Australasian Association for Research in Education (AARE) Sydney 2000

Raising standards in mathematics through effective classroom practice

Authors: Sonia Jones: sonia.jones@swan.ac.uk

Dr Howard Tanner: h.f.tanner@swan.ac.uk

Dr Mike Treadaway mtread@rmplc.co.uk

Address: University of Wales Swansea, Department of Education, Hendrefoelan, Swansea SA2 7NB, Wales, UK.

Abstract

This paper reports some of the results of the "Raising Standards in Numeracy" project which was funded by the Welsh Assembly during the period 1999/2000. Schools in which pupils achieved standards significantly higher than would have been expected from their prior attainment were identified using value-added analyses in order to explore factors and strategies which might contribute positively towards standards in mathematics. Two primary and two secondary schools were identified in each of five LEAs. Pedagogical factors contributing to high attainment were then explored through extended interviews with LEA advisors, head teachers, mathematics subject leaders and classroom teachers. These factors were triangulated at classroom level through participant observation. This paper focuses on features of classroom practice, making only brief mention of factors at the level of whole school and subject leader. These features are contrasted with findings from other projects and analysed to provide a rationale for good practice.

Introduction

This paper reports on the "Improving Standards in Numeracy" strand of a collaborative research project which involved five Welsh local education authorities (LEAs) and the University of Wales Swansea, and funded by the Welsh Assembly. As part of the project, evidence from 20 particularly successful schools in the five LEAs was analysed to identify factors which appeared to contribute positively towards standards in numeracy and mathematics. This paper focuses on features at the level of classroom practice and discusses why the observed practices should enhance standards in mathematics.

School effectiveness and value added

The project was built around earlier work by the Vale of Glamorgan LEA involving the development of value-added analyses linking National Curriculum data to prior attainment scores, and the development of an approach to target setting based on pupil-level data.

A considerable body of research suggests that individual schools can exert an influence above and beyond the qualities of their incoming pupils, their families and even the social determinants of gender, class and race (Mortimore et al, 1994, p316). Debate now centres on how the effectiveness of individual schools should be measured.

In relation to the insights they provide, league tables based on raw assessment data are rarely worth the paper they are printed on as they say more about intake than school effectiveness (Murphy, 1997, p33). Value-added methods, based on the progress pupils make rather than raw assessments, should be used to evaluate the differential effectiveness of schools (Mortimore et al, 1994; Nuttall, 1990; Saunders, 1999). Value-added is not a well defined term, however, (see, for example, Schagen, 1998) but, in general, the term is used to represent a fairer way of measuring pupils' performance, stripping away those factors which are associated with attainment but are not related to institutional quality (Saunders, 1999), and that is the definition used here.

Methodology

Value-added analyses, supplemented with other longitudinal data, were used to identify schools where pupils obtained significantly higher than expected scores in statutory tests (fuller details are given in Jones et al, 2000 and Tanner et al, 2000). Two primary and two secondary schools which were particularly successful in developing the mathematical abilities of their pupils were identified in each LEA. These schools were then visited by the researchers, who interviewed head teachers, subject leaders and teachers in order to ascertain the factors to which they attributed their success and, in particular, to identify and to investigate factors contributing to the effective teaching of mathematics. Lessons were observed to examine the classroom processes in practice. Common factors of good practice were then identified at the levels of whole school, subject co-ordinator, and individual teacher.

The intention of this research was to determine the nature of those common factors rather than to consider whether the educational experiences of pupils in those schools measured up against some pre-existing notion of what "good practice" might be. Clearly, as the researchers we did have pre-existing notions as to the nature of good practice, and our observations were coloured by the lens of our previous experiences as teachers, teacher trainers and researchers. However, we tried at all times to maintain a degree of reflexivity in our observations and to be aware of our own prejudices during observation and interview. Wherever possible, respondent and observer triangulation was used to confirm results which were then subjected to respondent validation.

A further note of caution should be added here in relation to the hidden assumptions in the methodology. Success is being judged here in terms of external examination results in mathematics. This target should not be accepted uncritically, as there are greater goals for education than the passing of examinations. For example, improved value-added scores in mathematics might be achieved by increasing the proportion of curriculum time and resources allocated to it. This might be unacceptable in terms of other goals held for education. The analysis represents one instrument of evaluation amongst many; not everything which is desirable in education is measurable and vice-versa. However, success in mathematics examinations is one significant goal for education and the results should be read in that context.

The statistical model

The models, developed initially as part of a pupil-based approach to value-added and target-setting within the Vale of Glamorgan, use non-linear regression techniques to estimate the probability that pupils will achieve at or above a given level at the end of each key stage. Overall, the current models estimate attainment at individual pupil level to an accuracy of 85%, eg: of the pupils estimated to achieve level 4 or above at Key Stage 2 (using Key Stage 1 data) around 85% actually achieved level 4 or level 5.

