New Literacy’s in Mathematics: Implications for Teacher Education

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Abstract: School mathematics has changed over the last number of decades from pre-Sputnik curricula, through "modern math", back-to-basics, and various "Standards" adaptations. Recently, as we crossed over into the new millennium, calls for new literacy’s in mathematics have once again emerged. These calls tend to be multi-faceted suggesting new or altered school content, perhaps for statistical reasoning, or for more spatial sense but less proof. These calls also provide deliberate focus on both understanding of computational strategies as well appropriate application; more focus on process components such as modeling and problem posing; and technological applications to include roles for both computers and graphing calculators (Steen, 1998; Charles & Lobato, 1998; Organisation for Economic Co-operation and Development, 1998). Additionally many argue that mathematics learning must provide opportunities for group and project work that enable students to make connections among various components of the mathematics and beyond mathematics curriculum. In light of these "new literacy" components, significant concerns result. Do (beginning) teachers need courses in advanced pedagogical content knowledge? (Usiskin, 2001) How can teachers respond to the "mathematics for all students" challenge? (Kilpatrick, in press)

Literacy and New literacy’s

Mathematical literacy, numeracy (Steen, 1998) and, most recently, proficiency (Kilpatrick, Swafford & Findell, in press) are phrases that have all been used to describe in some way an individual’s capacity to function mathematically at some competent level in a particular society or culture. This capacity generally is understood to include various facts and skills, processes, and applications essential to daily living and working. (Edge, 2001) This construct of ‘mathematical literacy’ has, however, been subject to much debate for a number of reasons in response, for example, to the Russian ‘Sputnik’ program, to the ‘modern mathematics’ implementations, and to back-to-basics movements. Most recently educators have taken advantage of the move into the new millennium to evaluate once again the suitability of our mathematics programs. Educators thus question whether or not we are adequately preparing our students to meet their future needs. Calls for new literacy’s abound. (Burton, 1996; Steen, 2001; Lee, (in press)).

An important question, given the ‘subject to change’ view of mathematical literacy as noted above, then becomes "What mathematical teaching and learning changes can we reasonably expect within the next ten to twenty years?" To examine this question, I structured my research by using a Singapore curriculum model that has problem solving as its core and complementary components of skill, concept, process, attitude and metacognition. (Ministry of Education, Singapore, 2000) Conclusions from those investigations included the following points:

• mathematics must be learned with an intent to understand, with a view that it can be used, and that it be seen as a field itself worthy of study;
• with respect to content changes, in primary mathematics classrooms, pupils will study more and different statistics (fairness and chance), have more of an activity based geometry program that includes transformational geometry and IT applications, and experience an algebra where the focus will be on pattern seeking and prediction behaviours;

• with respect to both oral and written forms of language, pupils will be expected to present reports, write in journals, and complete projects with graphics, text, illustrations and web references; and

• student evaluation will be enhanced by utilizing alternative forms of assessment. (Edge, 2001)

As a mathematics educator, the rationale for the next question to address seemed fairly obvious. If I understood and accepted these potential changes to students’ curriculum, "what changes should I implement in my own teaching of pre-service and graduate education teachers?". The purpose of this paper is thus to present and discuss a few resulting consequences.

Teacher Education Implications

The implications for teacher education can be grouped under principal headings of knowledge of content, knowledge of pedagogy, and curriculum issues. However, to appreciate the discussion of the implications, several points need some consideration. It is fair to say we now live in a global environment where many factors, including education, are characterized by rapid growth and uneven expansion. Needs and wants of any one group may very well be different from those needs and wants of another group. Perhaps these expectations are culturally or socially driven; they may be economic. Some jurisdictions may have implemented certain initiatives. In others, implementation has not (yet) been considered. Hence, depending on our own situations, we may think of a thought or recommendation as something novel or we may find ourselves asking if anything is really new. Have we not been told to teach for understanding before (Brownell, 1935), to incorporate statistical thinking into early grades curriculum (Shulte, 1981), and to use assessment strategies other than pencil and paper testing (Clarke, 1988; NCTM, 1989)? We may find evidence of people asking for what appears to be well implemented elsewhere. These comments notwithstanding, there are areas where change within the next decade appears likely. Let us begin with mathematics content knowledge.

Knowledge of mathematics content:

Two types of mathematical knowledge that we have long associated with teacher knowledge of mathematics are the formal academic courses teachers studied in university and those courses especially mounted to address topics associated with school mathematics. In this latter category one finds textbooks with names such as "Mathematics for elementary school teachers" and which include topics such as "Babylonian mathematics" and "number theory" where concepts such as prime / composite, perfect / non-perfect, and factor / multiple are reviewed.

