Putting meaning behind bars:

Children's interpretations of bar graphs ®

Brian Doig
Australian Council for Educational Research

Susie Groves
Deakin University

A glance at any newspaper shows that graphs play an important part in presenting data to the public. It appears self-evident that children need to develop "graphical literacy" as part of their mathematics education. As part of a series of mathematically orientated science activities in the Practical Mechanics in Primary Mathematics* *project, 102 upper primary children measured the distance travelled by a falling ball for different time intervals, presented their data in a bar graph and commented on what they thought was happening to the speed of the ball. Children's graphs and written comments were analysed from two perspectives: the degree to which the graphs conformed to graphical conventions (including their accuracy in representing the data), and what children inferred about the motion of the ball from their graphs. Results of the first analysis show that the majority of children understood bar graph conventions, while the second analysis reveals that the children's "graphical literacy" is at various stages of development.*

Introduction

Recent calls for improvement in both curriculum coverage and student capabilities in what has come to be termed "graphical literacy" have recounted the problems that students have with many aspects of graphical work (see, for example, Pereira-Mendoza, Watson & Moritz, 1995). Curcio (1989, pp.5-6) presents three levels of graph comprehension: reading the data (directly from the graph); reading between the data (to interpret and integrate the data); and reading beyond the data (to predict or infer from the data). In order to improve primary children's levels of comprehension, Curcio (1987, p. 391) argues that they should "collect 'real world' data to construct their own simple graphs ...[and] verbalise the relationships and patterns observed ... e.g. larger than, twice as big as, continuously increasing".

This paper arises from collaborative research that was carried out by the authors in the *Practical Mechanics in Primary Mathematics* project, together with Julian Williams the director of the *Mechanics in Action Project* at the University of Manchester.

Research in the *Practical Mechanics in Primary Mathematics* project was designed to investigate ways in which practical activities can be used to foster links between upper primary children's spontaneous concepts and Newtonian mechanics. The underlying approach used in many of the activities in both projects was to replicate some of the motion experiments carried out by Galileo, but in a simplified form.

Key parts of these modifications in some of the activities used in the *Practical Mechanics in Primary Mathematics* project were to use a metronome to measure one second time intervals, and to use paper strips (party streamers) to measure the distance travelled by a moving ball in each second. These strips were then used to make bar graphs (strip graphs) where each strip of streamer represented distance per unit time. In this way speed was made "concrete" and the graph enabled the dynamic acceleration (or in some activities deceleration) to be made static.

A commonly heard claim is that "the most important value of graphs ... is that [they help] children to see relationships which may have passed by unnoticed" (Nuffield Mathematics Project, 1967, p. 3). It is with this in mind that the children were asked to explain the motion of the ball, using their graphs as referents. We believe that this way of representing measured observations and using them to interpret physical activities takes mathematics "into" science.

An earlier paper (Doig, Groves & Williams, 1997) discusses project children's use of such streamer graphs to interpret and explain the motion of a ball rolling along a track in a variety of situations which include uniform motion, acceleration and deceleration due to gravity, and deceleration due to friction. Theoretically, these situations should result in linear graphs. However the significant errors in the data obtained by the children frequently provoked them to formulate spurious explanations for apparently erratic changes in speed.

In the Falling Ball activity, which is the focus of this paper, a ball with an inbuilt stop watch reading to hundredths of a second was used to find the height required for a half second drop, followed by a quarter second drop. Based on this information, children were asked to predict and find the height for a one second drop, present their data in a bar graph (no longer using strips of paper) and comment on what they thought was happening to the speed of the ball. Unlike the rolling ball activities using a metronome, this activity produces data remarkably close to the theoretical values, with no danger of acceleration not being observable. Doig, Groves and Williams (1996) discuss this activity in terms of the opportunities it provides for children to engage in mathematical modelling at an intuitive, informal level and the need for teachers to take account of children's existing models for motion. That paper, however, makes no attempt to examine children's work in detail or from the point of view of graphical literacy.

