Mathematical analogs and the teaching of fractions ®

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Abstract: The literature has noted that some mathematical analogs are more effective than others for the teaching of fractions. This study aimed to evaluate the efficacy of seven mathematical analogs commonly used in the teaching of the partitive quotient fraction construct. A sample of twelve purposively selected Year Three children were presented with partitioning (partitive quotient) tasks in a simulated restaurant setting where they assumed the roles of waiters and waitresses serving pizzas, pancakes, pikelets, icecream bars, apple pies and licorice straps to the customers. Clinical interviews, talk-aloud protocols and non-participant observations were conducted within an interpretative methodology incorporating the Hermeneutic Dialectic Circle. The educational efficacy of the analogs was evaluated according to: (i) ecological validity, (ii) abstraction-ability, and (iii) ease of partitioning. Great variance was found in the efficacy of the seven mathematical analogs. This has significant implications for the initial teaching/learning of the partitive quotient fraction construct.

Introduction

In this study, we set out to investigate how mathematical analogs influence young children's learning of the partitive quotient fraction construct. In the partitive quotient fraction construct, the fraction *x/y* refers to the quantity represented by one of the resulting shares when the quantity *x* is partitioned into *y* equal parts. For example, if two whole cakes are partitioned into three equal shares, then each share is two thirds of a cake. Understanding of the partitive quotient fraction construct is central to the development of fractional number knowledge (Kieren, 1993).

An analogy is "a mapping from one representation to another" (Halford, 1993, p.249) in which knowledge from one domain, the *source*, is mapped to another, the *target*, in such ways that it conveys that a system of relations which is true for the source objects is also true for the target objects. For instance, "human is to house as dog is to kennel" (Halford, 1993). Gentner (1983) defined good analogical matching "as one in which there is significant overlap in the relations between objects in two domains" (p. 3). Analogs are mental models which are generally based on concrete properties drawn from life experiences. However, analogs also possess the power to deal with abstract concepts (English & Halford, 1995).

According to English and Halford (1995), a mathematical analog assigns a concrete model as the *source* and the concept to be constructed as the *target*. They identified two categories of mathematical analogs commonly used in the teaching of fractions: unstructured analogs and structured analogs.

Set models such as collections of counters or other discrete objects form the basis of unstructured mathematical analogs. When used in the teaching of whole number concepts, unstructured analogs have been very successful. However, the use of unstructured analogs to facilitate the learning of fractions has not been so successful. This is because the complex nature of the mapping between the discrete objects in the source and the target fraction concept (English & Halford, 1995). In order to carry out this mapping, a child first must recognise the total number of discrete objects in the source as being the whole in order to generate the fraction name. For example, if in a collection of seven counters, there were three red counters and four blue counters, then 3/7 of the counters would be red and 4/7 of the counters would be blue. To construct these two fractions from the concrete model, the child would have to recognise that there were seven counters in the whole collection and therefore that each fractional part could be expressed as sevenths. In addition to involving complex mappings, the unstructured analogs when used to facilitate the teaching of fractions also tend to impose a high information processing load on young children (English & Halford, 1995).

Region models and length models (Smith, Booker, Cooper & Irons, 1988) form the basis of structured mathematical analogs used in the teaching of fractions. A region model is a two-dimensional geometric form (e.g., square, rectangle, circle) whilst a length model is a one-dimensional geometric form (e.g., line). Because of the complex mappings and the high information processing loads associated with unstructured mathematical analogs, structured analogs based on continuous region and length models generally are considered as being more appropriate for the initial learning of fractions (Steffe & Olive, 1993).

According to English and Halford (1995), structured analogs have in-built structure which exhibits specific numerical relationships and properties and thus are most suitable for the initial learning of fraction concepts . However, they cautioned that in the application of structured analogs for the teaching of fractions, several considerations were important: (a) analogs alone cannot impart meaning, (b) children may not make the appropriate mappings from the concrete to the abstract, and (c) the accompanying mapping language and processes must be appropriate. English and Halford therefore reiterated Gentner's (1982) assertion that in order for the analog to be effective, it is important that:

· the source should be clearly displayed and explicitly understood by the child (*Clarity of source principle*)

· the analog should facilitate clear, unambiguous mappings from the base concrete model to the target fraction concept (*Clarity of mapping principle*)

· relations mapped from source to target should form a cohesive structure (*Principle of conceptual coherence*)

· an analogy be transferable (*Principle of scope*).