Once individual pupil estimates had been calculated, a whole-school estimate (the mean of each individual pupil's estimate) was then compared with actual results. Taking into account the standard deviation of the prediction and the number of pupils involved, it was possible to identify schools where achievement was significantly higher than predicted.

The selection of the schools

Following discussions with LEA advisors, two secondary schools and two primary schools were selected from those schools in each LEA which showed the highest added-value. Matched cohort data did not yet exist for Key Stage 1 (KS1) to KS2. However, 3 years of KS1 data (95/96 to 97/98) and 3 years of KS2 data 95/96 to 97/98) did exist so estimates averaged across 3 years of KS 1 data were used to provide a reasonable basis for comparison.

Unfortunately, a full sample in which the difference was significant at the 5% level could not be generated in this manner and an alternative criterion based on improvement over the three year period was used to complete the sample. Fuller details are given in (Jones et al, 2000 and Tanner et al, 2000)

The primary schools

Schools were selected on the basis of two criteria:

> A. Attainment in mathematics at level 4+ (averaged over the three years for which data is available) is significantly higher (<5%) than the estimate (in either all 3 years or in the last 2 years) provided by analysing Key Stage 1 data.
>
> B. Attainment at Key Stage 2 has shown an improvement over the three year period with attainment in 97/98 being higher than the estimate from Key Stage 1 data.

Four schools were identified within each LEA of which two were selected following discussion with each LEA advisor.

The secondary schools

Two criteria were used:

> A. Matched data from Key Stage 3 (95/96) to GCSE (97/98) was evaluated so that performance in Mathematics could be compared with overall school performance.
>
> B Consistent improvement from 95/96 to 97/98 in Key Stage 3 Level 5+ attainment in Mathematics.

The nature of the sample

It should be noted that the final selection of schools included a wide range of contexts. Primary and secondary schools from industrial, post-industrial, urban and rural contexts were represented. Economically, they ranged from affluent, professional, suburban districts to some of the most socially deprived areas in Europe.

Interviews and classroom observations

The selected schools were then visited by the researchers who conducted detailed, open ended, semi-structured interviews with the head teachers of the schools who were invited to identify the features of their schools of which they were most proud and any features which they felt might have contributed to their success. The interviews were wide ranging and although they all included a common core relating to management style, organization, ethos, policies, staff training and qualifications, resources, and parental involvement. However, head teachers were encouraged to lead the interview in the direction they felt was most significant. The interviews varied in length and character, but lasted at least an hour and sometimes as much as three hours.

Detailed, open ended, semi-structured interviews were also conducted with subject leaders (mathematics coordinator or head of department) in each school. Again, the interviews were wide ranging and the subject leader was encouraged to identify the features they felt were significant in their school, but a common core relating to management style, organization, ethos, policies, staff training and qualifications, implicit theories of teaching and learning, teaching styles, classroom organization, and numeracy was included in each case. These interviews lasted between one and two hours each.

In a further visit, the researchers then observed a sample of mathematics lessons (between 4 and 6) in each school. Classroom processes were analyzed and with the aim of validating aspects of

the interview data and interpreting some of the classroom processes described. Some excerpts of lessons were videotaped to provide stimulus material for discussion at a network meeting.

All interviews were audio-tape recorded and transcribed. Immediately following each visit for interview or lesson observation, the researchers sat in their car in a convenient lay-by and audio-tape recorded a debriefing discussion to triangulate impressions and interpretations and provide a record of emergent theories and ongoing focussing questions.

An action research network group was then set up consisting of 5 primary mathematics coordinators and 5 secondary heads of department to validate the findings of the researchers, discuss classroom processes and develop recommendations on the nature of good practice in the teaching of numeracy.

Triangulation of data was provided by the range of respondents and the 2 researcher/observers. Data gathered from the head teacher was validated by comments made by subject coordinators, teachers and pupils. Teaching styles and strategies nominated by teachers and subject leaders were validated and interpreted through classroom observation and discussions with pupils.

Where observed behaviour in the classroom seemed in the opinion of the researchers to be at odds with approaches nominated by teachers or subject coordinators, follow up interviews were used to clarify the complex issues of style and strategy involved. Several of the teachers observed found it difficult to describe their approaches and beliefs in interview, seeming to lack an adequate language to do justice to the complexity of their skills and behaviours. In such cases, detailed discussions about the particular lesson observed after the event were used to clarify issues for the researchers.