In this paper we examine the bar graphs created by 102 children together with their explanations of the motion of the falling ball and look at the extent to which Curcio's (1989) levels of graphical comprehension are evident.

Background

The idea of using strips of paper to make a graph is not new. Such "concrete" graphs are usually seen as a form of bar graph, a graphical form familiar to children of 10 to 12 years of age. In Australia, the so-called *National Profile* in mathematics (Curriculum Corporation, 1994a) suggests that by the end of Level 4 (the end of primary school) children should be able to "display data in bar graphs ... [i]nterpret and report on information in tables and bar graphs ... interpret and report on information in line graphs, informally describing trends in the data" (p. 77). In the parallel science document (Curriculum Corporation, 1994b), the suggestion is that by the end of Level 4 children should be able to "measure everyday motions and represent them in graphs and tables, using ideas of position and speed" (p. 59).

In the United States, the National Council of Teachers of Mathematics' standards for mathematics are supported by booklets known as the *Addenda*, which focus on particular aspects of mathematics. In the grades K - 6 booklet *Making Sense of Data* (Lindquist, Luquire, Gardner & Shekaramiz, 1992), virtually all the data are discrete and the main graphical representation is the bar graph. Typical activities involve (bar) graphing responses to questions such as "What is our favorite color?" or "How do we like our chicken cooked?"

(p. 42). In the solitary instance of a line graph there is no suggestion that points may be interpolated, or that the slope of the line could provide information.

The same type of questions were suggested in Britain thirty years ago. For example, in the Nuffield Project's booklet on graphs, topics for graphs for upper primary school children are based exclusively on data collected in discrete categories (Nuffield Mathematics Project, 1967, pp. 22-23).

Kerslake (1981) noted that there appears to be a large gap between children's ability to read a graph for information and their ability to discover and use relationships between the variables. This observation is borne out by a more recent study in Victoria, where some fifteen hundred 10-year-old children were asked to interpret a graph of the patterns of recent migration to Australia (Doig, Piper, Mellor & Masters, 1994). Only 5 per cent of 10-year-olds were able to "synthesise information by indicating changes in trends over the period" (p. 96). The largest percentage (65%) of these students were categorised as offering no response at all to this question, while 30 per cent of students were able only to make responses about a single aspect of the graph.

The Falling Ball activity described in this paper makes considerable demands on children's ability to synthesise information in that they not only need to recognise trends, but also need to interpret the meaning of the data in terms of faster speed corresponding to longer "bars" on the graph.

Early activities devised for the project focused on speed as distance per unit time in order to establish a basis on which to introduce acceleration under gravity. At the outset it was thought that the children would be familiar with, and understand, the concept of speed. This proved not to be the case as some children thought that a larger distance travelled per unit time meant a slower speed!

An introductory activity was devised to remedy this situation. The activity required that a number of children walk in a straight line while their positions, at each second, were marked by other children placing blocks on the ground beside each walking child. Paper streamers were then cut to the length between these blocks to provide a record of the distance walked each second. The questions put to the children were "Who walked the fastest?" and "In which second did the child walk fast (or slow, faster, or slower)?" This walking activity established a direct link between the observation (fast or slow speed of walking) and the length of the measured strip. Thus the activity provided a referent in which a longer strip indicated greater speed than a shorter strip. It was felt that all children now had a shared understanding that a longer distance travelled per unit time means greater speed.

This walking activity and the ones where children used streamer graphs to represent the motion of the rolling balls depended on children cutting strips of paper to the length travelled in equal time intervals and interpreting that longer or shorter strips represented faster or slower speeds, respectively. Hence acceleration is represented by increasing strip lengths.