English and Halford's warnings about the need for structured analogs based on region and length models to facilitate clear, unambiguous mappings have been lent support by the corpus of research findings about the teaching and learning of fractions. Streefland (1991), for example, found that ecological validity of the mathematical analog greatly influenced the quality of children's learning. He found that children presented with problems in realistic contexts scored higher than children engaged in a mechanistic approach where there was a powerful focus on rules and algorithms. Streefland's findings seem to be in accordance with Gentner's (1982) "Clarity of Source Principle".

Another factor which has been found in previous research studies to influence the educational efficacy of structured mathematical analogs is ease of partitioning. Ball (1993) initially had the young children in her study investigate the sharing of circular cookies. However, the partitioning of the circular cookies into equal shares proved to be too difficult. When Ball asked the children to partition rectangular cookies, the partitioning problems no longer were evident. Streefland (1991) and Pothier and Sawada (1983) also noted that young children in their studies had more difficulty in partitioning circular models than rectangular and length models.

Most of the previous studies which have investigated the use of structured mathematical analogs in the teaching/learning of fractions have been conducted in foreign countries with cultures and social practices different from those in Australasia. However, as Lamon (1996) pointed out, the strategies utilised by young children with problems associated with fractions are situationally specific, demonstrating a strong adherence to social practice. Therefore, in this study, we set out to investigate how the learning of the partitive quotient fraction construct by a sample of young Australian children was influenced by seven different structured mathematical analogs which had been the focus of research in other countries and which subsequently had been included in mathematics curricula and teacher source books in Australia.

The seven different structured analogs investigated in the study were: pizzas and apple pies (based on large circular region models), pancakes (based on medium circular region models), pikelets (based on small circular region models), icecream bars (based on long and narrow rectangular region models), rectangular cakes (based on short and wide rectangular region models), and licorice straps (based on length models).

The analogs were evaluated in terms of their:

1*. ecological validity* (i.e., how realistic for the young Australian children was the sharing context engendered by the analog);

2*. abstraction-ability* (i.e., how well they facilitated the abstraction from the source concrete model to the target partitive quotient fraction construct); and,

3. *ease of partitioning* (i.e., how easy was the concrete model to physically partition).

How well the analogs facilitated the abstraction from the source concrete models to the target partitive quotient fraction construct was judged with respect to: (a) the quality and quantity of interrelated tasks generated by each analog object, and (b) the quality of partitioning strategies promoted by the analog object.

Criteria 1 was based on Gentner's Clarity of Source Principle and Streefland's notion of "realistic contexts," Criteria 2 was based on Gentner's Clarity of Mapping Principle, Principle of Conceptual Coherence and Principle of Scope. Criteria 3 was based on the work of Ball (1993),Streefland (1991) and Pothier and Sawada (1983).

Method

The study was couched in the constructivist paradigm. The data gathering technique used was the clinical or mixed method technique of Ginsburg, Kossan, Swartz, and Swanson (1983) which is a combination of Piaget's clinical interview and the talk aloud procedures of Ericsson and Simon (1984). According to Ginsburg et al., this technique enables researchers to not only elicit complex intellectual activity but also to identify the internal symbolic mechanisms underlying the complex intellectual activity.

Participants

Twelve third grade students (aged between 7.9 to 8.3 years) from a primary school located in Eastern Australia participated in this study. The children were chosen through serial, contingent, purposeful and exhaustive sampling (Guba & Lincoln, 1989) to produce maximum variation in cognitive functioning. Each participant was chosen after consultation with the students' classroom teacher in order to maximise the probability that as many strategies as possible would emerge during the course of the study. When it was found that no new partitioning strategies or other insights about young children's partitioning emerged during the interviews with the eleventh and twelfth participants, data collection ceased. The size of the sample thus was influenced by Guba and Lincoln's criteria of redundancy.

Instruments

A set of thirty realistic partitioning tasks was developed for this study (see Appendix 1). In each of the tasks, the children were asked to assume the roles of waiters/waitresses serving pizzas, pancakes, pikelets, icecream bars, apple pies or licorice straps to a number of customers sitting at a restaurant table.

Three task variables were taken into consideration during the development of the set of partitioning tasks: (i) types of analog objects, (ii) number of analog objects, and (iii) number of people. The analog objects used were circular region models, rectangular region models and length models (cf., Streefland, 1991). The number of analog objects ranged from one to six and the number of people sharing the objects ranged from two to six. The circular region models for the study included: pizza, pancake, pikelet and apple pie. The rectangular region models included: cake and icecream barcake. The length model was represented by the licorice strap (cf., Streefland, 1991).