We observed a sample of mathematics lessons (usually between four and six) in fifteen of the twenty schools. Classroom processes were analysed with the aim of validating aspects of the interview data and interpreting some of the classroom processes described.

Where observed behaviour in the classroom seemed, in the opinion of the researchers, to be at odds with approaches nominated by teachers or subject leaders, follow up interviews were used to clarify the complex issues involved. Several of the teachers observed had, indeed, found it difficult to describe their approaches and beliefs in the interview, seeming to lack an adequate language to do justice to the complexity of their skills and behaviours.

Validation procedures

A copy of the interim report was sent to head-teachers and subject leaders of all the schools involved for respondent validation purposes. A second draft was then discussed in detail at a network meeting with representatives from a primary and secondary school in each LEA for further validation. The report was also discussed with LEA representatives at a project meeting.

The results

In nearly every case, we considered that our observations confirmed the validity of the selection process through statistical analysis. We consider ourselves privileged to have visited so many exceptional schools which are making a significant and positive impact on mathematical attainment above and beyond the qualities of their incoming pupils, their families and even the social determinants of gender, class and race (Mortimore et al, 1994, p316).

The majority of schools showed aspects of the positive features we have identified at all levels in the institution- whole school, subject leader and classroom teacher. However, in the case of one secondary school we considered that an exceptional head of department was succeeding against the trend in an otherwise unexceptional school. In another secondary school we considered that an exceptional head had generated a very positive ethos which was driving an otherwise unexceptional mathematics department. In one primary school we considered that most of the features of good management which were common to the other institutions were missing and that the mathematics coordinator was making progress in spite of the head teacher. In two of these cases we considered that there was evidence to suggest that the baseline data was suspect, having been artificially depressed due to earlier contextual factors.

The key features which we have identified as representing Agood practice@ were remarkably consistent from school to school and from phase to phase. We had anticipated originally that we would need to report separately for primary and secondary schools, but have been struck by the common characteristics of effective schooling at all ages and the comments which follow apply to all ages.

There is not room here to consider in detail all the features which were identified at each level. This paper will focus mainly on those features dealing with interaction in the classroom, making only brief mention of some of the major features at other levels which facilitated the development of such approaches. In nearly every case, we considered that our observations confirmed the validity of the selection process based on the statistical analysis.

The majority of schools showed aspects of the positive features which we identified at all levels in the institution- whole school, subject leader and classroom teacher. However, in the case of one secondary school we considered that an exceptional head of department was succeeding against the trend in an otherwise unexceptional school. Similarly, in one primary school we considered that most of the features of good management which were common to the other institutions were not evident and that the mathematics co-ordinator was making progress in spite of the head teacher. In another secondary school we considered that an exceptional head had generated a very positive ethos which was driving an otherwise unexceptional mathematics department. In two of these cases we considered that there was evidence to suggest that the baseline data was suspect, having been artificially depressed due to earlier contextual factors.

The key features which we have identified as representing "good practice" were remarkably consistent from school to school and across phases. We had anticipated originally that we would need to report separately for primary and secondary schools, but found that the characteristics of effective schooling were common to all ages, and the comments which follow apply to both the primary and secondary phases.

There is not room here to consider in detail all the features which were identified. This paper will focus on those features relating to interaction in the classroom.

Effective Classroom Practices

The following features were identified during discussion with the subject leaders and during observation of lessons in the vast majority of the project schools.

Planning

In nearly all schools we observed detailed planning, both of the overall scheme of work and of individual lessons. Most of the lessons which we observed exhibited clear learning objectives, an obvious overall structure, and a variety of learning activities. Nearly all the teachers planned to end their lessons with plenaries and noted possible assessment opportunities. Unfortunately, plenaries did not always occur as planned and several teachers talked about the plenary they had intended to have if only the bell had not rung. The majority of the lessons did end in some form plenary however, and in the best cases we considered the plenary to have contributed significantly to learning.

Expectations

The teachers held high expectations of their pupils. Whole class lessons or sequences were pitched at the upper quartile with many of the teachers expressing the beliefs that "the less able pupils benefited from listening to the explanations of the more able" and that "a good way to develop your understanding was to try to explain your work to others". Several teachers professed that they were continually surprised by how much pupils were able to "pick up" from others, even on topics which the teachers would have considered to be too hard for them. Although classes seemed to be operating at a higher level than might usually be expected, we saw no evidence of children being "lost" or of work "going over their heads". Through effective techniques of mixed ability questioning - targeting pupils at an appropriate level - the pupils were engaged on tasks at all times.