The Falling Ball activity differed from these activities in two major ways. Firstly, because the distances travelled by the falling ball in the time intervals ranged from about 30 cm to 5 metres, it was impractical to cut streamers to length. Instead, children were required to complete a table to record their data and then transfer that data to a graph, where the axes and divisions were already provided (see Figure 1 for an example of a completed bar graph). Secondly, in this activity the distances travelled were obtained for *differing* time intervals collected on *different occasions*. That is, data for the height required was initially collected for a 0.5 second drop and then for a 0.25 second drop. After this, children were asked to predict and find the height for a 1 second drop. In most cases, this led to an incidental additional data point for a 0.75 second drop. Thus instead of showing the distances travelled by a moving object in successive seconds, the graphs produced in this activity showed distances travelled for different time intervals on different occasions. In order to interpret the graph as representing acceleration, children needed to recognise, albeit intuitively, its essentially exponential nature - a big ask for children at this level. The conceptual complexity of the task needs to be kept in mind when examining the extent to which children exhibited Curcio's (1989) levels of graphical comprehension.

Methodology

The data was collected from the workbooks used by the children to record their predictions, data, graphs, explanations and comments for the Falling Ball activity. A section of the activity required the children to record the distances for drops of varying time intervals, to graph these data, and to explain the motion of the ball. Figure 1 shows an example of a bar graph and the child's explanation of the motion of the ball based on their interpretation of the graph. A different section asked children to "Write what you found out in this lesson".

A total of 102 children's workbooks were read and the graphs and their explanations of the ball's motion categorised, using both their explanations of what they thought was happening to the speed of the ball and their comments on what they had learnt in the lesson, when these were relevant.

The data were analysed in two distinct phases. In the first phase, the graphs were analysed with respect to their conformity to the collected data and their correctness with respect to the conventions of graphical representation. In the second phase, the children's explanations of the falling ball's motion were categorised according to the nature of the explanation. All those comments that displayed the same interpretation (in a qualitative sense) were placed into the same category. Each time a different interpretation was encountered, a new category was formed. In this manner every interpretation was placed into one of several mutually exclusive categories. The process was iterative and followed the procedure described in Adams, Doig and Rosier (1992). The categories that were used to fully classify the data are described below.

Accuracy of graphs

In phase one of the analysis the children's graphs of the Falling Ball, data were placed into four mutually exclusive categories. Descriptions of the four categories are given below, while the number of graphs in each category is shown in Table 1.

Table 1

Accuracy of graphs (N=102)

Degree of accuracy	Accurate	Difficulties with vertical axis	Difficulties with horizontal axis	Other
Number of graphs	67	26	6	3

A total of 67 graphs accurately represented the data. All of these graphs used the vertical axis and its scale accurately, approximating values when necessary. While there had been an expectation that the graphs be drawn as bar graphs, three of these graphs were drawn as line graphs, which accurately plotted the data points but used a line to join them. All of these children used the conventions of graphs correctly, and demonstrated their ability to interpolate values on the y-axis when necessary.

