—Later in Josh's Example)</u>

Josh: Okay, Chuck, what do you think?

> Chuck: So the one [inaudible] you're subtracting, it'll be a positive here, and if you're adding, it's gonna be negative?

> Josh: You can formulate your own rules if you want to, yeah. I mean, that's one way of thinking about it. If this, if this *x*, if it's 4 + *x*, what would it have to be, though?

Chuck: It's gonna be—It'd be negative four

> Josh: Negative four because four plus negative four is zero. Whatever *x* value sets it equal to zero. […]

In this example, Chuck noticed that to find the solution sometimes you had to take the opposite of the operation given (e.g., in *x* + 3 you are adding, so it would result in –3). Josh responded by telling Chuck he could "formulate your own rules if you want to." Unlike Karla's exploration of the rules Mark offered, Josh tried to provide a counterexample. The instance that he gave, however, did work and affirmed Chuck's rule. I think Josh meant to say 4 –x as he tried to get Chuck to focus on "whatever *x*-value sets it equal to zero" rather than emphasizing the rule Chuck had offered where one might only attend to the operation that was given in the problem.

While neither teacher fit "neatly" into one of these two categories (i.e., recitation or discussion), Karla tended to have more of the "discussion" attributes in her classroom explorations and Josh tended to follow more of the "recitation" format. For example, Karla made space for student comments, but the discourse was not more balanced. Karla talked markedly more than her students did. There were fewer teacher questions in Karla's classroom than in Josh's, but the "big ideas" in Karla's classroom were used to guide the discussion. In Josh's classroom, the Initiate-Respond-Evaluate format was pervasive but the actual flow of the lesson was not completely defined. Josh acknowledged that he did not always know how he was going to get where he was going, but he did know where he wanted to end up. Even though Josh proceeded through a "checklist" of topics, the discourse did not feel choppy and incoherent—it was all tied to the big idea that was emphasized in the lesson and that was where he focused the classroom talk.

<u>"Mathematical discussion": What would this lens say about what students are learning?</u>

When classifying episodes of discussion, Pirie and Schwarzenberger used three parameters: "the focus of discussion, the kind of language used, and the type of statements being made" (p. 466). These parameters are described as:

First parameter: What is it that gives the speakers something to talk about?

a. They have a task or concrete object as the focus of their talk.

b. They do not have an understanding of something but they know this and it gives them something to talk about.

c. They have some understanding and this gives them something to talk about.

Second parameter: What level of language is being used?

f. They lack appropriate language; they do not have the right or useful words.

g. They use ordinary language.

h. They use mathematical language.

Third parameter: What kind of statements are being made?

p. Incoherent statements; that is, incoherent to other participants.

q. Operational statements about what to do or how to do it.

r. Reflective statements offering explanations or attempts to move beyond the immediate task (p. 466-467).

<u>What does this lens tell us about Karla's classroom?</u> The students have explored and discussed quadratic functions in a previous unit and again earlier in this unit. They have

some understanding of these functions and this gives them something to talk about. The task that Karla has set before them is to think about particular points on the graph—not the entire graph. The points they are to focus on are where the *x*-values are zero which are the solutions or *x*-intercepts of the function. Students use some information that they already knew (e.g., that the equation $y = x^2$ has one *x*-intercept of (0,0)) and use their knowledge of technology (e.g., how to find points on a table or graph) to make inferences about the solutions from the expanded form of the equation.

The level of language varies quite a bit. Students seem to use a large range of mathematical language, including "classroom-generated language," language associated with various representations, as well as "official mathematical language" (for more information about these various types of language, see). Some of the official mathematical language used includes: *x*-intercept, negative, *x*-squared, equation and parabola. In addition, Karla revoiced many of the student contributions and used terms like coefficient, parabola, and quadratic. She did not correct students who chose to use classroom-generated language. Some examples of this included the phrases "*x*-two" and "hill-thing." When a student used a word that did not have as common a meaning as these two phrases (i.e., upside down thingymajigger), Karla pushed her to say what she meant. In response, Charity showed her using her fingers by tracing a parabola that opened upward in the air. Barbie then helped Charity name what she was talking about in official mathematical language: a parabola. The language early on is very much exploratory talk. Students are trying to figure out what is going on in the mathematics. When Karla turned to "figure all this out," student talk became more final draft. For example, Dale no longer said "*x*-two"; he now used the more mathematically appropriate "*x*-squared". In addition, Mark again made the mistake and substituted "*y*-intercept" for *x*-intercept. However, at this point he corrected himself and made his statement more mathematically correct. Karla again helped students make their contributions more mathematically precise and general when she revoiced Mark's statement, "if it was a positive, like *x*, positive number and *x*, it'll be at negative that number" to "opposite of that coefficient of *x*?" Karla also pointed out the discrepant event that took place when the squared value was negative; that is, the "opposite coefficient stuff fell apart right here." As the discussion ensued, students moved in and out of all levels of language. At times students lacked appropriate language. When they did, Karla sometimes revoiced making the contributions more mathematically appropriate. In other instances, students offered the words or definitions. Students also used ordinary language and mathematical language.