Problem solving and investigation

The majority of the lessons observed were underpinned by an investigative approach to mathematics. We did not often observe pupils Adoing investigations@ for their own sake. Rather, mathematics was viewed as essentially a problem-solving activity. Investigative approaches were often used to teach new content with understanding. Pupils were required to focus on the mathematical structure being taught, for example, by listening to another pupil explain a different approach to a task, and to choose an appropriate strategy for the problem. They were expected to apply their knowledge of mathematics in new contexts with teachers emphasising the use of "twists" and "variations" in the questions set rather than requiring merely the repetition of an algorithm or technique.

Teaching approaches

No standard format for the teaching of mathematics was discerned in either phase. The teaching approaches varied according to the age and ability of pupils, the topic being taught, the constraints of the physical environment and the pedagogical preferences of the teacher. However, many characteristic strategies and themes were apparent across the majority of the lessons observed.

Secondary heads of department often began interviews with the claim "We're very traditional in our teaching here..." However, the meaning of such a descriptor needed probing. One "traditional" teacher led a lesson in which Y10 pupils investigated the impact of varying **a** in curves such as: sin **a**x, **a**cosx, tan(x+**a**), etc, by using a computer graph plotter to try to find the correct function to fit a given curve. "Traditional" clearly means something quite novel when applied to a computer-based investigation and it would be unhelpful to use the term to describe the rich and varied range of activities and strategies which we observed.

"Traditional" seemed to mean *teachers leading lessons* with *clear objectives* and *good pace* using a balance between periods of *individual work, small group work*and *interactive whole class teaching* with a *high level of pupil involvement*. It seemed to be distinguishing *active teaching* from a situation in which a commercial scheme was used for individualised learning with the teacher as facilitator. These characteristics were common to lessons observed in both the primary and secondary phases, and the phrases in italics above could be used to describe nearly all of the lessons we observed in the successful schools.

There were, however, a number of other significant features of their teaching strategies which were common to the successful teachers. Many lessons in primary began with a session of whole class mental activity, although a few teachers chose to place it elsewhere in the lesson. Mental mathematics was less common as an introduction in secondary classrooms, although both mental and oral activity was a significant part of the work in each week. However, in both the primary and secondary classrooms, the main teaching activity often began with an explanation of the aims of the lesson.

The practices described in the National Numeracy Strategy (DfEE, 1999) were clearly impacting on practice in primary schools. The teachers in our study were not new converts to such techniques, however, claiming to have developed their own comparable strategies and approaches over the period we were considering (1995 to 1998).

Mental and oral work

Mental and oral work formed a significant and planned part of the mathematics curriculum in both primary and secondary classes. This contrasts with the findings of a recent all-Wales survey of mathematical practices in secondary schools where only half of the heads of mathematics departments reported mental arithmetic practice to be a regular feature in their Year 7 classes with the frequency of such practice declining sharply after Year 8 (Jones & Tanner, 1998).

Although mental work was often used as a "warm up" activity at the start of the lesson in primary classrooms it was also used naturally during other periods of teaching and learning and it was clear that both teachers and pupils expected to be able to engage with simple mathematical situations mentally and there were situations when it was considered inappropriate to reach for pencil and paper or a calculator. Mental mathematics was required by teachers, not just mental arithmetic, particularly in secondary schools.

It should be noted that much of the mental activity seen was not of the traditional mental test form (although this was also seen occasionally). Very often the questions asked required a degree of explanation following solution. Even quite simple arithmetical questions such as "225-177= ?" were used as opportunities for pupils to explain their approaches to others and form a starting point for discussion. We often saw such an example lead to explanations of approaches from several different pupils.

A number of different techniques were used to facilitate the use of whole class mental sessions in mixed ability and mixed age sessions in order to avoid public humiliation and to encourage whole class participation. Questions were asked and then targeted differentially to ensure participation and yet facilitate success by less able pupils. "Show-me" cards were used in most primary schools. Open questions were used very effectively in most of the primary classrooms to allow pupils to self-differentiate work and set their own highest targets. For example, "How many different ways can we get the answer 101?" was used in a primary classroom where they had just read 101 Dalmatians. Answers varied from the use of simple addition eg: 99+2, through subtraction with large numbers: 1001-900, to multiple operations: 500)5+1. Interestingly, once one pupil had used the strategy 500)5+1, others quickly followed: 600)6+1, 300)3+1; demonstrating how pupils are able to assimilate each other's ideas during a social discourse.

Questioning and pupil involvement

Questioning was used with a range of styles and purposes by both primary and secondary teachers, but we regularly observed effective use made of open questions, questions which required an extended response, and probing questions which demanded further explanation and justification, in addition to the more widely used traditional forms of short question for the purposes of recall, feedback and accountability. Interactive whole class teaching techniques were used in every lesson we observed and although these varied somewhat in character, they were always more like interactive explanations rather than clearly explained lectures. At the very least questions asked by the teacher were used to develop an interactive mathematical argument with the pupils. In the majority of lessons we observed, the interaction demanded far more from pupils than a simple response to a question. Most lessons were characterised by significant participation by pupils.