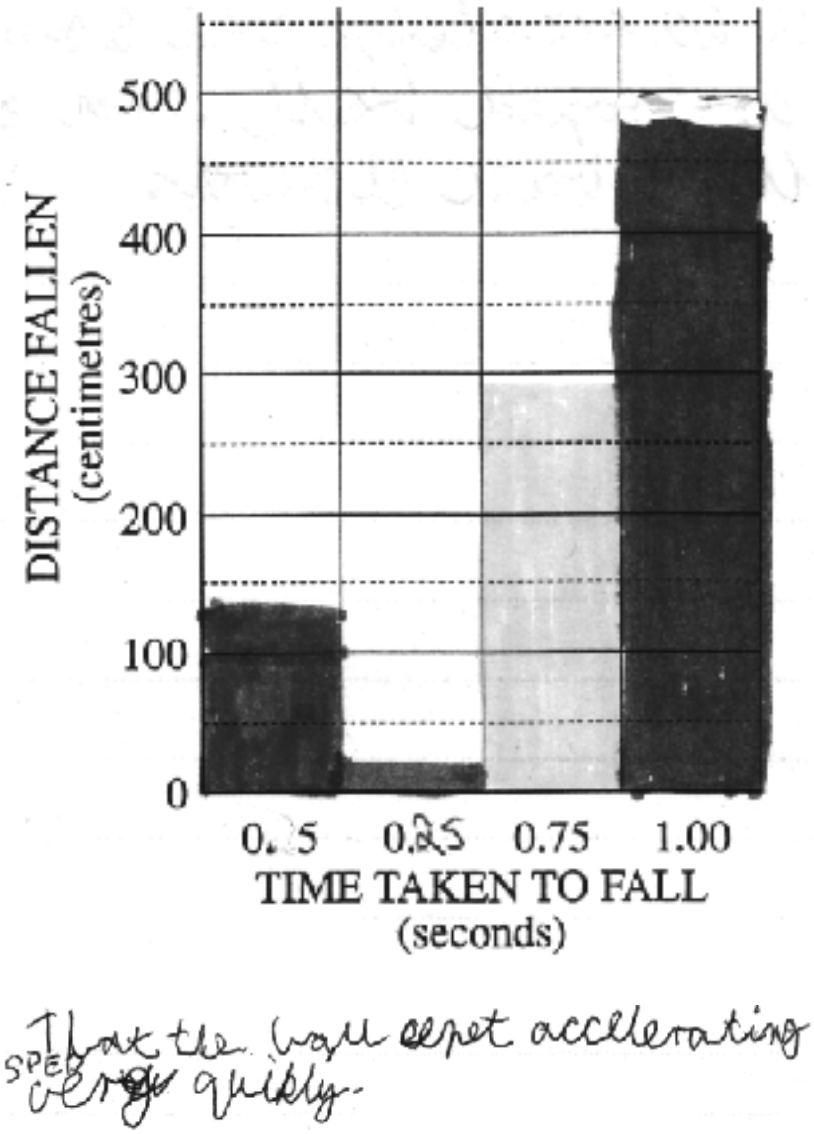


Figure 2. Graph with horizontal axis changed

The second largest category, 26 graphs, were those which exhibited problems with the vertical axis. These problems were of two types. The larger group, 21 graphs, showed that some children were unable to consistently read the scale, whilst a smaller group of five graphs showed no use of intermediate values on this axis. These five graphs were completed simply using the nearest "block" of the supplied grid for the data value.

Surprisingly, as the horizontal axis was a simple four-category arrangement, there were six graphs that exhibited problems with this aspect of the graph. Some of these graphs appear to be the result of a misreading the order of the data from the data table, while the remainder are unexplainable re-arrangements of the order of the data categories. In either case problems with the horizontal axis have caused the graph to be incorrect.

The three graphs in the "other" category consisted of one graph where all four data points were recorded and graphed as approximately 33 cm (see Figure 9) and two graphs where the first two data points had been reversed, but this had been noticed by the children and the horizontal axis changed accordingly (see Figure 2 for an example of such a graph).

Children's explanations

In the second phase, children's responses to the request to "Write down what you think is happening to the speed of the ball" were analysed. In some instances, where children had made comments pertinent to the speed of the ball in their general comments about the whole activity, these were included in the analysis.

Because there had been no explicit request for children to refer to the graph in their responses, the analysis was carried out in two stages. In the first instance, those responses which made explicit mention of the graph (or could be implied to be referring to data points on the graph or its extrapolation) were separated from those which made no such direct reference. A total of 38 children referred directly to the graph, while 64 children did not.

The two sets of responses were analysed separately as described below.

Explanations which referred directly to the graph

The 38 explanations which referred directly to the graph were placed into eight mutually exclusive categories based on the type of explanation given as described below. Table 2 shows the number in each category, further subdivided to indicate the accuracy of the graphs used. This table also shows the classification of the explanations based on an expanded version of Curcio's (1989) levels of graphical comprehension, as will also be described below.