The set of partitioning tasks began with problems that used the "pizza" circular region model (cf., Streefland, 1991), then proceeded to the rectangular region model and the length model.

The literature noted that children have sound informal knowledge of a half (Ball, 1993; Behr, Harel, Post, & Lesh, 1992) and powerful strategies for halving (Pothier & Sawada, 1990) so an obvious starting place was to present tasks which produced shares of one half and one quarter (Task 1 & Task 2). A natural progression from here was to investigate children's knowledge of partitioning for three people (Tasks 3-6). Tasks 4 and 5 were included to investigate Ball's (1993) contention that partitioning of length models is easier than the partitioning of rectangular region models. It was expected that children who had difficulty partitioning one circular region model among three people may be able to successfully partition one length model among three people.

Tasks 7-11 were included to further explore children's partitioning strategies for four people, while Tasks 12-14 investigated children's partitioning strategies for five people.

Tasks 15-30, explored children's partitioning strategies when interacting with the length model and the rectangular region model. A variety of tasks were included here to explore children's knowledge construction of quarters, thirds, fifths and sixths. Particular note was taken of children's partitioning strategies for the rectangular region model as Streefland (1991) noted that this model supported both one-directional (i.e., vertical or horizontal) and two-directional (i.e., vertical and horizontal) partitioning.

Procedure

The children were removed from the classroom and interviewed individually in a mobile research vehicle which had two sections: an interview room with two video cameras, and an observation room with monitors and a videomixer. The interviewer sat with the children and two observers watched and communicated with the interviewer via earphones. Each of the observers was an experienced teacher or a researcher with expertise in mathematics education. During the course of an interview, the two observers were able to have immediate input into the questioning and into the selection of each succeeding task administered to the child during the course of an interview.

The interview and analysis of children's responses operated within a Hermeneutic Dialectic Circle (Guba & Lincoln, 1989). Each of the children received a series of interviews. For each child in turn, the interviewer began by administering Tasks 1-3, asking the children to think aloud as they attempted to solve the problems. The interviewer and the observers used the children's actions and verbalisations to identify strategies and inform the selection of problems administered later on in Interview 1. At the end of Interview 1, the interviewer and the observers developed a sequence of problems suitable for that child which they administered in later interviews. Changes to this sequence of problems were made as the interviews progressed. Each child was thus administered a unique sequence of partitive quotient fraction problems.

In order to produce a more dynamic assessment of the children's knowledge structures, the clinical interviews were often but not always extended to include limited teaching episodes. According to Hunting (1980), the inclusion of limited teaching episodes within clinical interviews takes the clinical interview a step beyond merely assessing the status of a child's cognitive functioning. During these limited teaching episodes, there was no direct teaching of partitioning strategies. The teaching episodes instead focused on: (1) exploring and extending children's knowledge construction, or (2) helping the children to overcome cognitive impasses (VanLehn, 1990) encountered while attempting to solve a partitioning task by asking appropriate focusing questions.

Data analysis

Data analysis methods which allowed children's knowledge constructions to "crystallise" from the data were a critical aspect of the interpretation process. *Open coding, axial coding* and *selective coding* as proposed by Strauss and Corbin (1990) was employed to facilitate this process

Results

The analogs were evaluated in terms of their ecological validity, abstraction-ability, and ease of partitioning. Each of the seven analogs will now be discussed in turn.

Pizza analog

The pizza analog tasks had ecological validity for the children. They perceived a meaningful connection between the context created by the pizza analog and their "real-world" experiences with fractions. They were familiar with the analog and identified with the generally accepted tradition that pizzas are shared fairly among people. For them, the context engendered by the pizza analog was very realistic.

In fact, for Ben, the pizza analog engendered such realistic connotations that he partitioned the pizza into many "realistic" but unequal pieces just as it were served to his family at his local pizza restaurant. This gave rise to some early concerns that the pizza analog might not be conducive to the development of sound partitioning practices, but as the study proceeded, this concern dissipated as it became evident that this was an isolated case.

With this analog, the children were able to explore a large number of interrelated (by number of objects and number of people) tasks. Furthermore, the pizza analog also engendered contexts which promoted the adoption and learning of effective partitioning strategies. The adoption of these effective partitioning strategies indicated that the context facilitated the process of mapping from the source to the target and thus greatly facilitated the process of abstracting the *partitive quotient* fraction construct (i.e., that 2/3 means 2 wholes partitioned into 3 equal shares) from the concrete activities and thus promoted the construction of knowledge about this important fraction construct.