One of the characteristics which was common to nearly all the schools was "getting pupils out to the front", often to write on the board or flip chart. Of course, it is not likely to be the act of writing on a board itself which facilitates learning - it is far more likely to be the demands made of the pupils while they are at the front and the contributions made by teachers and other pupils to assist the development of the mathematics.

In both primary and secondary schools pupils were frequently required to explain and justify their work to other pupils. One secondary teacher regularly gave out "the pen of doom" several times in the lesson and demanded that its recipient talk and explain while writing their method on the board. When the researcher asked the pupils why their teacher did this they replied "If you can explain it then you know you understand it." Several of the teachers expressed their aim in such situations as "getting the point over without telling" or to "leave the pupils the joy of working it out for themselves - but providing the support they need to make sure they get there".

On another occasion three pupils were chosen to solve a trigonometry problem on the board as a Y10 class entered the room and settled down. By the time the class had settled three different solutions were on the board. The solutions were then thrown open for discussion by the class and their authors challenged to explain by members of the class. The authors and the class then engaged in self and peer assessment of the work eventually judging how many marks would have been gained by each pupil in a GCSE examination. Clearly this was a teacher-led activity, and although the mathematical contributions all came from members of the class, it would not have occurred without teacher intervention, leadership and control.

Plenaries

Plenaries were planned in advance in most cases and offered opportunities for pupils report back on progress and to reflect on their progress and learning. Unfortunately, plenaries did not always occur as planned and several teachers talked about the plenary they had intended to have if only the bell had not rung. The majority of the lessons did end in some form of plenary however, and in the best cases we considered the plenary session to have contributed significantly to learning.

The plenary was usually driven by focussing questions from the teacher, and in many lessons the class had obviously come to anticipate such a session. For example in one Year 6 class were not surprised when their teacher asked towards the end *"Well, how was it for you?"* She accepted comments relating to the difficulty of the work, personal concerns about success and failure, mathematical patterns that had emerged, and the design of the worksheet. However she used supplementary questions such as *"Which bits were hardest? "Why? What mistakes did you make that you won't do again?"*, and *"What have you learned?"*. Other teachers asked questions like "*What was important about today's lesson that you are going to remember?*" Others made links to ongoing work and set connecting homework tasks like "*Tomorrow we will go on to... so tonight I'd like you to think about..."* These questions prompted pupils to reflect on their work and to analyse their learning.

Discussion

The teaching and learning practices described here were based on high expectations of pupils. Lessons had clear objectives and were planned to ensure progression and continuity of learning. The classroom culture was disciplined, supportive and required pupils to take an active part in their learning.

Pupils were expected to articulate and discuss their own methods and tentative conjectures, however, the teacher's role was the most significant in the classroom discourse. The teacher was in control at all times, leading and driving the lesson forwards, challenging the pupils, supporting, probing, encouraging, and perhaps most important of all, focusing the attention of the class on what was significant in the activity and what they were learning through their participation in the lesson.

Many of the above features may be identified within the recommendations from other initiatives designed to improve standards in mathematics learning. Studies of countries which out-perform England in international comparisons (see, for example, Andrews, 1997, 1999, Graham et al 1999, Keys et al, 1996) and the recommendations of the National Numeracy Strategy (DfEE, 1999), indicate the importance of high expectations, effective questioning, and articulation by pupils of their methods and reasoning.

Such features have been subsumed under descriptors such as "direct teaching" which describe only part, and we would argue not necessarily the crucial part, of effective teaching practices. There is a danger that the current emphasis on lesson structure will overshadow more fundamental issues. The teachers observed as part of this research followed no set lesson format. Indeed, the TIMSS studies found no single factor relating to classroom practice had a significant and consistent effect (Brown, 1999). As Andrews (1999) suggests, we need to move beyond the rhetoric of whole-class interactive teaching towards a deeper understanding of how mathematics may be taught effectively. We now attempt to justify why the processes outlined above should lead to more effective learning by pupils.

Analysis of the teaching and learning strategies

From a socio-constructivist viewpoint, there are four main reasons why such processes should lead to progress in mathematics. They are associated with constructing knowledge and testing its viability; the development of corporate meaning; the development of metacognitive awareness; and the development of a disposition to construct.