Table 2

Summary of explanations which referred directly to the graph (N=38)

Interpretation of graph	Accurate	Difficulties with vertical axis	Difficulties with horizontal axis	Other	Level of comprehension
Exponential growth (strong)	5	1	0	0	3
Exponential growth (weak)	5	2	0	0	3
Linear growth (strong)	1	0	0	0	2
Linear growth (weak)	1	1	0	0	2

Extrapolation (strong)	1	0	0	0	2
Extrapolation (weak)	1	0	0	0	2
Comparison across horizontal axis	4	4	0	0	1.5
Discussion of data points only	11	1	0	0	1

Curcio's (1989) first level of graphical comprehension requires children to read the data directly from the graph. While the focus of the graphing part of this activity was on children constructing an accurate graph, there was evidence in the children's explanations of them reading data points and connecting these with their experience of the activity. The example shown in Figure 3, however, urges a note of caution as we cannot really be sure whether the child was reading the point from the graph or directly from the table constructed during the data collection. In any case, the explanations which referred only to data points, without any attempt to integrate the data, were classified at level 1, as shown in Table 2. Of the 38 children who appeared to refer to the graph, 12 responses were classified at this lowest level.

While Curcio (1989) describes three levels of comprehension, when grouping the data we found it necessary to insert an extra level between her levels 1 and 2 (this level is referred to as level 1.5 here). Level 1.5 corresponds to children's responses which, while not fully integrating the data, attempt to compare data values across the horizontal axis in some sense.

Figure 4. Examples of responses involving comparison across the horizontal axis

Figure 4 shows two examples of children's responses classified at this level. A total of 8 responses were classified this way. It is difficult to be certain whether the second response refers to the graph. However, we are more confident about the first child's statement that "I expected the ball would go frister because it pics up speed so the graph would be higher to get the ground over longer desten" (sic) -at least if we assume that one of "higher to get the ground" or "over longer desten" refers to time. This highlights some of the difficulties in interpreting the children's responses. The process was indeed an iterative one!

The responses categorised as being at Curcio's level two were characterised by the children interpreting the data as representing a linear model of growth. Although there were only five responses classified at level two, these differed in the way the children presented the explanations and in terms of their clarity of explanation. Three children explicitly stated a linear model. Figure 5 shows a "strong" explanation classified as linear growth. The other two children extrapolated on the basis of a linear model. The "stronger" response (in terms of clarity of explanation) states that "if you dropped the ball from the Rielto it would only take

5 seconds because every second is about 500 cm, like on the graph". The weaker response appear to state that "the ball increased 5 times a 1 sec .... In 2 sec go so fast it will brack".

As discussed earlier, interpreting acceleration in this activity requires children to recognise, albeit intuitively, the essentially exponential nature of the graph. We judged this to be at Curcio's level three - i.e. as showing an ability to read beyond the data in order to predict or infer from the data. A total of 13 children's responses were classified as interpreting the graph to show exponential growth - as in the previous level, these were split according to the clarity of the responses.

Figure 6. A "strong" response based on an exponential interpretation of the graph

Although not strictly speaking correct, Figure 6 shows an example of a "strong" explanation of the exponential nature of the graph. Since the correct interpretation of the graph is that the height quadruples every time the time doubles, this interpretation fails to model the data accurately, however it certainly captures the exponential nature of the data. A much weaker explanation - probably the weakest in the "Exponential growth (weak)" category - is shown in Figure 7.

Figure 8. A "weak" response based on an exponential interpretation of the graph

The children whose responses are in these categories clearly demonstrate their ability to make inferences from the data and therefore exhibit Curcio's third level of graphical comprehension.

Explanations which did not refer directly to the graph

The 64 explanations which did not refer directly to the graph were also placed into eight mutually exclusive categories, based on the type of explanation given. Table 3 shows the number in each category, subdivided to indicate the accuracy of the graphs used.

Table 3

Summary of explanations which did not refer directly to the graph (N=64)

Interpretation of motion of ball	Accurate	Difficulties with vertical axis	Difficulties with horizontal axis	Other
Acceleration (faster and faster)	21	8	3	1
Acceleration (faster at a point)	5	0	0	1

Higher = faster	6	3	2	1
Higher = longer	3	2	0	0
Deceleration (slower and slower)	0	3	0	0
Science only	1	1	0	0
Other	1	0	1	0
Blank	1	0	0	0

Since the explanations do not refer to the graphs, it is not possible to deduce anything about these children's levels of graphical comprehension. Nevertheless, the first two categories of responses imply acceleration and so these roughly correspond to the first two categories in Table 2 in terms of the explanations given, although in this case children do not refer to the graphs to support their explanations.