The type of statements being made were never "incoherent"; that is, they did not appear to *not* make sense to other participants in the conversation. Some statements were "operational" or were about what to do or how to do it. Examples of these would include segments of transcript that were grounded in particular equations, rather than generalizing a rule. For instance, when Charity suggested Dale and Mark were on to something because she tried their method with another equation. This may have indicated that Charity thought she could generalize from just two equations. Also, Karla asked Charity to ground her idea "if it's a negative *x*, it's gonna be positive" in an example. This would be more operational. Other statements were "reflective" in that they offered explanations or attempted to move beyond the immediate task. An example of this would include Dan's suggestion that they needed to consider the solution (0,0) when the coefficient was one. He connected what they were doing to previous knowledge and suggested why that solution made sense. Also, students eventually abstracted "rules" that they could follow for various forms of the equation. For example, when they discussed equations of the form $y = ax^2 + bx$, Mark said that they could take the opposite of the number with the *x* and divide it by the number in front of the x^2. Karla helped to connect this to the general form of the equation and showed them that they could write this as –b/a. I am unsure if they ever got to a point in their discussion, however, where they contemplated why this method worked. They did eventually connect these methods with factoring the expanded form to find the solutions.

<u>What does this lens tell us about Josh's classroom?</u> The focus of the discussion is determined when Josh called on Sandy to read from the textbook. After Sandy read, Josh inquired about things they had already learned about quadratics in previous units (i.e., *Moving Straight Ahead* and *Frogs, Fleas, and Painted Cubes*). By doing so, Josh defined the parameters of the classroom talk, using the textbook to determine what would be talked about and when. The talk moved from remembering what they had learned about quadratics in other units to factoring quadratics using an area model to solving quadratics to being focused on using symbolic manipulation to solve the problems. Each of these topics was defined and introduced by Josh. In some cases, he also defined the process for doing what he requested. For example, when he asked students to factor, he reformulated the question to ask them what the rectangle's dimensions would be. Some of the processes were defined by students. For instance, Josh asked students to find the *x*-intercepts or roots and told them, "I don't care how," insinuating that they could solve it symbolically, use the graphing calculator, etc. Jaylen used the graphing calculator and described what he did to solve the problem.

The range of language used in this transcript is fairly consistent. Josh and the students typically used official mathematical language. In fact, Josh may have set this stage by immediately referring to the textbook and by modeling this type of language right away when he asked, "How do you know it's a quadratic?" Some examples of this type of language included: quadratic, linear, parabola, *y*-intercept, *x*-intercept, minimum, maximum, line of symmetry, root, solution, expanded form, factored form. Josh further supported this type of talk in his revoicing of student contributions. For instance, when Jennifer said, "And 3 times *x* is 3*x*, so put *x*+3 in parenthesis," Josh revoiced her answer as, "…*x* times the quantity *x*+3," making her contribution more mathematically precise. Another example of this would include when one student gave the *x*-intercepts as –3 and 0. Josh first restated what he said and then revoiced himself, saying, "Or (-3, 0) and (0, 0) as far as points are concerned." There were also many times when Josh used official mathematical language and then continued by describing the process or definition that went with the term. For example, after he asked students to "translate the book," Sandy offered that they were trying to find the "solutions." Josh revoiced this as "the roots or *x*-intercepts" followed by what that meant: "In other words, I'm setting *y* equal to zero."

In attending to the type of statements being made, there is one instance of an "incoherent" statement. When Chuck tried to describe how he had come up with the solution he said, "Uh, what you need to get zero at *y*, the three is zero and the *x* is zero." However, the statement appeared to only be incoherent to other participants, but not to Josh. In fact, Josh pointed this out when he said, "I-I think I understand what you are saying, but I'm not sure anybody else does." He confirmed his suspicions by calling on Jonah and asked Chuck to "clear it up." Chuck tried to do this once and then Josh tried to help him make his contribution more clear by asking a directive question (e.g., What are you looking at, the expanded or the factored form?"). Josh may have attempted to have another student help with the explication of this method when he brought Mindy into the conversation by asking her what she thought. It was not clear to me whether Mindy was trying to help Josh understand Chuck's contribution or if she was offering her own method for solving the equation. However, it was clear that both Mindy and Chuck were attempting to use the expanded form to describe their solution. Josh pointed out that they needed to think about how to solve when they did not know what the *x*-intercepts were and proceeded by asking students to use what they knew about solving linear functions symbolically. He talked them through the process, filtering out the previous solutions methods that had been offered by students. He directed their attention to the factored form and how to use that to solve the quadratic equations. In fact, at one point, Josh read a question from the textbook that asked, "Which form makes it easier to find the roots?" Although the question is never answered explicitly, it became fairly clear that the factored form was the approved method.

