

The Graphic Calculator as a Thinking Tool: Perspectives from the Classroom

Susie Groves

Deakin University

grovesac@deakin.edu.au

Deodato Obregon

**Western Visayas College of Science
and Technology, Philippines**

dino23@gurmail.com

For over a decade the graphic calculator has been promoted not only as a computational tool but also as a thinking tool — for example, as an aid to enhance conceptual understanding, as a problem-solving tool and as a means of enabling students to engage in meaningful investigations. However, research studies focusing on these aspects have shown mixed results and have mostly focused on graphs and functions. This paper reports on one aspect of a case study in a year 10 mathematics classroom — the role of the graphic calculator as a thinking tool. Data from observations of nine statistics lessons and interviews with the teacher and five students are analysed from three perspectives: the teacher's intentions with respect to the use of the graphic calculator as a tool to promote conceptual understanding as opposed to procedural competence; the opportunities afforded during the lessons for student investigation; and students' views of how the graphic calculator enhanced conceptual understanding. The results provide insights into ways in which students perceive the graphic calculator as promoting conceptual understanding, as well as some of the difficulties encountered in practice in a classroom.

Introduction

Ideally, a cognitive tool should not only be capable of assimilation to established modes of thinking but also be able to support cognitive growth and change on the part of the user. (Ruthven, 1992, p. 95)

There is a substantial body of research into the use of graphics calculators — for extensive reviews of such research, see, for example, Dunham and Dick (1994), Ruthven (1996), Pengalese and Arnold (1996), Heid (1997), Asp and McCrae (2000), and Dunham (2000).

Much of the research into the effectiveness of the use of graphic calculators in enhancing student achievement appears to show positive results. However, studies which use experimental and control groups are, in general, fraught with difficulties in that they fail to take into account the relationship between the use of the graphic calculator and issues of pedagogy (Dunham & Dick, 1994; Pengalese & Arnold, 1996). Thus research needs to move beyond effectiveness studies and address more fundamental questions about the

actual processes of teaching, learning and thinking, and how the graphic calculator functions as a tool for learning (Ruthven, 1995, p. 239; Berger, 1998, p. 13).

Adopting a Vygotskian framework, Pea (1985) distinguishes between the potential for technology to act as a *cognitive amplifier* — i.e. to "change how effectively we do traditional tasks, amplifying or extending our capabilities" — and as a *cognitive re-organiser* — i.e. as a tool whose use can "fundamentally restructure the functional system for thinking" (Pea, 1995, pp. 168; 170). According to Pea (1997, p.99), the fundamental question is "How can technology support and promote thinking mathematically?"

Acknowledging Pea's distinction, Salomon, Perkins and Globerson (1991) further distinguish between the effects *with* technology use — i.e. the effects while people are working with technology — and the effects *of* technology use — i.e. the "subsequent cognitive spin-off effects for learners working away from machines" (p. 2). According to Salomon, Perkins and Globerson (1991), intelligent technology — i.e. technology that is able to "undertake significant cognitive processing on behalf of the user" (p. 3) — has the potential for the formation of an *intelligent partnership* where the division of labour between the human and the technology allows the "partnership ... [to] be far more 'intelligent' than the performance of the human alone" (p. 4). Jones (1993; 1996) proposes that, when assessing mathematical intelligence, we need to consider whether it is the performance of a student alone or the partnership that needs to be assessed.

Findings of studies into the cognitive effects *of* using technology have been mixed (Salomon, Perkins & Globerson, 1991, p. 6; Pengalese & Arnold, 1996). A critical factor in achieving cognitive effects *of* using technology (or Pea's *cognitive re-organisation*) is to provide students with situations which require "mindful abstraction" (Salomon, Perkins & Globerson, 1991, p. 6). However, the graphic calculator has not been specifically developed for such a purpose, but rather the purpose of solving particular problems (Jones, 1993). According to Jones (1996), the challenge to teachers is to develop instructional strategies that promote the formation of intelligent partnerships.

Pengalese and Arnold (1996) urge researchers not to treat the graphic calculator in isolation from the context in which it is being used — the particular learning environment. Tharp, FitzSimmons and Brown Ayres (1997), in a four month study of 261 mathematics and science teachers, found that while participants' views changed significantly in favour of viewing the graphics calculator "as a 'thinking tool' to enhance conceptual understanding and expand exploration" (p. 551), there was a difference between those teachers they classified as holding a *rule-based* view of mathematics learning and *non-rule-based* teachers. In particular, rule-based teachers quickly abandoned inquiry approaches, were more likely to feel that the calculators were a hindrance to learning, and were more concerned about students' emotional reactions than indications of conceptual understanding.

After three years of using graphic calculators in the teaching of introductory calculus, Jones (1993) found that, while students generally enjoyed using them, there had been no great change in mathematical understanding. He states that "there is a need to go beyond the immediate issues of curriculum and classroom practice, and focus on the fundamental issue of the nature of the user's interaction with the technology (Jones, 1993, p. 212).

Galbraith, Renshaw, Goos and Geiger (1999), like many of the previously cited authors, use Vygotsky's notion of the zone of proximal development to underpin their research. They examine "interactions between people, technology and mathematics" in a classroom setting and propose "four roles for technology in relation to student learning: Master, Servant, Partner and Extension of Self" (Galbraith, et al, 1999, p. 223).

In a study of 20 mathematics major students in a first year course in South Africa, Berger (1999) found that "the graphic calculator functioned primarily as an amplifier of the zone of proximal development" (p. 17) with little evidence of cognitive re-organisation. As well as other reasons for this finding, Berger (1999) also suggests that the socio-cultural setting may *privilege* one form of reasoning (in this case algebraic) over another (graphical).