Typical comments in the first category describe the motion of the ball in terms of acceleration, using words such as "faster and faster" or explanations such as "I found out that as the ball gose down it picks up speed" (sic). Over half of the responses which made no direct reference to the graph (a total of 33) were grouped in this category. For example, the graph and explanation shown in Figure 2 were classified in this way, using the category of "other" for the accuracy of the graph, since the first two horizontal values were exchanged, and "acceleration (faster and faster)" for the explanation, since the child explained "that the ball cepet [kept] accelerating sped very quickly".

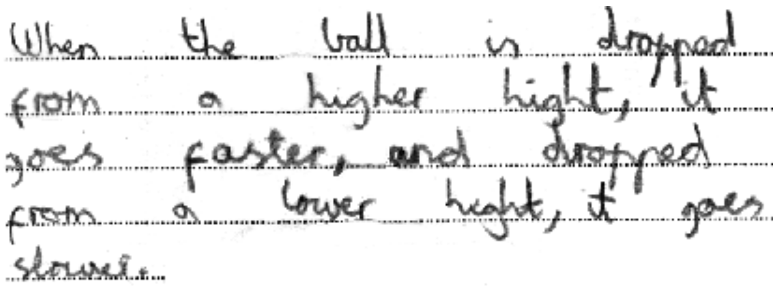
Figure 1, on the other hand, shows an explanation that "the ball gets faster at a certain point then keeps going at about the same speed", which was classified in the category of "acceleration (faster at a point)". Only 6 of the responses which make no direct reference to the graph fit this category. While it was difficult at times to distinguish between these two categories and both were classified under "acceleration", the use of the key words "gets faster at ..." as opposed to "goes faster and faster" or "picks up speed" were used to separate the categories. The notion of an accelerating ball starting to go faster "at a point" was also evident in several of the other activities - in fact, many children claimed to be able to identify the point where they believed that they saw a rolling ball "suddenly speed up" and were often able to persuade other children to "see" the same thing.

The graph and explanation shown in Figure 9 illustrate how difficult it is to decide the extent to which children drew inferences from their graphs. The graph is completely consistent with the data table, which supposedly shows the distances recorded for the different time intervals. However, there is no possible connection between the graph and the explanation given. This graph and the explanation were coded as "acceleration (faster at a point)" and "other".

The next group of three categories correspond to a variety of explanations, some correct, some possibly correct and some clearly incorrect.

The first of these categories, containing 12 explanations, involves children interpreting a higher drop as faster speed. An example of such an explanation is shown in Figure 10.

The second of these categories, containing five explanations, is characterised by children interpreting a higher drop as corresponding to a longer fall.



Figures 11 and 12 show examples of such explanations. The example in Figure 12 is interesting because it somehow reverses the usual description and almost gives the ball animate qualities, although, strictly speaking, time *is* the independent variable.

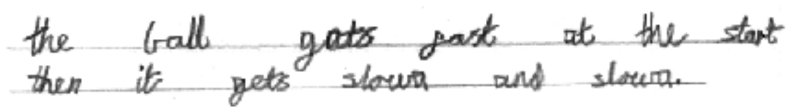


Figure 13 shows the explanation of one of the three children who interpreted their graphs as showing deceleration and who described the motion observed using words such as "the ball gets slower and slower". This indicates that, despite the early walking activities, some children still believe that a longer "bar" represents slower speed.

All of the explanations in these three categories correspond to some sort of comparison being made "across the horizontal axis" as suggested also for the second last category of Table 2.

The remaining five explanations were classified as follows: two gave a scientific explanation only with no mention of speed or acceleration (see Figure 14 for one example); two could not sensibly be categorised here (see Figure 1.5 for one example) and one which was blank.