There is a third area of mathematics though, described as mathematics that teachers need to know yet have not encountered (Usiskin, 2001), that needs to be considered. For example, prospective teachers may well know how to add and subtract, yet do not have any sense of place value. Do they understand why an algorithm works? Do they know why one algorithm might be preferable to another? One former student of mine lamented angrily that had she not come to teachers college she would never know what "borrowing in subtraction"
meant. Parenthetically we may argue that a non-teacher does not need to have knowledge of regrouping procedures, but the same argument cannot be made for teachers. Teachers require concept analysis abilities to effectively plan lessons, to diagnosis pupil errors, and to plan for effective assessment. Usiskin categorized what he termed this "teachers’ mathematics" into three kinds of mathematics "not found in typical college mathematics courses" (Usiskin, 2001, p. 86). The three kinds are mathematical generalizations and extensions, concept analysis, and problem analysis. This body of mathematics includes, he argued:

- explanation of new ideas,
- alternate definitions and their consequences,
- wide ranges of applications and how they have changed over time, and
- how problems encountered in school may relate later to mathematics study. (p. 9)

Usiskin illustrated his *generalizations and extensions* category by examining a collection of \( \frac{1}{2} \) "area of triangle" proofs that extend from the simple \( A = \frac{1}{2} bh \) to one requiring knowledge of trigonometry: \( A = \frac{1}{2} ab \sin C \). For *concept analysis*, he described examples relating to
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\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1, \quad \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \]

explanations for why \( 0.999\ldots = 1 \) [\( 0.333\ldots = \frac{1}{3} \); \( 0.666\ldots = \frac{2}{3} \); and since \( \frac{1}{3} + \frac{2}{3} = 1 \), and \( 0.333\ldots + 0.666\ldots = 0.999\ldots \), therefore \( 0.999\ldots = 1 \). And so on.] One of my pre-service students looked at me incredulously as I tried to make the point that \( 0.999\ldots \) and \( 1 \) had the same value. Did our subsequent brief discussion help?

The *problem analysis* activity that Usiskin discussed is too complex to outline here but begins with a simple average question found in beginning secondary school texts and concludes with an interesting situation where one may use a graphical model to predict who might finish a basketball season with a higher scoring record. [Particularly interesting - if you are a basketball fan and follow the careers of O’Neal and Jordan!]

Usiskin is however only one of many to highlight the need for teachers to acquire content knowledge different from the kind they normally receive in college level instruction. For example, Steen (2001) differentiated between numeracy and mathematics with a clear implication that teachers require numeracy as well as mathematical knowledge. "Not calculus but numeracy is key to understanding in our data-drenched society". For Steen, elements of numeracy include cultural appreciation, mathematics in context, and number and symbol sense. Armstrong et al. (undated report, circa 1995), in a report on preparing middle school teachers to teach multiplicative structures, made a number of recommendations relating to the need for teachers to make sense of what they were to teach – teachers needed to see mathematics as a sense making activity itself, to use both conventional and unconventional notational forms to express their reasoning about quantitative relationships, and to mathematize simple and complex situations. Further, Devlin (2000), stressing that teachers need further specialized forms of mathematical knowledge, argued that "four faces of mathematics" be developed in young people: mathematics as computation, formal reasoning and problem solving; as way of knowing; as a creative medium; and applications. In schools we concentrate on the first, perhaps some on the fourth. We need to concentrate on all four! Teacher preparation and curriculum revision are key. (p. 26)
Before leaving this topic, it might be suitable to make one other point. Fennema and Franke (1992) in their discussion of teachers' knowledge of mathematics discussed collected "evidence" that suggested that there was no relation between teacher mathematical knowledge and student learning. However, they noted that "measure of mathematical knowledge" was "number of college level mathematics courses taken by the teacher" and concluded that "no attempt was made to measure what the teachers knew about mathematics nor to ascertain accurately what the mathematics covered in the various courses completed". (p. 148) Perhaps the original research should not have counted number of college level mathematics taken but questioned instead the kinds of mathematics being studied. More 'teacher mathematics' thus may, in fact, be beneficial.

Knowledge of pedagogy (learning and teaching):

Although some have argued there is little researched theory of mathematics education (see an interesting analysis in Steen, 1999), others have taken a different view. Bransford, Brown and Cocking (2000), have suggested there is an emerging theory (of mathematics education) coming into focus that provides, for example, new knowledge about competent performance, young children's acquisition of reasoning capabilities, and learning in a variety of social contexts. They also noted that much of this theory is being developed by researchers now working in schools with classroom teachers. Within this development of the science of learning, Bransford et al. focused on selected 'hallmarks':

- learning with understanding - Knowledge of facts alone will not produce learning; 'usable knowledge' has the facts somehow embedded in structure. Students, to become competent learners, must have deep foundations of factual knowledge, understand these facts and concepts in the context of a conceptual framework, and organize knowledge in ways that facilitates retrieval. (p.16)

- notions of pre-existing knowledge – Children come to formal education with ranges of prior knowledge, which in turn affects their ability to acquire new knowledge. Generally described as a 'constructivist position', this constructivism, however, must be clarified to acknowledge the role of lecture and class discussion. If children have appropriate structures in place, within the context of pre-existing knowledge, it must be recognized that not all learning has to begin with children using manipulative materials.