The processes involved in the construction of knowledge suggest that students should have opportunities to articulate their tentative constructions, and to test them for viability against the shared, corporate understanding of the class. Whilst all pupils were expected to participate, the teacher was the most significant arbitrator in such discussions as the "mathematical expert" setting the parameters for mathematical acceptability.

Articulation does more than provide an opportunity for pupils to test their understandings for viability against corporate meaning: it also contributes to the generation of corporate meaning by providing a further opportunity for construal to other members of the class. Whilst listening to pupils describe their methods, individuals may contrast the interpretation being offered with their own thoughts (Clarke, 1994). The teachers facilitated such reflection by asking questions such as "Which do you prefer, Sandra's method or John's method? Does Ann's method always work? When would you use Jo's method?"

Many of the techniques used by the teachers to support pupils during such interactive whole class teaching might be described as scaffolding, however, scaffolding is a problematic construct for application to classrooms. It was originally designed to explain learning in individualised situations, however, research (Wood & Wood, 1996 p7) suggests that scaffolding does not have to be optimal for each pupil in order for learning to occur. Criteria for scaffolding to occur successfully include a classroom culture where teacher and pupils can work jointly on problem-solving activities in a conjecturing atmosphere (Mason, 1988); and teachers who are able to draw on their subject knowledge to identify more than one way to achieve the desired learning outcomes and hence to follow the learner's path (Askew et al, 1995; Wood & Wood, 1996). It is suggested that such processes were observed in these successful schools.

It is possible to distinguish between two very different forms of interaction which might be described as scaffolding: funnelling and focusing (Bauersfeld, 1988; Wood, 1994). In funnelling it is the teacher, as the person with the expert knowledge, who selects the thinking strategies and controls the decision process to lead the discourse to a predetermined solution. In focusing, the teacher's questions draw attention to critical features of the problem which might not yet be understood. The pupil is then expected to resolve perturbations which have thus been created (Wood, 1994, p160). Both forms of scaffolding were observed in the successful schools, but the emphasis was on focusing.

The scaffolding we observed was dynamic in character with the teacher drawing on ideas articulated by the pupils but using focusing questions to control the direction of the discourse. Teachers were continuously assessing responses in order to evaluate pupils' understandings and to decide whether to adapt their teaching to provide further help, or whether to move on to the next teaching point. For such scaffolding to result in learning, the task must be more difficult than could be accomplished by the pupils unassisted. Teachers in the successful schools described setting the level of work to be "challenging but attainable with the help of the teacher".

The scaffolding described so far could be characterised as encouraging reflection in action (Schön, 1983) as pupils and the teacher responded to ongoing discussions. The scaffolding observed also had a reflective character as teachers generated opportunities for evaluation and reflection. During plenary sessions, focusing questions were often used to make pupils' explanations the object of discussion. When pupils attempt to explain their methods to others the explanation itself becomes the focus of their thinking and the explicit topic for class discussion. Such objectification of thought through articulation is associated with bringing the subconscious into the conscious and hence the development of reflective awareness and conscious control (Prawat, 1989).

To control a mental function, Vygotsky (1962) claims that a student must be conscious of it, but suggests that unconscious self-regulation should precede conscious self regulation, presumably appearing first on the social level between people (interpsychological) and then inside the child (intrapsychological) in an unconscious form (cf: Vygotsky, 1978). The shift to reflective awareness and deliberate control of cognition would then be achieved through a transition to Averbalised self observation@ which denotes Aa beginning process of generalisation of the inner forms of activity@. This is a shift to a higher type of inner activity opening up new ways of seeing things and new possibilities for handling them (Vygotsky, 1962, p91).

> In perceiving some of our own acts in a generalising fashion, we isolate them from our total mental activity and are thus enabled to focus on this process as such and to enter into a new relationship with it. In this way, becoming conscious of our operations and viewing each as a process of a certain kind... leads to their mastery

(Vygotsky, 1962, p91-92).

Awareness of one's own mathematical knowledge is a pre-requisite for its application to problems or, indeed, for its use in the learning of new mathematics. Such metacognitive awareness was developed during the plenary sessions as pupils reflected back on what they had learned prior to presenting their conclusions to their peers, assisted by teachers' focusing prompts.

When reflective discourse is encouraged within a classroom, teachers can be pro-active in encouraging construction, focusing the attention of students on significant aspects of the discourse for collective reflection. AWhat was previously done in action can become an explicit topic of conversation@ and thus Aparticipation in this type of discourse constitutes conditions for the possibility of mathematical learning@ (Cobb et al, 1997). The social character of the discourse may be arranged to lend social status to Athe disposition to meaning construction activities@ which is a Ahabit of thought@ that can be learned (Resnick, 1988) and which contributes to a classroom culture in which scaffolding is achievable.