This paper reports on the role of the graphic calculator as a thinking tool as one aspect of a case study in a year 10 mathematics classroom. Data from observations of nine statistics lessons and interviews with the teacher and five students are analysed from three perspectives: the teacher's intentions with respect to the use of the graphic calculator as a tool to promote conceptual understanding as opposed to procedural competence; the opportunities afforded during the lessons for student investigation; and students' views of how the graphic calculator enhanced conceptual understanding.

Overview of the study

The study was carried out by one of the authors (Obregon) during Term 3 of 2000 in a Year 10 mathematics class at an all-girls Catholic secondary college located in the eastern suburbs of Melbourne.

In choosing a teacher and class, the main criteria were that the teacher be reasonably knowledgeable regarding the uses of graphic calculators in mathematics lessons and that the students have ready access to graphic calculators and be familiar with the basic functions of the calculator.

The mathematics teacher selected for the study, who will be referred to here as Mr D, had been teaching mathematics for over a decade, and had started using the graphic calculator in his tertiary and secondary classes in the early 1990s. He had also participated in professional development activities relating to the use of graphic calculators in mathematics classes.

For practical reasons, Mr D selected his Year 10 class to take part in the study. He initially described them as "very good" students who usually make use of graphic calculators in lessons. It was later found that, with a few exceptions, the class was a fairly homogeneous, accelerated group in terms of mathematical ability. All of the 25 participating students owned a graphic calculator and had used them consistently since Term 1 of 2000.

There were three main sources of data: a teacher interview; observations of nine consecutive lessons; and an interview with five of the 25 girls in the class.

The teacher was interviewed in the school for about 30 minutes, several days before the commencement of the classroom observations. A semi-structured interview schedule was developed and used to elicit his views on the role of graphic calculators in teaching and learning. The interview was audiotaped and transcribed in preparation for later analysis. Preliminary analysis of this interview led to a tentative conceptual framework for the observations that followed.

The class was then observed during the last three weeks of Term 3. These observations, which were conducted unobtrusively in order not to disturb the students and their teacher, were recorded using a written observation schedule. The first page of the schedule was used to record background information about the class, school, date, etc, as well as a lesson summary, the seating arrangement, and resources used during the lesson. The second and subsequent pages provided columns to record time, teacher actions (what the teacher says or does), student actions (what the students say or do), and observer comments. While it is

clearly impossible to take note of everything that happens when using such an observation schedule, the use of video or audiotaping was not regarded as a viable option for this study.

A total of nine lessons were observed, the first two of which served the purpose of trialling a draft observation schedule. The succeeding lessons were all part of a unit in Statistics, focusing on simple data analysis. Prior to the observations, Mr D had been requested to supply a copy of the unit outline and other resources for the series of lessons to be observed. Reading the handouts in advance helped the researcher in recording activities in the subsequent classes. Moreover, there were usually opportunities for the researcher to talk informally with the teacher after each lesson about what had transpired in the lesson. Salient points from these conversations were recorded in the observer comments section of the corresponding observation schedules.

Towards the end of the classroom observations, five of the students were selected for individual interviews focusing on their perceptions of the effects of graphic calculator use. While there was no attempt to find a representative sample from the 25 students, it was felt to be important that a range of views be sought. Based on the classroom observations and advice from the teacher, five students, who will be referred to here as Cathy, Debbie, Joan, Trish, and Susan, were selected.

A semi-structured interview schedule was again developed and used for the student interviews. The five selected students were interviewed at the school for about 30 minutes each during the last two days of the observation period. The interviews were again audiotaped and transcribed in preparation for later analysis.

The analysis of relevant portions of the data from these three sources is described in the next three sections of this paper.

The teacher's intentions

Questions in the teacher interview were grouped into five sections: background information and general philosophy of mathematics teaching; teaching experience with graphic calculators; features of the graphic calculator used in lessons and their relative importance; planning for effective use of graphic calculators; and a general summary of the role of graphic calculators.

The interview transcript was coded by italicising portions of the teacher's responses that illustrated his specific views and comments in a particular section. For example, under the heading of *Philosophy of Mathematics Teaching*, the phrase "to learn as much to become critical thinkers" was italicised as a key component of the teacher's philosophy. This statement was then coded as BCT to represent "be critical thinkers". (These italics are retained in the quotes which follow later in this section.)

After coding statements by the teacher within each section of the transcript, five themes or categories emerged. These were: philosophy of mathematics teaching; most useful features of graphic calculators; how graphic calculators are used in class; extent of using the graphic calculator; and preparing and conducting a graphic calculator lesson. For a full list of the codes and categories, see Obregon (2000, Appendix N).

Due to restrictions of space and the specific focus of this paper, these codes and categories will not be explored in full here. Instead, only the analysis of responses related to the focus of this paper — namely the teacher's intentions with respect to the use of the graphic calculator as a tool to promote conceptual understanding as opposed to procedural competence — will be discussed and this discussion will omit explicit reference to the

various codes which were used in the analysis. Responses used here are taken from three of the five categories: philosophy of mathematics teaching; how graphic calculators are used in class; and extent of using the graphic calculator.

Mr D's philosophy of mathematics teaching was elicited through a direct question asking him to describe his philosophy, with additional probes about his beliefs about the use of technology in teaching and the role of the graphic calculator.

The analysis of Mr D's responses indicate that his philosophy of mathematics teaching is premised on the goal of developing students to become critical thinkers, rather than emphasising the learning of procedural details. A procedural emphasis can be partly evidenced by the use of test items focusing on the application of algorithms or mathematical procedures, while it follows that if students' critical thinking is one's main teaching objective, problem solving and other tasks that require higher order skills should permeate one's classes. The graphic calculator is promising in this aspect, in that it can take on the procedural matters, thus giving time for mathematical thinking. Discriminately using the graphic calculator itself also involves critical thinking. Mr D hinted at this role when teaching with graphic calculators:

I'd really like them all *to learn as much to become critical thinkers*, ... I see these as the important things I want to convey *rather than learning a lot of procedural details*. I'd like them to become good critical thinkers and I think maths has got a role to play in that area. ... I think it [the calculator] has got a lot of potential. (IntM, p. 1)

One of the things I learned early on is that *kids will teach each other about the procedural things and one of the best ways to encourage them to develop their skills is to give them tasks that are more open-ended*.. (IntM, p. 7)

Mr D acknowledges the fact that, at this time, there is still need for ideas on how best to teach in a graphic calculator environment. But he makes it clear that the graphic calculator has a role to play in his classes.

