

Teachers' beliefs about successful teaching and learning in mathematics

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In the dead of an Antarctic winter, relations at an Australian base went mutinously wrong

He (the station leader) was not the sort of person who expected to win popularity contests. As a leader he thought things through, took a decision and then made an announcement. He believed in rules, pinned notices on board and liked working alone. He seemed not to grasp the idea of collegiate or consultative management.

Being disliked didn't worry him and it was almost as though he thought that better leaders made tougher decisions which upset people and made you unpopular. In his mind, effective management was inextricably linked to unpopularity. (Sydney Morning Herald, October 16, 1999)

Introduction

Teachers' beliefs and practices

Teachers' days are taken up with myriad exchanges with students, fellow teachers, administrative staff, and parents. These exchanges usually are not one-to-one, but one-to-twenty or one-to-thirty. Decisions about what to say or how to behave must be made in an instant. What forms the basis of these decisions?

There is a lot of evidence that teachers' decisions tend not to be based on thoughtful application of a body of professional knowledge acquired during teacher preparation courses, in-service days, or post-graduate study (Korthagen & Kessels, 1999; Nespor, 1987; Pajares, 1992). In fact, teacher preparation courses with their emphasis on theoretical frameworks exert little influence on teaching practice. Rather, decisions are based on deeply held beliefs about teaching that were formed when teachers themselves were students, or, as beginning teachers, assimilating the attitudes and behaviours of their more experienced colleagues. Beliefs, once firmly established, are difficult to change: like everyone else, teachers selectively choose information that confirms their beliefs, even to the point of distorting evidence to make it fit.

An emerging theory about the way knowledge is constructed helps to explain why professional development courses exert little influence on teachers' beliefs and practices (Airasian & Walsh, 1997; Brown, 1997). Knowledge is not transmitted and accepted *in toto* by passive recipients. Rather, people construct new knowledge by aligning it with, or fitting it into, their existing beliefs and experiences. Following from this, knowledge is tentative, subjective, and personal. If teachers' beliefs about activities or attitudes that motivate students and help them to learn are not unearthed and explored, then attempts to challenge them or to encourage different attitudes and behaviours probably will be futile. From another perspective, examining teachers' thinking may provide advances in theory.

The causal link between beliefs and practices is not clear cut. One tends to think of beliefs affecting practice. In the newspaper excerpt printed at the beginning of this paper, the station leader's beliefs about the role of a leader do appear to have a direct influence on his behaviour. In other situations, however, the link is less clear. Novice teachers might adopt the practices of more experienced colleagues because they want to fit into the school, not because of any deliberate intention to translate their own beliefs into practices (that is, apart from the belief that people who want to be accepted by the locals should adopt their practices). There are many stories of novice teachers being told to forget anything they were taught at university or teachers' college because it had no bearing in the real world of schools. Being able to articulate a link between practice and belief may come later. Similarly, enforced changes to behaviour, such as a change in assessment practices, may result in practice followed by beliefs about its efficacy. As an interviewer, it is easier to ask teachers about their teaching practices and *then* to ask them why they behave this way rather than to ask them their beliefs about effective teaching and to show how they put beliefs into practice. Perhaps we are looking for a clearcut beliefs-practices link that does not exist:

"We are also trained to assume an unnatural clarity and tight coherence in what and how people 'believe', and so tend to excise contradictions and conceptual blurriness as indicative of inadequacies in informants or 'the record', instead of being how people (including ourselves) think" (Clendinnen, 1991, p.11).

Beliefs and practices in teaching mathematics

The present study focuses on links between beliefs and practices in the teaching of mathematics at both the primary and the secondary level. Currently there is considerable interest in mathematics teaching. The mathematics reform movement has been calling for

major shifts in teachers' beliefs about the nature of mathematics leading to corresponding major changes in their teaching practices (Battista, 1994; Cobb, Wood, Yackel, & McNeal, 1992; Ernest, 1989; Gutierrez, 1996; Secada et al., 1999). As Battista (1994) points out, *"school mathematics was seen as a set of computational skills; mathematics learning was seen as progressing through carefully scripted schedules of skill acquisition"* (p. 463). That is, there was a set of numerical problems, each with one right answer, and it was the role of

the teacher to show students a set of procedures that would reveal that answer. Many adults recall, as students, following procedures demonstrated to them by teachers, getting the right answer, and passing examinations with no real understanding of what they were doing. There was little attempt by most teachers to understand why some students could not provide the right answer - mostly they were labelled low ability.

The reformists wanted teachers to focus on students' *conceptual understanding* of problems in everyday life that involved mathematics (eg, working out a cricketer's batting average, setting out a vegetable garden, managing a shopping budget). Then teachers would encourage students to try to work out ways in which these real-life problems might be solved. Teachers would not provide students with a set of how-to procedures. The point of mathematics was to help students make sense of everyday life. The reformers acknowledge, however, that the changes they call for will not come easily. The notion of mathematics as a set of procedures to arrive at a right answer is so deeply ingrained in most mathematics teachers, and in teachers of mathematics teachers, that a re-focus on mathematics as a way of making sense of the world will be hard won.