A number of observations can be made about these classifications. Firstly, a much smaller percentage of those children who did not refer directly to their graphs had actually drawn accurate graphs than those who did refer directly to their graphs. Moreover, all of the difficulties related to the horizontal axis occurred in this group who did not refer directly to their graphs. In addition, it is clear from the example shown in Figure 9 that children did not necessarily rely on their graphs in order to infer what was happening to the speed of the ball.

Discussion

The performance of the children in terms of the accuracy of their graphs is compatible with that suggested by Swatton and Taylor (1994), who found that many upper primary school children are competent in working with bar graphs. The fact that just over a quarter of the responses were categorised as displaying problems with the vertical axis would also confirm their view that scales play a major part in increasing the complexity of graphical tasks. However, in terms of graphical literacy, competency with drawing bar graphs is but a beginning. Reading a graph to extract information is a crucial ability, and probably the most important in everyday life out of school. Thus the second phase of the analysis was aimed at determining the level of competency of these children in the interpretative aspects of graphical literacy.

The analysis of the children's explanations, however, did not support Swatton and Taylor's (1994) claim that "less than 10% of pupils ... satisfactorily described any relationship between the variables" (p. 238). Certainly, there were a number of children whose comments reflect little or no interpretative competency. However, over a third of those children whose explanations directly referred to their graph (13 of the 38) were regarded as being at Curcio's highest level of graphical comprehension, with a further five children classified as being at the second level, which requires reading between the data to interpret and integrate it. Even when taken as a percentage of the total number of responses, this represents considerably more that 10 per cent of children.

Unfortunately it is not possible to comment on the level of graphical comprehension of most of the 64 children who made no direct reference to their graph. However, many of the children's comments do suggest that the emphasis on graphical representation of motion throughout the entire suite of activities provided a vehicle for interpreting their experiences.

Conclusions

A goal of graphical literacy must be to avoid school-based problems of the type described so well by Duggan, Johnson and Gott (1996, p. 473), who state that "without an understanding of the meaning and purpose of continuous data, pupils will either quite sensibly opt for the easier qualitative definition of variables or resort to the ritual collection of numbers". In the *Practical Mechanics in Primary Mathematics* project, children made graphs from their own data gathered during practical science experiments, and were given the opportunity to use their graphs to assist them in interpreting and explaining the observed motion. The results of the analysis of the accuracy of their graphs show that most children were able to construct suitable graphs of the ball's motion. However it was more difficult to find evidence that they were able to draw plausible inferences from these graphs.

Curcio's (1989) three levels of graphical comprehension guided our interpretation of children's explanations of the motion of the Falling Ball. It was not expected that the third of these levels be present in this group of responses, considering both the limited nature of the children's experiences with kinematics and the (assumed) lack of experience they would have had with this aspect of graphical work. In the event we were pleasantly surprised at the number of children whose responses showed the ability to make inferences from the data at or near this highest level.

If our goal is for children use graphs to guide their thinking and build explanations, it is essential that "rather than suggest to students that graphs are not pictures [we should suggest] that graphs are pictures, but that we must understand what it is that they are pictures of" (Chasan & Bethell., 1994, p. 183). A step towards achieving this goal is to be able to find ways to support the development of graphical abilities.

Practitioners and researchers alike have called for the use of "real" data in children's graphical experiences. We agree that this is necessary, but believe that it is not sufficient. Whilst the results of the Falling Ball activity were better than expected with respect to graphical literacy, it must be remembered that this activity was the last of several, where graphs had initially been concretized (with paper streamers) and care had been taken to develop children's use of these graphs as devices for explaining their observations. Simply using the Falling Ball activity alone, we believe, would not have been as productive.

It is inevitable that graphs will continue to be used in everyday life for a wide variety of reasons such as to provide a reason for action, prove a point, explain an issue, describe a situation or justify a decision. If we expect children to develop high levels of graphical literacy at school, curriculum experiences must be designed so that they include opportunities for children to develop an understanding of different types of data, different graphical "types", and the interpretive possibilities of each.

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