I believe that *there is potential with technology to assist students to learn*.... I think my belief is probably that *we don't have a lot of ideas about how best to teach in a technology environment*. (IntM, p.1)

I mean it's been a help in that way, I think, *because it's meant that we can look at a broader range of functions; we've got an investigative tool that we can use more frequently* than we have in the past ... and often those sort of *tasks just became a bit onerous*; you know, they might be possible but they would have involved *a lot of time wasting* in the construction of the graphs and the sort of testing in conjectures. (IntM, p. 7)

An analysis of the transcript of the teacher interview, shows two general categories of ways in which Mr D saw the calculator being used in the class: as a "result-generator" and as a "thinking, doing, teaching tool".

As a result-generator, Mr D saw the teacher or the students using it to get quick results for mathematical tasks. He regarded this as being helpful as students are not distracted from the main lesson by lengthy calculations and other routine tasks.

So that's another form, *sometimes they're just result-generators*, you know, *just quick things*. (IntM, p.3)

They [students] need a *testing tool* rather than get bogged [down] in the graphing, manually graphing. It can actually give them the table of values and the graphs *very efficiently*. (IntM, p. 2)

Of more significance is the second category, that can be seen as building on the first one. That is, the results quickly and accurately generated by the calculator are often ingredients for exploration of concepts or for mathematical investigations. As a student tool for thinking about and doing mathematics, the graphic calculator may be thought of as a visualisation tool, a conceptual one, or an investigation tool. These sub-categories emerged from the terms used by the interviewed teacher.

As a visualisation tool, the calculator was seen by Mr D as

a tremendous visualisation tool ... I tend to use it in conjunction when I teach theory just because it gives kids that benefit from the visual side, a little bit more of the thing of getting a hold of sort of the overall picture of a function. (IntM, pp. 2; 4)

Mr D emphasised the potential of the calculator as an investigation tool, in keeping with his philosophy of helping the students "become critical thinkers" and achieve the "higher aims of mathematics".

It's also good for *setting up tasks that involve the students doing some of their own investigations* particularly where they relate to things like functions and stats, *you can just ask big questions that require investigations*. (IntM, p. 2)

And sometimes they're *conceptual tool*, like one of the programs generates a quick discrete probability distribution graph and you can [then] use that as *an investigation tool* because you then say, 'Ok I want you try a different p value and then see how that affects the shape of the distribution. Try what happens if you get the same ... p-value around the centre and you choose a large n-value; *what does it do to the shape of the distribution?*' (IntM, p. 3)

In exploring Mr D's perceptions of the effects of graphic calculator use in his teaching, he was asked to what extent he makes use of the graphic calculator as a computational tool or an investigatory tool. His responses generally indicated the value he sees in the calculator as both a teaching and learning tool.

I think it's very good as a *tool for demonstrating things* so that you can actually have students come out and explain how they did something and show the rest of the class that you can spend a bit of time *using it like an electronic whiteboard*. So you project it onto the screen and then you actually write over the top of it with a white board marker. (IntM, p. 2)

While Mr D emphasised the value of the calculator for demonstration and computation, he maintained that investigations arise more, and the calculator is consequently used more for such purposes, when doing higher or later topics in different units.

As we get higher up we'll probably have more and use other functions other than linear. We'll probably spend a bit more time on function investigations. We did a trig one when we looked at a trig rule; in fact, they originally had the data for it and they found the curve for them and they did some further analysis. *So the more that we get into the function area ... but also in the data analysis, there'll be some scope there for doing some investigations.* (IntM, p. 7)

In summary, the analysis of the teacher interview reveals that Mr D saw the potential of the graphic calculator in assisting his students' learning $\frac{3}{4}$ understanding mathematical ideas and achieving higher aims of mathematics. The graphic calculator was seen as having this potential because of certain things it makes possible $\frac{3}{4}$ e.g. the calculator can take on some of the "onerous" procedures or tasks so that students can focus on the concepts. Mr D saw his role in a graphic calculator environment as one of *guiding* the students' use of calculators. His aim was to encourage the critical and effective use of these tools.

The lessons

This paper reports on the last seven of the nine lessons observed. All lessons were of 45 minutes duration, except Lesson 5 which was a double period, referred to here as Lessons 5a and 5b. Students were seated in three rows, each with five adjacent tables seating two people. Approximately 20 students were present in each of the observed lessons. The teacher kept a graphic calculator with an overhead display in the classroom during all lessons observed.

Using the completed written observation schedules, each lesson was analysed and summarised using five categories: a brief description of each of the main lesson segments; the mode of instruction; the graphic calculator features used; who used the calculator and how it was used; and the nature of use of the calculator (see Tables 1 – 4). The first and third of these categories are relatively self-evident. The remaining three are discussed in slightly more detail below — for a full discussion see Obregon (2000, pp. 70–73).

Three modes of instruction were observed. These are described here as: whole class, teacher-directed; individual, teacher roams; and small group, teacher roams.

The analysis of the observation schedules revealed six ways in which the teacher and students used the calculator:

- *Teacher models, students reproduce.* The teacher used the overhead graphic calculator to illustrate how a particular feature or combination of features may be used, usually in the context of a specific mathematical idea or task, and expected students to follow. Most students were able to reproduce the calculator displays or results of the teacher.
- *Teacher demonstrates, some students reproduce.* The teacher explained a mathematical idea or showed how to carry out a mathematical task, but did not expect students to follow step by step. Some students reproduced calculator displays even when not expected to do so. Others focused on the explanation or illustration without using their own calculator.
- *Teacher suggests, students use.* The teacher suggested which calculator feature may be used to students who were working on a mathematical activity and could not find the appropriate calculator procedure. The students then used the suggested features.