Pancake analog

The pancake analog tasks were found to lack ecological validity with the children in this study. In Australia, pancakes generally are not shared in fractional pieces but like pikelets, they are eaten whole. Thus, many of the children did not make meaningful connections between the context engendered by the pancake analog and their "real-world" partitioning experiences. The partitioning of a pancake into fractional pieces was not realistic for them. This was epitomised by Caitlin when she said, "You can't really share one pancake amongst three people!"

The pancake analog also did not engender contexts which promoted the adoption and learning of effective strategies for partitioning. For example, Claudia cut the pancake horizontally into three unequal-sized pieces. As such, the context engendered by the pancake was not considered effective for facilitating the initial construction of knowledge about the *partitive quotient* fraction construct.

Pikelet analog

Like the pancake, the pikelet analog tasks lacked ecological validity. The children did not make meaningful connections between the context engendered by the pikelet analog and their "real-world" experiences with fractions. When asked to partition three pikelets among four people, Ben partitioned each pikelet into eighths and shared 2/8 to each person. Each share was comprised of six absurdly small pieces. These actions by Ben seemed to indicate that the context established by the pikelet analog was not realistic for him. The investigators (interviewer and non-participant observer) attributed the pikelet analog's lack of ecological validity to a traditional Australian custom which predicates against the partitioning and sharing of individual pikelets. Pikelets are usually shared as whole pikelets not as fractions of a pikelet. Therefore, when the children were required to partition a pikelet into fractional pieces, the whole activity tended to lose relevance and meaning for the children.

If the pikelet was partitioned at all, its small size predicated against its partitioning into more than four pieces (i.e., sharing among two, three and four people). Accordingly, with this analog, children were able to explore only a limited number of interrelated (by number of objects and number of people) tasks. Furthermore, the pikelet analog did not engender contexts which promoted the adoption and learning of effective strategies for partitioning and as such, was not considered an effective analog for facilitating the construction of knowledge about the *partitive quotient* fraction construct.

Apple pie analog

The apple pie analog had ecological validity for the children. They perceived a meaningful connection between the context created by the apple pie analog and their "real-world" environment. They were familiar with the analog and identified with the generally accepted tradition that partitioned apple pies are a regular part of Australian cuisine. In contrast to pikelets and pancakes, apple pies are rarely eaten whole. The context engendered by the sharing of apple pies thus was very realistic for the children. Furthermore, the children found the apple pie tasks very interesting and they were motivated to actively seek solutions for the tasks.

With this analog, the children were able to explore a large number of interrelated (by number of objects and number of people) tasks. Furthermore, the apple pie analog also engendered contexts which promoted the adoption and learning of effective strategies. The adoption of these effective partitioning strategies indicated that the context established by this analog facilitated the process of mapping from the source to the target and thus greatly facilitated the process of abstracting the *partitive quotient* fraction construct from the concrete activities.

Icecream bar analog

Real icecream bars were not used in this study. Instead, the children were presented with long rectangular strips of cardboard and asked to imagine that the strips of cardboard were icecream bars. Because icecream bars were not familiar to the children, they did not perceive a meaningful connection between the context created by the icecream bar analog and their "real-world" environment. For instance, several children "folded" the icecream bar to confirm their estimation of equal-sized pieces before they partitioned the analog object. Even after a reminder from the investigator that it was an icecream bar that they were folding, they continued folding. This action would appear to indicate that, for some children, the partitioning tasks engendered by the icecream analog did not carry with them the realistic connotations normally associated with an icecream bar.

However, despite this slight problem, the analog did engender some very sound partitioning strategies and children were motivated to become involved with the problem and to actively seek solutions. Interestingly, children invariably found the rectangular shape of this analog much easier to partition into thirds and fifths than the circular region analogs. Each time a child experienced difficulty partitioning a circular region, a rectangular region such as the icecream bar was offered in its place and the object was successfully partitioned and shared. For instance, when Thomas was asked to partition two circular region analog objects among three people, he partitioned each object into six pieces. In a subsequent task, when sharing two icecream bar analogs among three people, he partitioned each rectangular object into three pieces. In the next task, he was required to share two circular region models among three people and he, once again, partitioned each object into six pieces! When asked why he adopted different strategies for the circular region model and the rectangular region model he replied, "Because if you tried to cut these [circular region models] into thirds they wouldn't come out equal but the bars come out equal."