Articulation, dynamic scaffolding and reflective discourse were identified as key characteristics of the classroom processes observed in the successful schools. The significance of such features are confirmed by other research studies, for example, several studies have included articulation by students as significant aspects of their approach (eg: Gray, 1991; Wheatley, 1992; Cobb et al, 1992; Tanner & Jones, 1994; 1995; Galbraith, 1995). The effectiveness of flexible scaffolding and reflective discourse for metacognition and mathematical development in the secondary school was demonstrated in The Mathematical Thinking Skills Project in which accelerated mathematical development was observed (Tanner, 1997).

The effective teaching approaches identified here required explication of pupils' methods and discussion of their thinking strategies. The prompts by the teacher alerted pupils to dead-ends or unprofitable approaches and focused attention on those aspects which would lead ultimately towards the desired learning objectives for the lesson. The classroom climate was such that all pupils were expected to contribute ideas and to participate by contrasting the suggestions offered by others with their own ideas. It is conjectured that by so doing pupils were able to draw on all the ideas as scaffolding for their own thinking, and whilst this scaffolding would not necessarily be optimal, it was sufficient for learning to occur. Such learning opportunities were enhanced by the classroom culture in which mathematics was construed as a problem-solving activity to which every pupil could contribute ideas. Such a classroom culture resulted in increased intrinsic motivation and increased self esteem.

Conclusion

The teaching approaches observed in the schools which were identified as particularly successful in value-added terms accord with recommendations for good practice from research elsewhere (eg: Ayers et al, 1999; Cobb et al, 1997, Tanner & Jones, 1999; 2000). In particular, they further validate some of the approaches suggested by the National Numeracy Strategy (DfEE, 1999) and the Mathematics Enhancement Project (CIMT, 1997). However, it is clear that even teachers in some of the most successful schools found it difficult to express the complexities of their approaches in the abstract. The key elements of their approaches identified here bear little resemblance to the traditional teaching and tables testing beloved by back-to-basics movements. Communicating the subtleties of effective approaches to teachers in other schools is going to be a difficult task. However, the exceptional progress demonstrated by these successful schools dictates that we must.

Acknowledgements

We consider ourselves privileged to have visited so many exceptional schools which are making a significant and positive impact on the mathematical attainment of their pupils and wish to thank the staff who gave so generously of their time.

References

Andrews, P. (1997). A Hungarian perspective on mathematics education. *Mathematics Teaching*, *161*, 14-17.

Andrews, P. (1999). Looking behind the rhetoric. *Mathematics Teaching*, *167*, 6-10.

Askew, M., Bliss, J., & Macrae, S. (1995). Scaffolding in mathematics science and technology. In P. Murphy, M. Selinger, J. Bourne, & M. Briggs (Eds.), *Subject learning in the primary curriculum* (pp. 209-217). London: Routledge/OUP.

Ayers, P., Dinham, S., & Sawyer, W. (1999). *Successful teaching in the New South Wales higher school certificate*. Sydney: NSW Department of Education and Training.

Bauersfeld, H. (1988). Interaction, construction and knowledge: alternative perspectives for mathematics education. In D. Grouws, T. Cooney, & D. Jones (Eds.),*Effective mathematics teaching* (pp. 27-46). NCTM, Reston, VA: Lawrence Erlbaum.

Brown, M. (1999). Is more whole-class teaching the answer? *Mathematics Teaching*, *169*, 5-7.

Centre for Innovation in Mathematics Teaching (CIMT). (1997). *Mathematics Enhancement Project (MEP)*. Exeter: University of Exeter.

Clarke, D. J. (1994). The metaphorical modelling of "coming to know". *Proceedings of the eighteenth international conference for the psychology of mathematics education, Lisbon, Portugal*, *2*, 193-200.

Cobb, P., Wood, T., Yackel, E., & Perlwitz, M. (1992). A follow-up assessment of a second-grade problem centred mathematics project. *Educational Studies in Mathematics*, *23*(5), 483-504.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for research in mathematics education*, *28*(3), 258-277.

DfEE. (1999). *The National Numeracy Strategy: framework for teaching mathematics*. Cambridge: Cambridge University Press.

Galbraith, P. (1995). Modelling, teaching, reflecting - what I have learned. In C. Sloyer, W. Blum, & I. Huntley (Eds.), *Advances and perspectives in the teaching of Mathematical modelling and applications* (pp. 21-46). Yorklyn, DE: Water Street Mathematics.

Graham, T., Rowlands, S., Jennings, S., & English, J. (1999). Towards whole class interactive teaching. *Teaching Mathematics and its Applications*, *18*(2), 50-60.