- *Student demonstrates, other students reproduce.* Similar to the previous category, except that suggestion came from a student volunteer or one who was asked to demonstrate.
- *Students use freely.* Students used the graphic calculator in their own ways to carry out the given task. The teacher did not prescribe the use of any feature; nor suggest how these may be used.
- *Graphic calculator not used at all.*

The descriptions under the last category of the lesson summaries in Tables 1 – 4, the nature of use of the calculator, were derived from the teacher’s categorisations of the ways in which the calculator was used (see previous section).

Table 1

Summary of Lessons 1 & 2

Main Lesson Segments	Mode	Calculator Features Used	Who Uses the Calculator & How	Nature of Use of Calculator
<ul style="list-style-type: none"> • Introduction to statistical investigation <i>Yothus Lost at Sea</i>² 	Whole class, teacher-directed	NA1	NA	NA
<ul style="list-style-type: none"> • Students input set of data into calculators 	Whole class, teacher-directed	LIST	Teacher models, students reproduce	Result-generating
<ul style="list-style-type: none"> • Brief discussion of mean, median, mode 	Whole class, teacher-directed	1-VAR STATS	Teacher models, students reproduce	Conceptual
<ul style="list-style-type: none"> • Introduction to a box plot 	Whole class, teacher-directed	BOX PLOT	Teacher demonstrates, some students reproduce	Demonstrating
<ul style="list-style-type: none"> • Teacher sets homework for students³ 	Whole class, teacher-directed	NA	NA	NA

<ul style="list-style-type: none"> Brief discussion on range, interquartile range, and mode⁴ 	Whole class, teacher-directed	BOX PLOT ZOOM TRACE	Teacher models, students reproduce	Focusing Result-generating Conceptual
<ul style="list-style-type: none"> Teacher explains ideas about centredness 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Introduction to notion of spread (deviation) 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Derivation of average square deviation for one set of data 	Whole class, teacher-directed	1-VAR STATS LIST COMPUTATION	Teacher demonstrates, some students reproduce	Demonstrating Conceptual
<ul style="list-style-type: none"> Teacher sets homework for students⁵ 	Whole class, teacher-directed	NA	NA	NA

1 NA indicates that graphic calculators were not used in this segment.

2 Data for arm length for students of different ages were provided. Students were asked to organise and display the data in different ways in order to decide whether or not an arm found in the stomach of a shark could belong to one of three children of different ages lost at sea.

3 Students were asked to produce box plots for the remaining two sets of data in the same handout.

4 Discussion was based on results of the previous investigation *Youths Lost at Sea*.

5 Students were asked to derive average square deviation for two other sets of data.

The statistical investigation *Youths at Sea* formed the basis of Lesson 1, the first lesson in the Year 10 Statistics (data) unit. As can be seen in Table 1, both Lessons 1 and 2 exclusively used a whole class, teacher-directed mode of instruction. While in Lesson 1 distribution of the handout *Youths at Sea* signalled an investigatory emphasis as opposed to direct instruction, the teacher directly intervened in much of the lesson, with calculator

features such as LIST, 1-VAR STATS, and BOX PLOT being introduced. Students needed to rely on the teacher's directions.

In Lesson 2, basic statistics such as range and interquartile range were introduced. Mr D modelled the use of BOX PLOT, ZOOM, and TRACE, with most students being able to reproduce his actions. Students were learning to determine and understand these statistics by using the calculator features themselves. In the later interviews, students identified some of the ways in which they did this as working backwards, visualisation, and exploring concepts. The teacher acted as the main resource and motivator for the class activity. As opposed to the investigative thrust of the previous lesson, the bulk of Lesson 2 appeared to be the teacher's derivation of a "spread" statistic, the average square deviation. Mr D explained after the lesson that he wanted to show the students how this can be derived from first principles, thus introducing this mathematical concept clearly. As can be seen in Table 1, some students tried to reproduce what was being demonstrated by the teacher. As the concept of average square deviation seemed at first not easily understood through the demonstrated computations, learning to use the calculator features may have got in the way of students' attempts to understand the concept. In the next lesson, for instance, some students were still not able to understand why a particular computation was used in deriving the average square deviation.

In Lesson 3, students were expected to use the calculator features learned so far. This is suggested by the teacher simply demonstrating the use of a particular feature (in contrast to modelling it) and by the opportunity in these lessons for students to freely use the calculator in investigations while the teacher acted as a consultant. Students were also given the opportunity to familiarise themselves with the use of particular features while deriving the standard deviation.

The main focus of Lesson 3 was a statistical investigation of the relationship between two variables. The graphic calculator was a valuable tool in generating results to aid this investigation. Yet the tool doesn't substitute for student thinking, as the discussion following students' individual work (see Table 2) saw some students setting aside the calculator in order to make suggestions for resolving the questions in the investigation.

Lesson 4 differed in some respects from Lesson 3. While Lesson 3 saw instances where students participated in a full investigation, Mr D shifted back to a whole class, teacher-directed mode Lesson 4. In the first segment of this lesson only a few students were able to reproduce what the teacher had modelled. While the notion of cumulative frequency is not too complex, the production of a cumulative frequency graph, central to this lesson, was a source of difficulty for most students in the initial exercise. This apparent difficulty could be one reason that the teacher took on a more directive, whole class approach.