- active learning – Learners must come to understand for themselves what they know and what strategies they can use to determine their levels of understanding. Metacognitive skills such as prediction, explanation to self, and error analysis hence are critical activities.

In addition to developing a better understanding of learning, we are also expanding our knowledge of teaching. For example, by studying teachers in Japan and Germany and comparing them to teachers in the United States, Stigler and Hiebert (1999) concluded that teaching methods, not teachers per se, were the critical factor in promoting student learning. American teachers tended to have a narrow band of procedural skills when compared to their Japanese counterparts. The Japanese teachers spent more time helping pupils solve challenging problems and discussing concepts rather than allowing them to practice skills. Specifically Stigler and Hiebert wrote:

These [teacher] differences, which appear so large within our [American] culture, are dwarfed by the gap in general methods of teaching that exists
across cultures. We are not talking about a gap in teachers’ competence, but a gap in teaching methods. (p. x)

Stigler and Hiebert suggested more study of the Japanese teachers’ approach to lesson planning as perhaps one way of examining or developing a theory of teaching. By using an eight step model that included planning, implementation, reflection, and so on, these teachers were seen as contributing to the development of knowledge about teaching.

In this section I considered a few ideas about our knowledge of pedagogy. The point, however, was not to discuss these ideas in particular detail but, relating to the purpose of this paper, rather to give the reader some sense of the type of changes we might expect within the next decade and to consider implications for how teachers might be better prepared.

Curriculum issues:

In terms of ‘new literacy’s and teacher education implications, there is another set of issues or concerns that we must also address. As well as having to come to terms with adjustments to ways in which we teach, we may need to learn new mathematics content to meet curriculum change requirements. In Singapore, for example, in primary schools, ‘probability and statistics’, except for a short unit on finding average, is operationally defined as drawing bar, circle, picture and line graphs. In Canadian schools, however, children are likely to experience mathematics instruction that also includes notions of chance and fairness as well as with other graphing activities with stem-and-leaf plots and box-and-whisker graphs. (Ministry of Education and Training, Ontario, 1997). Some teachers will need to learn new content. Transformational geometry is another example where additional studies may be needed. Decisions related to curriculum content change will affect teachers and thus must affect those involved in teacher education. Broadly speaking, we shall all have to come to terms with “life long professional development”.

Other areas likely to experience change are information technology, language use, and assessment strategies, to name a few. Graphing calculators and expanded uses of computers, asking children to read and write reports in mathematics, and incorporating alternative assessment options such as project or portfolio evaluation will all require teachers to further their own pedagogical learning. To provide an example related to programming and assessment changes, a year ago the Singapore Ministry of Education announced that students would be required to do project work as a component of evaluation for their "A-level" examinations. Mostly recently the Ministry announced that some schools might in fact forgo having students write "O-level" examinations entirely. Schools should feel free to propose six-year curriculum structures culminating directly in "A-level" qualifications for students. (Davie, 2001). Reasons for the changes include providing more program flexibility for students, more experiences requiring elements of creativity (acknowledged as more time consuming), and so on.

There are other directions emerging in mathematics education all which have implications for teachers’ professional development, in-service, and graduate education. Equity and social justice issues, alternatives to public education, and global influences resulting from research, cross-cultural experiences and international tests are all examples of these emerging directions. All have the potential to influence teacher education activities.
Conclusion

Now, to ask if I have answered the original question: If there are new literacy's in mathematics, what changes must I, as a mathematics teacher educator, consider? Before answering, understanding the question suggests that any mathematics educator must first contextualize his or her answer. What country do I live in? (What are cultural norms and values?) What is the local school curriculum and is there some interest in, or need for, change? What are my students like? (What is their background – academic content, abilities, and attitudes?) What kind of students will they be asked to teach? And under what kind of constraints must we all work (economic, political, professional, and so on)?

With these constraints in mind, in this paper, three general implications were addressed:

- Teachers need to learn more mathematics that allows them to develop deeper understanding of the mathematics they will be asked to teach.
- A clearer idea of how children learn is beginning to emerge – to understand the need for children to develop complex mental structures and their need to be able to monitor this development.
- A theory of teaching is beginning to be formulated – one that promotes the development of these mental structures through process and content learning and that incorporates principles of understanding, consolidation and application of mathematics.

How do we change our (teacher education) classroom instruction? That is the real question, of course! Each of us will have to answer that question personally but the principal question will have to be centered on whether our students leave our class with a ‘bag-of-tricks’, so to speak, or whether they have notions of a theory of teaching that includes an integrated set of models of instruction. When and how do you promote understanding? What forms of drill and practice are effective, and when best implemented? What type of application and problem solving activities are appropriate for my students? These foreshadowing questions, are, however, somewhat simplistic. Better, perhaps, will my students know that ‘understanding’ is complex and involves schematizing and pattern seeking activities? Will they know which manipulative materials might be helpful to promote effective learning? Will they select different modes of instruction to reflect learning demands (direct-, activity-, cooperative-based, and so on.)? This list is but the tip of the iceberg! Critically, will they know what to think about when they don’t know what to do!

References