In another instance, Sally was requested to share one apple pie among five people. She declared that she would cut it in half then try to get five pieces. She also stated that it was, "Pretty hard." When asked if an icecream bar would be easier, she replied, "Yes" and proceeded to partition it into five equal pieces so that each person would receive 1/5.

This analog promoted contexts which enabled the children to explore a large number of interrelated (by number of objects and number of people) tasks. Furthermore, the context engendered by this analog did not promote the application of the "repeated halving/repeated sizing" strategies which only could be successfully applied to tasks with an even number of shares. Instead, the icecream bar analog promoted the adoption and learning of effective strategies. The adoption of these effective partitioning strategies indicated that the icecream bar analog facilitated the process of mapping from the source to the target and greatly facilitated the process of abstracting the *partitive quotient* fraction construct from the concrete activities and thus promoted the initial construction of knowledge about the partitive quotient fraction construct.

Cake analog

The cake analog tasks had ecological validity. The children made many meaningful connections between the context engendered by the cake analog and their "real-world" environment. It was realistic for them and they were able to identify with the tradition of partitioning and sharing large, wide rectangular cakes. The cake analog also motivated the children to become involved with the problem and to actively seek solutions.

With this analog, the children were able to explore a large number of interrelated (by number of objects and number of people) tasks. However, because of its size (30cm x 21cm), the cake analog did not always engender contexts which promoted the adoption and learning of effective strategies for partitioning. This strategy did not promote ease of partitioning into thirds, fifths or sevenths. The most commonly adopted strategy for partitioning this analog was to perform both horizontal and vertical partitioning. This strategy led to outcomes which resulted in shares which were unequal and unquantifiable. Accordingly, this analog was not considered an effective analog for the initial construction of knowledge about the *partitive quotient* construct.

Licorice strap analog

The children did not perceive a meaningful connection between the context created by the licorice strap analog and their "real-world" environment. They were not familiar with flat, linear licorice straps used in this study. In the real world, most licorice straps are circular, thick and non-linear. Therefore, the context engendered by the flat, linear licorice straps was not realistic and lacked ecological validity for the children.

However, despite the fact that the licorice strap analog lacked ecological validity, it did engender some very sound partitioning strategies and children were motivated to become involved with the problem and to actively seek solutions. Interestingly, children invariably found the narrow rectangular shape of this analog much easier to partition into thirds and fifths than the circular region analogs. When a child experienced difficulty partitioning a circular region and then was presented with the linear licorice analog in its place, the object was successfully partitioned and shared. For instance, Mark stated, "Yeah, but it's a lot harder with something...it's a lot easier with something that's in a strip 'cause you can cut across but with the circle it makes it quite hard."

This analog promoted contexts which enabled the children to explore a large number of interrelated (by number of objects and number of people) tasks. Furthermore, the contexts engendered by this analog object did not promote the application of the ill-fated "repeated halving/repeated sizing" strategies. Instead, the licorice strap analog engendered contexts which promoted the adoption and learning of effective strategies. The adoption of these effective partitioning strategies indicated that the context facilitated the process of mapping from the source to the target and greatly facilitated the process of abstracting the partitive quotient fraction construct from the concrete activities and thus promoted the construction of knowledge about this fraction construct.

Discussion

In this study, we evaluated how the learning of the partitive quotient fraction construct by a sample of young Australian children was influenced by seven different structured mathematical analogs. A summary of this evaluation is presented in the Table 1 below.

Table 1

Evaluation of the analogs

	Ecological validity	Abstraction-ability		Ease of partitioning
		Supports interrelated tasks	Supports effective strategies	
Pizza	Yes	Yes	Yes	No
Apple pie	Yes	Yes	Yes	No
Icecream bar	No	Yes	Yes	Yes
Licorice straps	No	Yes	Yes	Yes
Cake	Yes	Yes	No	No
Pikelet	No	No	No	No
Pancake	No	No	No	No

The pikelet and pancake share the same deep structure as the pizza and apple pie (i.e., they are all circular region models). However, the pikelet and pancakes' lack of ecological validity seriously predicated against them being as effective as the pizza and the apple pie. Within the Australian context, pikelets and pancakes are generally shared whole; they are not partitioned into fractional parts prior to sharing. Therefore, the contexts engendered by the pikelet and pancake promoted unstructured analogs based on set models rather than structured analogs based on region models. The pikelet and pancake thus did not engender effective analogs for the teaching/learning of the partitive quotient fraction construct.