The main focus of the first segments of Lesson 5 was on computing the mean, and later the median, of weighted and grouped data. Mr D first discussed the basic statistical attributes of weighted data. As indicated in Table 3, he did this without using the graphic calculator. He then used the calculator to show the class how the mean of weighted data is calculated. Mr D then suggested that students individually use LIST and COMPUTATION to compute the mean of grouped data. Students chose their own methods, although some students heeded the teacher's suggestion. The fact that the class median was not a new concept for the students probably led Mr D to allow students to work in groups using the calculator freely.

Table 2

Summary of Lessons 3 & 4

Main Lesson Segments	Mode	Calculator Features Used	Who Uses the Calculator & How	Nature of Use of Calculator
<ul style="list-style-type: none"> A student explains her homework (Lesson 2) 	Whole class, teacher-directed	LIST COMPUTATION	Student demonstrates, others reproduce	Conceptual Focusing
<ul style="list-style-type: none"> Introduction to standard deviation 	Whole class, teacher-directed	1-VAR STATS	Teacher demonstrates, students reproduce	Result-generating
<ul style="list-style-type: none"> Students familiarise themselves with sequence of commands for st dev6 	Individual, teacher rooms	1-VAR STATS LIST COMPUTATION BOX PLOT	Students use freely	Conceptual
<ul style="list-style-type: none"> Introduction to statistical investigation <i>Comparing two sets of data</i> 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Students investigate relationship of two variables (same handout) 	Individual, teacher rooms	LIST 1-VAR STATS BOX PLOT	Students use freely	Investigating
<ul style="list-style-type: none"> Teacher initiates construction of a table, to compare variables 	Whole class, teacher-directed	1-VAR STATS	Teacher demonstrates, some students reproduce	Result-generating

<ul style="list-style-type: none"> Brief discussion about results of investigation 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Teacher sets homework for students⁸ 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Class exercise on producing cumulative frequency graph⁹ 	Whole class, teacher-directed	STAT PLOT (Line Graph) LIST	Teacher models, (few) students reproduce	Demonstrating Visualising
<ul style="list-style-type: none"> Discussion on cumulative frequency¹⁰ 	Whole class, teacher-directed	STAT PLOT (Line Graph) LIST	Teacher demonstrates, some students reproduce	Demonstrating Result-generating Conceptual
<ul style="list-style-type: none"> Finding for quartiles and the median graphically 	Whole class, teacher-directed	GRAPHING (Main) Line Graph TRACE	Teacher models, students reproduce	Demonstrating Visualising Conceptual
<ul style="list-style-type: none"> Introduction to the modal class 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Teacher sets reading assignment¹¹ 	Whole class, teacher-directed	NA	NA	NA

6 st dev denotes the standard deviation, which was obtained directly from average square deviation.

7 Based on a handout entitled *Smoking and Birth Weights*.

8 Students were asked to address other questions in the handout.

9 From their textbook.

10 Discussions focused on grouped data.

11 Students were asked to read about finding the mean of grouped data.
A graphic calculator booklet was given to students during the unit.

In the later segments of Lesson 5, Mr D reinforced the meanings of class attributes such as mean, median, and mode. Again, as evident from the second column of Table 3, he addressed the whole class and directed the remaining activities in the lesson. By this time, all of the statistical features relevant to the whole unit had been introduced and used. Lesson 5b is representative of most of the lessons in this unit in terms of the predominance of the "teacher models or teacher demonstrates, students reproduce" use of the calculator.

Table 3

Summary of Lessons 5a & 5b

Main Lesson Segments	Mode	Calculator Features Used	Who Uses the Calculator & How	Nature of Use of Calculator
<ul style="list-style-type: none"> Teacher asks students about computing mean of weighted data 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Brief discussion on mean, median, mode of weighted data 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Teacher explains the formula for computing mean of weighted data 	Whole class, teacher-directed	COMPUTATION	Teacher demonstrates, some students reproduce	Result-generating

<ul style="list-style-type: none"> Students do an exercise on grouped data¹² 	Individual, teacher roams	LIST COMPUTATION	Teacher suggests, students use	Result-generating Demonstrating
<ul style="list-style-type: none"> Teacher clarifies notions of class mean¹³ 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Students do another exercise^{3/4} solving for class median 	Small group, teacher roams	LIST STATPLOT GRAPHING TRACE	Students use freely	Result-generating
<ul style="list-style-type: none"> Teacher details the procedure for obtaining the class median and class mean 	Whole class, teacher-directed	LIST ZOOM / TRACE STAT PLOT COMPUTATION	Teacher demonstrates, some students reproduce	Demonstrating Result-generating
<ul style="list-style-type: none"> Teacher illustrates the concept of modal class 	Whole class, teacher-directed	STAT PLOT (Line Graph)	Teacher demonstrates, some students reproduce	Demonstrating
<ul style="list-style-type: none"> Discussion on the meanings of class median and IQR¹⁴ 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Use of a box plot to obtain the class 	Whole class, teacher-directed	LIST BOX PLOT	Teacher models, students	Demonstrating Result-

median		TRACE	reproduce	generating
<ul style="list-style-type: none"> Teacher suggestions on comparing sets of data 	Whole class, teacher-directed	NA	NA	NA

12 A list of exercises on grouped data the textbook.

13 "Class" mean corresponds to the mean of grouped data or data specified in intervals and

frequencies. Similarly for other "class" attributes.

14 IQR denotes interquartile range.

Lesson 6 differed from the previous lessons in that it was a problem-solving lesson. Mr D posed the *Midge Problem* in order to allow students to apply and appreciate what they have learned thus far. For most of the lesson, students were using the graphic calculator freely or as suggested by the teacher (see Table 4). In this lesson, the value of having a tool that could "process" lists and yield useful results for problem solving can be seen. As some students later claimed, the graphic calculator acted as a problem-solving tool, or, as Mr D claimed, a tool for investigation, confirming the value of graphic calculators in higher-order activities.