Gray, S. S. (1991). Ideas in practice: Metacognition and mathematical problem solving. *Journal of Developmental Education*, *14*, 24-28.

International Association for the Evaluation of Educational Achievement (IEA). (1996). *Mathematics Achievement in the middle school years: IEA's third international mathematics and science study*. Boston MA: Centre for the Study of Testing, Evaluation and Educational Policy.

Jones, S., Tanner, H., & Treadaway, M. (2000). Raising standards in mathematics through effective classroom practice. *Teaching Mathematics and its Applications*,*19*(3), 125-134.

Keys, W., Harris, S., & Fernandes, C. (1996). *Third International Mathematics and Science Study: First national report, Part 1: Achievement in mathematics and science age 13*. London: NFER.

Mason, J. (1988). Modelling: What do we really want pupils to learn. In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 201-215). London: Hodder & Stoughton.

Mortimore, P., Sammons, P., & Thomas, S. (1994). School effectiveness and value added measures. *Assessment in Education*, *1*(3), 315-331.

Murphy, R. (1997). Drawing outrageous conclusions from national assessment results: where will it all end? *British Journal of Curriculum and Assessment*, *7*(2), 27-39.

Nuttall, D. (1990). Differences in examination performances. *London research and Statistics Branch*, *RS1277/90*, ILEA, London.

Prawat, R. (1989). Teaching for understanding: three key attributes. *Teaching and Teacher Education*, *5*(4), 315-328.

Resnick, L. B. (1988). Treating mathematics as an ill structured discipline. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 32-60). Reston, VA: National Council of Teachers of Mathematics.

Reynolds, D. (1998). *The implementation of the National Numeracy Strategy: the final report of the numeracy task force*. Sudbury: DfEE publications.

Rutter, M., Maugham, B., Mortimore, P., & Ouston, J. (1979). *Fifteen thousand hours*. Milton Keynes: Open Books.

Saunders, L. (1998). *Value added measurement of school effectiveness: an overview*. Slough: NFER.

Saunders, L. (1999a). A brief history of educational value added: How did we get to where we are? *School effectiveness and school improvement*, *10*(2), 233-256.

Saunders, L. (1999b). *Value added measurement of school effectiveness: a critical review*. Slough: NFER.

Schagen, I. (1998). Adding value with value added. Paper in the proceedings of NFER's annual conference. (http://www.nfer.ac.uk/conferences/value.htm) (accessed at 3.9.99).

Schön, D. (1983). *The reflective practitioner*. New York: Basic Books.

Straker, A. (1997). *National Numeracy Project*. Reading: National Centre for Literacy and Numeracy.

Tanner, H. F. R. (1997). *Using and Applying Mathematics: developing mathematical thinking through practical problem solving and modelling*. Unpublished Ph.D. Thesis, University of Wales Swansea, Swansea.

Tanner, H. F. R., & Jones, S. A. (1994). Using peer and self assessment to develop modelling skills with students aged 11 to 16: A socio-constructive view.*Educational Studies in Mathematics*, *27*(4), 413-431.

Tanner, H. F. R., & Jones, S. A. (1995). Teaching mathematical thinking skills to accelerate cognitive development. *Psychology of mathematics education conference (PME-19), Recife, Brazil*, *3*, 121-128.

Tanner, H., & Jones, S. (1999). Dynamic scaffolding and reflective discourse: the impact of teaching style on the development of mathematical thinking. *Proceedings of the 23rd conference of the International Group for the Psychology of Mathematics Education, Haifa*, *4*, 257-264.

Tanner, H., & Jones, S. (2000). Scaffolding for success: reflective discourse and the effective teaching of mathematical thinking skills. In T. Rowland & C. Morgan (Eds.), *Research in Mathematics Education Volume 2: Papers of the British Society for Research into Learning Mathematics* (pp. 19-32). London: British Society for Research into Learning Mathematics.

Tanner, H., Jones, S., & Treadaway, M. (2000). The role of middle managers in raising standards in mathematics. *Australian Association for Research in Education*.

Vygotsky, L. S. (1962). *Thought and language*. Cambridge, Mass: The MIT Press.

Vygotsky, L. S. (1978). . In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind in Society: The development of higher psychological processes*. Cambridge, Mass: Harvard University Press.

Wheatley, G. H. (1992). The role of reflection in mathematics learning. *Educational Studies in Mathematics*, *23*(5), 529-541.

Wood, D., & Wood, H. (1996). Contingency in teaching and learning. *Learning and instruction*, *6*(4), 391-397.

Wood, T. (1994). Patterns of interaction and the culture of mathematics classrooms. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 149-168). Dordrecht, Netherlands: Kluwer Academic Publishers.