The cake had ecological validity for partitioning into fractional parts. However, it did not support interrelated tasks and was not easy to partition into fractional parts such as thirds and fifths. Its dimensions encouraged both horizontal and vertical partitioning. Such partitioning led to the adoption by many of the children of strategies which did not facilitate the process of abstracting the partitive quotient fraction construct from the concrete activity of partitioning. Because of these two factors, the cake analog did not promote effective abstraction from the concrete model to the partitive quotient fraction construct target. Therefore, the cake did not engender an effective analog for the initial teaching/learning of the partitive quotient fraction construct.

The pizza, apple pie, icecream bar and licorice strap supported interrelated tasks and promoted effective strategies for partitioning. However, the analogs engendered by each of these models had restrictions. The pizza and apple pie had ecological validity but were not easy to physically partition into thirds and fifths. The icecream bars and licorice straps lacked ecological validity but were easy to partition into thirds and fifths. This difference has significant implications for the initial teaching/learning of the partitive quotient fraction construct. Ease of physical partitioning was found to be a more salient factor than ecological validity. If the analog object was easy to partition into a variety of fractional parts, then the children were afforded opportunities to explore a wide variety of tasks, and to apply partitioning strategies which facilitated the abstraction of the partitive quotient fraction construct from the learning activity.

The icecream bar and the licorice strap's lack of ecological validity did not appear to hinder the process of abstracting the partitive quotient fraction construct. However, the difficulties associated with partitioning the pizza and the apple pie into thirds and fifths reduced the number of tasks that could be readily explored by the children. This had the effect of limiting the degree of generalisation of the abstraction that could be achieved from the repertoire of tasks. Being able to explore tasks involving thirds and fifths seemed to be a necessary condition for the abstraction of the partitive quotient fraction construct. Accordingly, the icecream bar and the licorice strap were found to engender more effective analogs for the teaching/learning of initial partitive quotient constructs than the pizza and the apple pie.

Implications for teaching

The findings from this study indicate that if structured mathematical analogs such as those used in this study are to be educationally effective in the teaching of the partitive quotient fraction construct, then teachers need to address the factors which limited their effectiveness. Table 1 summarises the limitations of each of the seven analogs used in this study. For example, the efficacy of the pizza and the apple pie analogs were limited by the fact that circular region models are very difficult to partition into thirds, fifths and sevenths. This problem could be overcome by teaching interventions which provide children with opportunities to first experiment with the estimation of these fractional parts and then develop effective strategies for partitioning circular region models into thirds, fifths and sevenths. Once the children are able to accurately measure and cut out thirds, fifths and tenths, teaching efforts might focus on the creation of templates for use in subsequent tasks. The educational efficacy of the icecream bar and the licorice strap analogs was limited by their lack of ecological validity. However, the long, narrow rectangle and the length models provided high levels of abstraction-ability and ease of partitioning. This suggests that teachers should attempt to devise analogs based on these two types of models which have ecological validity for the children. For example, the children in this study had had much recent experience in dividing up streamers for a craft project. Therefore, in hindsight, it probably would have been better if we had used an analogy based on a length of streamer rather than the licorice strap analogy.

Teachers should ensure that the mathematical analogs used in the teaching of fractions have: (i) ecological validity, (ii) abstraction ability encompassing the quality and quantity of interrelated tasks and the quality of partitioning strategies, and (iii) ease of partitioning. Each proposed analog should be measured against these criteria because, as was demonstrated in this study, even seemingly similar concrete models such as pizzas, apple pies, pikelets and pancakes, can engender analogs which produce dramatically different levels of learning.

Furthermore, teachers also need to be aware that the concrete analogs and their associated contexts which engender effective mathematical mappings for the partitive quotient in some cultures may not do the same for children from other cultures. For example, the pikelet and pancake models which Streefland (1991) found were very effective in enhancing the leaning of young children in the Netherlands lacked ecological validity for children in Australia. It is a matter of concern that many Australian mathematics curricula and teacher resource books are founded on international research outcomes. This study bears witness to the dangers of mindlessly applying results from overseas research. Therefore, it is imperative that teachers adopt critical attitudes towards the use of mathematical analogs and apply appropriate criteria to the application of mathematical analogs in the teaching of the partitive quotient fraction construct.

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