Table 4

Summary of Lessons 6 & 7

Main Lesson Segments	Mode	Calculator Features Used	Who Uses the Calculator & How	Nature of Use of Calculator
<ul style="list-style-type: none"> Teacher sets problem solving task15 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Teacher gives hints on solving the problem 	Whole class, teacher-directed	NA	NA	NA

<ul style="list-style-type: none"> Teacher advises students to input sets of data for the given problem 	Whole class, teacher-directed	LIST	Teacher suggests, students use	Result-generating
<ul style="list-style-type: none"> Students attempt to solve the problem 	Individual & small group, teacher roams	LIST BOX PLOT 1-VAR STATS COMPUTATION	Students use graphic calculator freely	Visualising Investigating Result-generating
<ul style="list-style-type: none"> Teacher makes further suggestions 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Teacher explains the use of statistics in the previous problem 	Whole class, teacher-directed	NA	NA	NA
<ul style="list-style-type: none"> Students continue working on the <i>Midge Problem</i> 	Individual and small group, teacher roams	COMPUTATION LIST BOX PLOT	Students use freely	Result-generating Investigating
<ul style="list-style-type: none"> Teacher makes suggestions and gives explanations <p>16</p>	Whole class, teacher-directed	NA	NA	NA

15 The *Midge Problem* asked students to determine whether it is possible to distinguish between two species

of midges based sets of measurements of wing and antenna length for the two species.

16 The teacher explained the difference between using ratios and differences in determining whether the two

species of midges are distinguishable from each other.

The teacher suggested ways of writing up the solution to the problem. Students were due to submit their solutions for assessment shortly after this lesson.

Lesson 7 continued the problem-solving activity from Lesson 6. The individual and group work that again took the most time in this lesson involved students in a motivated search for solutions. Students had the opportunity to share ideas about conducting the investigation (an aspect of the problem) and on the use of the calculator's statistical features. It is important to note that Mr D was able to devise this problem in the first place because graphic calculators were available to facilitate various aspects of solving the problem.

The students' perceptions

Questions in the student interview were grouped into six sections: attitude towards mathematics; extent of use of graphic calculators; effect of graphic calculators on enjoyment, skills and understanding; features of the graphic calculator used in lessons and their relative importance; students' use of graphic calculators in the observed lessons; the teacher's use of the calculator; and a general summary of the role of graphic calculators.

The interview transcripts were analysed in a similar manner to the one from the teacher interview. A total of eight themes or categories emerged from the analysis of the transcripts: enjoyment of mathematics; mathematical skills; conceptual understanding; graphic calculator features used; purposes for which the graphic calculator was used; learning to use the calculator; the teacher's use; and the effect on thinking. The report below draws on most of these categories.

Three of the five students, Cathy, Debbie, and Susan, were identified by the teacher as consistently high performers in mathematics. During the observed lessons, Cathy and Debbie were vocal and active in class, while Susan was less vocal but contributed well to a discussion when asked by the teacher. Joan was also less vocal in class and was seen by the teacher as being average in performance. Trish was quiet throughout the lessons observed and was identified by the teacher as struggling in class.

All the five students indicated that they enjoyed mathematics more when the graphic calculator was used. Two main reasons emerged from the student responses: using graphic calculators removes the tedium of otherwise lengthy calculations and other demanding mathematical tasks, and using the calculator takes less time.

Table 5 shows that all five girls believed that using graphic calculators removes tedium. For example, Cathy stated:

I think it [the graphic calculator] definitely makes it [maths] more enjoyable, coz if I didn't have a calculator there, every move that I did, I'd have to be writing down, and rethinking that, instead of just knowing the method of how to do it. (Cathy, IntC, p.3)

Table 5

Students' perceptions of the effect of graphic calculator use on their enjoyment of mathematics

Code	Meaning	Cathy	Debbie	Joan	Susan	Trish
RT	Removes Tedium	√	√	√	√	√
LT	Takes Less Time		√			√

Trish and Debbie also mentioned the time factor. For example, Debbie stated that:

In Year 7 and Year 8 when we're doing plot graphs on linear ... you draw out the graph and it takes 3 hours But with the calculator it makes it so much easier coz you can just put it in and look at it and analyse it ... it's more enjoyable coz you don't have to sit there and draw them all out, (Debbie, IntD, p. 3)

Students were asked their views on the effect of graphic calculators on their mathematical skills. Table 6 shows that Cathy, Joan and Trish believed that with the constant use of graphic calculators in their classes, some mathematical skills were unused and then forgotten. According to Trish:

In Year 7 and Year 8, we [didn't use] ... the calculator much, and so my mind was very good with just simple stuff, but now I don't think I can really do them ... you just get the calculator coz it's quicker. You don't have to think. (Trish, IntT, p. 3)

Debbie and Susan, on the other hand, did not believe that the use of graphic calculators had affected their skills negatively, as can be seen from Debbie's comment below:

I don't find it hard so I can still draw graphs pretty well if you have to, I don't think it's [a problem]. (Debbie, IntD, p. 4)

Table 6

Students' perceptions of the effect of graphic calculator use on their mathematical skills

Code	Meaning	Cathy	Debbie	Joan	Susan	Trish
US	Unused Skills	√		√		√
NS	Neutral on Skills		√		√	√

In addition to being asked about the effect of the calculator on mathematical skills, the students were also asked their views on the effect on their understanding of mathematical concepts. Table 7 summarises the three ways in which students identified graphic calculator use as enhancing their understanding of mathematical concepts.

Table 7

Students' perceptions of the effect of graphic calculator use on their understanding of mathematical concepts

Code	Meaning	Cathy	Debbie	Joan	Susan	Trish
WB	Working Backwards	√			√	
VT	Visualisation Tool		√			√
EC	Exploring Concepts		√		√	

Cathy and Susan tried to explain how graphic calculator results gave them something to focus on, allowing them to "work backwards" by using the calculator display to trace back the meaning of concepts, a process that Cathy struggled to explicate.