Many of the student statements were "operational" in that they focused on how to do something. Sandy's response to Josh, in fact, was about how to solve and only focused on the factored form as Josh had indicated it should. In many cases, the process of how to solve the problem was the focus of the conversation. There were also some "reflective" statements made by Josh. At least three times, Josh tried to get students to use what they had learned in previous units, attempting to make the common knowledge more connected for students. He referred back to: how they knew the equation was quadratic, how they had solved linear equations symbolically, and how they could use the *x*-intercepts to find the minimum or maximum and the line of symmetry. Some of this information was related to what students had learned in previous units; other information was covered more recently in the current unit. In addition, explanations were given whenever Josh requested them.

<u>Looking across the classrooms.</u> More particularly, in Karla's transcript, all the components for a "mathematical discussion" outlined by Pirie and Schwarzenberger (1988) exist. In the example, the talk is centered on a mathematical subject: solving quadratic functions by using the expanded form of the equation. While the idea of solving quadratics was introduced by the teacher, the process of using the expanded form to find the solution was introduced by one student, Mark. In this way, it appeared that the content was defined by Karla, while the process was defined by Mark. Many students proceeded by offering and then exploring and discussing multiple cases that were taken up by other students as well as the teacher. In doing so, there were "genuine pupil contributions" and "interaction." In fact, a few times "cross discussion" took place; that is, students actually talked to one another instead of all of the classroom talk being filtered through the teacher.

In Josh's classroom, it appeared that many of the components for a mathematical discussion also existed. The talk in his classroom is also focused on solving quadratic functions. However, the textbook seemed to define what they were "supposed" to do. Josh and the textbook seemed to define both the content and the processes that would be used. This was different from Karla's students defining the processes to be discussed.

When looking at Josh's transcript in terms of "genuine pupil contributions" and "interaction," there appeared to be a further distinction between what happened in Josh's classroom as compared to Karla's. While students were contributing and answering Josh's questions, they sometimes seemed to be responding to rhetorical questions or tag-questions. In both instances, it seemed that a mere "okay" or "yep" was required. Other questions required students to focus on the "hows" and "whys" of the content. These are ones about which I am unsure. I would not consider these "mere passive response," yet they did not necessarily allow for the "introduction of new elements to the discussion." The answers to this type of questioning did help the "talk or thinking to move forwards," but they seem different from the contributions in Karla's classroom where students were offering many different cases of quadratics. One difference may be that Josh's student-talk was focused more often on operational statements; Karla's student-talk moved back and forth between operational and reflective. In Josh's excerpts, I only have evidence of him making reflective statements; students merely confirmed the statements that he made.

The movement of the talk in Josh's classroom was picked up by other students, so there were "interactions." However, many of these interactions were not instances of "critical listening" taking place; instead, they were cued elicitations (i.e., the answers were cued because the tag-form of the question only encouraged a positive response). In addition, there were no instances of cross discussion, as there were in Karla's classroom. Student responses were more often channeled through Josh. At one point, Josh may have wanted Mindy to respond to Chuck, but it was unclear to me whether Mindy understood that and was trying to explicate Chuck's idea or if she was introducing her own thinking about finding the solutions to the quadratic equation.

CONCLUSIONS

Both teachers have the same mathematical goal: to get students to solve quadratics. The mathematical ends are essentially the same. However, the processes are different—the means that get to the ends are different. The way that Karla is allowing students to engage in exploring mathematical ideas is different from the way Josh's students do.

The view of mathematics that Josh seems to be advocating is much more Platonic in nature. The view of teaching that Josh employs was very much a "*content focused with emphasis on conceptual understanding"* perspective. In one of the interviews I did with Josh, I had noticed that the majority of his comments focused on how the mathematics determined what he did/said in his classroom. I asked him if the mathematics tended to drive what he did, but that he kept the students in the back of his mind as he was teaching. He responded, "I don't think I could have said that better myself" (Interview J-2, 12/02/99). However, many of the connections that were being made were being made verbally by Josh rather than his students. Students were more often expected to affirm Josh's contributions and questions.

The portrayal of mathematics in Karla's classroom is a "problem solving view" of mathematics. The roles that she and her students play are quite different from those in Josh's classroom. Her approach is much more "*learner focused*." Student ideas and interest were of primary consideration, driving the movement of the discussion. In fact, even when Karla tried to show students that the method in the book could be applied to the many cases they had been exploring, the students returned the next day and continued the exploration of other cases that hadn't yet been considered. This pursuing of student ideas occurred frequently and often put Karla "behind" Josh in terms of how many investigations she could "get through."

From the perspective of the literature, it appears that two different epistemological stances are being operationalized in these two classrooms. I am left with the question as to whether students notice this or not. The students in each classroom appear to have implicitly understood the role that they are expected to play. However, do Karla's students feel empowered to generate mathematical ideas more than Josh's students? While I will not be able to ask students this question in an interview setting, I do intend to look at the interview data to see if the ways the students engage with my interview tasks parallel the ways I saw them engage in their classrooms. That is, do those roles permeate the ways in which they interact with mathematical ideas outside of the classroom? If so, what does that mean for their future mathematical experiences? What does it mean to their beliefs about what it means to know and do mathematics?