Yeah, I think it did [help me understand things]. Coz it gives me the answer and you can look at the answer and then you can figure out where it's come from. ... Well, I basically just put it in and see what happens and work from there, change it like, coz if I look at it long enough, then something comes free, like I'll pick up on something and push on, something will tell me. (Cathy, IntC, pp. 4; 6)

Susan's explanation of how she used the display to "work backwards" also reflected her way of exploring concepts such as the median .

It would help me like in understanding, in the sense that if I know the answer, I kind of think of that like if it shows the median, before I don't know what it means, and then it shows you the answer, what the median was for this particular range. So I look at the mean and I look at the median and realise that it's the middle number. So it helps me like that. (Susan, IntS, p. 5)

Debbie also mentioned an example of how her use of the graphic calculator helped her explore the meaning of constants in a linear equation.

Well, we did with the linear we're doing, different kinds of graphs and we did like the $y = x$, what you did to it, we can see what putting different numbers will change the graph or you can have them all up at the same time. You can see what happened if you change this, which makes it easier to picture it. (Debbie, IntD, p. 4)

Debbie's last comment shows that she also found it easier to visualise things with the calculator's graphic displays. Trish also considered the calculator helpful in her understanding of concepts because of her access to graphical aspects.

I think it does [help me understand] coz it's more visual with the box plots. And then, it has, shows bits of quartile range, and everything, I think it's better. (Trish, IntT, p. 4)

Debbie also saw the calculator as assisting her in understanding a problem.

It makes it a lot easier, it just helps me understand, to understand/visualise things better so I can actually see what the problem is, coz I need to see it on paper, you've got everything in front of you, but with the calculator you can actually like see the box plots and the graphs and things that really help you to get hold of everything. Coz sometimes, it's too much to focus on at one time, so the calculator helps just to break it down and to visualise everything. (Debbie, IntD, pp. 8–9)

Joan, on the other hand, gave no indication whether her use of graphic calculators enhanced her understandings. When asked what she perceived to be the effect of graphic calculator use on her understanding of mathematical concepts, she said:

I understand the main concept. It's just like when you go to graphing and you have to do like that. (Joan, IntJ, p. 5)

Students were also asked how they or their teacher used the graphic calculator. While all students commented on the use of the calculator as an "all round" tool — for example to draw graphs, carry out simple calculations, draw box plots — Debbie and Susan also mentioned their use in solving problems, where the distinction is made here between complex (problem solving) and less complex (all round) purposes (see Table 8).

Table 8

Uses of graphic calculators as reported by the students

Code	Meaning	Cathy	Debbie	Joan	Susan	Trish
AR	All Round tool	√	√	√	√	√
PS	Problem Solving tool		√		√	
TD	Teacher Demo tool	√		√	√	√
HO	Homework / Other	√	√		√	√

The girls maintained that the teacher used the calculator, with an overhead projector, primarily for demonstration. For example, Trish stated that:

He uses it to demonstrate things, or to show us a technique, how to do some problems or introduce new things ... stuff like that. (Trish, IntT, p. 4)

A theme emerging from some of the responses of the students concerning the usefulness of graphic calculators was the need to do mathematics without calculators. These comments were probed further in the interviews as they relate directly to current calculator issues regarding dependency and de-skilling.

Table 9

Students' views on the effect of graphic calculators on thinking

Code	Meaning	Cathy	Debbie	Joan	Susan	Trish
AT	Affects Thinking (neg)	√			√	√
HI	by Hand is Important	√			√	√
ET	Enhances Thinking		√			
NT	Neutral on Thinking			√		

It is clear from Table 9 that Cathy, Susan and Trish thought that, as a consequence of calculator use, they were not able to think enough. They wanted more time to be spent on doing things without the graphic calculator. It appears that, although these students enjoyed doing mathematics more with the calculator and found it very useful, they associate understanding with doing tasks manually. Trish commented that

I think it's good but we should move how to do it, just like manually, coz sometimes, you won't have the calculator on you all the time and you might do something and you'd be relying on the calculator and you wouldn't, don't know how to work it out. (Trish, IntT, pp. 5–6)

Joan appeared to take a neutral position, stating that

Maybe, it makes me think in everything, like I still do all the working out, it's just when it comes to getting the answers and I just type it in, and it does it all rather than doing it by hand (Joan, IntJ, p. 9)

Debbie however saw the calculator as helping her to get a wider view and to think significantly about problems.

But with the calculator you can actually like see the box plots and the graphs and things that really help you to get hold of everything, coz sometimes, it's too much to focus on at one time, so the calculator helps just to break it down and to visualise everything. (Debbie, IntD, pp. 8–9)

In summary, the analysis of student interviews shows that the use of graphic calculators in class were perceived to affect students' enjoyment of mathematics, mathematical skills, and conceptual understanding. All of the students claimed they enjoyed mathematics more, primarily because it removes the tedium of certain mathematical tasks (e.g. drawing graphs). Three students said that always using the calculators in tasks negatively affected their mathematical skills (e.g. mental computational). Two students, on the other hand, claimed that the use of graphic calculators didn't affect their skills negatively. They believed they could still accomplish mathematical tasks without the calculators. In terms of conceptual understanding, all of the students except Joan indicated three ways (working backwards, visualisation, exploration) in which graphic calculator use had helped them understand some concepts.

The students saw the graphic calculator as an all-around tool that can be used in most mathematical topics. Debbie and Susan went further, saying that they also used the graphic calculator as a tool for problem solving.

Regarding calculator dependency and consequent de-skilling, an earlier comment by one student on "thinking less" was probed. Three students complained about "not being able to think enough" when they consistently used the calculators for tasks and felt it valuable to "do things by hand" more often, which they presumed would make them think more and so learn better. Joan, on the other hand, felt that she would have to think regardless of whether the graphic calculator is used or not, while Debbie implied that the use of graphic calculators enhances her thinking.

Conclusion

Mathematics educators have written extensively about the educational utility of graphic calculators. Groves (1991), for example, focused on the conceptual and investigational uses of the calculator. The results from the interviews of both students and the teacher tend to validate such categories, and, to some extent, encourage rethinking the significance of further categories. In analysing the classroom observations, the ways of using graphic calculators reported by the teacher and interviewed students were integrated. From this analysis, the graphic calculator was found to be used as a visualisation tool for students, a demonstration tool for the teacher, a conceptual tool, a tool for investigation, a focusing tool, and a result-generating tool.

The analysis of the teacher's philosophy of mathematics teaching reveals that his main goal was to help students become critical thinkers when and after they study mathematics — i.e. he was interested in the effects of technology use (Salomon, Perkins & Globerson, 1991). This entailed de-emphasising mindless mastery of mathematical procedures and focusing more on the higher aims of mathematics — e.g. students' ability to solve genuine problems and model real situations. He devised or adapted "technology rich" mathematical tasks, not for the sake of using technology *per se*, but because he saw the potential of the calculator to provide an environment to promote mathematical thinking. However this did not always occur, for example when the calculator was consistently used as a result-generating tool. Moreover, it was only in the last two lessons observed, that substantial amounts of time were available for students to freely carry out investigations and solve problems, with the mode of instruction in the other lessons mostly being whole class, teacher-directed. In most instances, the use of calculators was either "teacher models, students reproduce" or "teacher demonstrates, some students reproduce".

While the students identified ways in which the graphic calculator enhanced their conceptual understanding, some were concerned about de-skilling effects of calculator use. This raises the issue of the role of exercising basic mathematical skills in achieving the goals of school mathematics. On the one hand, in light of technologies that obviate the need for certain mathematical skills, it would seem foolish for contemporary mathematics to invest heavily in the development and practice of such skills. An alternative focus, consistent with the problem solving and modelling thrust of contemporary school mathematics, is the interpretation and "making sense" of potential mathematical situations, rather than students exercising basic skills in accomplishing tedious tasks. On the other hand, we need to take seriously the notion of intelligent partnerships between humans and technology (Salomon, Perkins & Globerson, 1991; Jones, 1993) as pointing the way to the future.

References

- Asp, G. & McCrae, B. (2000). Technology-assisted mathematics education. In K. Owens & J. Mousley (Eds), *Mathematics education research in Australasia 1996–1999* (pp. 123–160). Campbelltown, NSW: Mathematics Education Research Group of Australasia.
- Berger, M. (1999). Graphic calculators: An interpretive framework. *For the Learning of Mathematics*, 18(2), 13–20.
- Dunham, P. H. & Dick, T. P. (1994). Research on graphing calculators. *Mathematics Teacher*, 87(6), 440–445.
- Dunham, P. (2000). Hand-held calculators in mathematics education: A research perspective. In E. Laughbaum (Ed.), *Hand-held technology in mathematics and science education: A collection of papers*. Columbus, Ohio: The Ohio State University.
- Galbraith, P., Renshaw, P., Goos, M. & Geiger, V. (1999). Technology, mathematics, and people: Interactions in a community of practice. In J. M. Truran & K. M. Truran (Eds), *Proceedings of the Twenty-second Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 223–230). Adelaide: Mathematics Education Research Group of Australasia.
- Groves, S. (1991). Using graphic calculators to support investigative learning. In O'Reilly, J. & Wettenhall, S. (Eds). *Mathematics: IDEAS*. (Proceedings of the 28th Annual Conference of the Mathematical Association of Victoria, pp. 356-359). Melbourne: Mathematical Association of Victoria.
- Heid, M. (1997). The technological revolution and the reform of school mathematics. *American Journal of Education* 106(1), 5–61.
- Jones, P. L. (1993). Realising the educational potential of the graphics calculator. In L. Lum (Ed.), *Proceedings of the Sixth Annual International Conference on Technology in Collegiate Mathematics*. (pp. 212–217). Reading, MA: Addison-Wesley.
- Jones, P. L. (1996). Handheld technology and mathematics: Towards the intelligent partnership. In P. Gomez & B. Waits (Eds). *Roles of calculators in the classroom*. (pp. 87–96). <<http://ued.uniandes.edu.co/roless-calc.html>> [Accessed 17 May 2001]
- Obregon, D. N. M. (2000). *Use of graphic calculators in upper secondary mathematics — A case study*. Unpublished Master's research paper, Deakin University.
- Pea, R. D. (1985). Beyond amplification: Using the computer to reorganise mental functioning. *Educational Psychologist*, 20(4), 167–182.
- Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89–122). Hillsdale, NJ: Lawrence Erlbaum.

Penglase, M. & Arnold, S. (1996). The graphics calculator in mathematics education: A critical review of recent research, *Mathematics Education Research Journal*, 8(1), 58-90.

Ruthven, K. (1992). Personal technology and classroom change: A British perspective. In J. T. Fey & C.R. Hirsch (Eds). *Calculators in mathematics education: 1992 yearbook*. (pp. 91–100). Reston, VA: NCTM.

Ruthven, K. (1995). Towards considered calculator use. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching — A bridge between teaching and learning* (pp. 231–256). Bromley, Kent: Chartwell-Bratt.

Ruthven, K. (1996). Calculators in the mathematics curriculum: The scope of personal computational technology. In A. Bishop et al (Eds), *International Handbook of Mathematics Education* (pp. 435–468). Dordrecht: Kluwer Academic Publishers.

Salomon, G., Perkins, D. N. & Globerson, T. (1991). Partners in cognition: Extending human intelligence with intelligent technologies. *Educational Researcher*, 20(3), 2–9.

Tharp, M., FitzSimmons, J. & Brown Ayers, R. (1997). Negotiating a technological shift: Teacher perception of the implementation of graphing calculators, *Journal of Computers in Mathematics and Science Teaching*, 16(4), 551–575.