Selecting a level of analysis

The present study seeks to examine links between teachers' beliefs and practices in teaching mathematics. It is difficult to choose the most appropriate level of analysis: the individual, the school or staffroom level, or the primary/secondary level? On entering any staffroom, differences among teachers in teaching approach, level of expertise, and level of enthusiasm are evident. Certainly, students have no trouble labelling teachers from the same school or the same staffroom as good teachers or bad teachers. These individual differences are evident too in statewide or district wide examination results, where the results for students of one teacher differ markedly from those of neighbouring classes. At the next level, schools or staffrooms generate distinctive cultures that are evident to an outsider who visits more than one school or staffroom. The more senior or more dominant teachers are confident enough to talk openly about their approaches to teaching and one can see student teachers or novice teachers taking careful note of the views of their more senior colleagues. At the primary/secondary level, other differences emerge, ones that seem to stem from the different structures of primary schools and high schools in NSW. This is the level of analysis for the present study, though further work at both the individual and the school/staffroom level is warranted.

In typical primary schools in NSW teachers take charge of a group of about 25 students for the whole school year. The teacher covers all the syllabus areas (English, mathematics, performing arts, science and technology, physical development/health/physical education, and human society and its environment), and has a home room where he or she can set up, permanently or semi-permanently, a wide range of equipment (eg, books, blocks, shapes, artwork, science experiments). In terms of assessment, the syllabuses indicate the outcomes that students are expected to achieve, and there is statewide testing of numeracy and literacy in Years 3 and 6. In comparison with high school teachers primary teachers appear to have more flexibility in planning, implementing, and assessing their work.

In typical high schools, mathematics teachers teach only mathematics to students from Year 7 to Year 12. So a teacher with three Year 7 classes, a Year 10 class, and a Year 11 class will see about 150 students four or five times a week. Teachers do not have a home room where they can have easy and secure access to physical equipment, so classrooms tend to be spartan places. Staffrooms are discipline based, so mathematics teachers spend most of their time with other mathematics teachers. Mathematics is formally streamed from Year 9, where students are assigned to Advanced, Intermediate, or Standard classes. Schools make their own decisions about streaming in Years 7 and 8. Finally, the upper levels of high

school are dominated by two statewide examinations, at Year 10 and Year 12. There is a lot of pressure put on teachers by the school and the parents for good results in these examinations.

Method

School principals gave permission to approach teachers and ask them to participate in a group interview at a time that suited them. The audio-taped interviews occurred during the lunch break or directly after school. The numbers of teachers involved in each interview varied from one to approximately six (during the lunch time meetings, some teachers left the group to do playground duty or other tasks, while other teachers joined the group). In all, four primary schools (17 teachers interviewed) and three high school mathematics departments (10 teachers interviewed) were visited. The interviews lasted from about forty minutes to about an hour and a quarter. The interviews with the primary teachers also contained questions about teaching English which are not reported here. The audio-tapes later were transcribed.

The interview questions were left rather open so as not "to lead the witness." The intention was to elicit teachers' spontaneous thoughts rather than to have them respond to predetermined areas of interest. The questions were:

Can you tell me about the strategies you use to teach mathematics, and explain why you think they are effective strategies?

Are there other strategies for teaching mathematics that you think would be effective but that you don't use in your classroom? What are those strategies and why don't you use them?

Why do you think some students don't do well in mathematics?

Do you think that it is important for a teacher to be confident that he or she will be able to teach all the students in the class?

The last question about teachers' confidence was removed from the later interviews. Teachers did not respond to it in the manner intended by the interviewer (to focus of teachers' sense of efficacy). Some spoke about the need for the teacher to have an air of confidence, while others said it was more important for students to have confidence in the teacher rather than for the teacher to have confidence in his or her ability.

Results

Teachers' responses were categorised in four ways: practices related to their epistemological beliefs; practices related to their beliefs about motivation; practices related to their beliefs about pedagogy; and attributional beliefs that were not tied to specific teaching practices. Of course there was some overlap among these categories, especially the motivational beliefs and the pedagogical beliefs. For example, teachers might get students to time themselves doing multiplication grids. This meant they were doing work at a level appropriate to their mathematical development (a pedagogical belief) and their success, doing the grid quicker and quicker, would motivate them to keep trying (a motivational belief).

As mentioned earlier, teachers were not asked directly about their beliefs about the nature of mathematics, the nature of motivation, or the nature of pedagogy. The questions moved from the strategies the teachers themselves nominated to the reasons for their use.

Epistemological beliefs and practices

Table 1 shows primary and secondary teachers' epistemological beliefs about mathematics and how they are reflected in their teaching practice. In the primary school, mathematics is seen as linked to everyday life, it is a way of making sense of objects in everyday life, it links with other aspects of the school curriculum, and it is tied closely to language. The beliefs emerge in practices such as playing in the sandpit and manipulating everyday objects to learn about their mathematical properties (eg, *"we try and teach wholistically, we're teaching the whole child, you don't try to segregate subjects as much as you used to", "we're doing maths now because we're measuring how many buckets of sand go into a big bucket...we're doing lots of things in our PLAY program which are purposeful learning activities", "the children look at any mathematical problem as a way to solve a real life problem...like we have little pictures of buses which are actually models of ten units...so we say to them, your bus drove along and five people hopped on at the first station, and then the bus drove a bit further and two more people hopped on. How many people are there now on the bus?"*). Mathematics also was incorporated with other parts of the curriculum into themes for the term (eg, *"the unit we've just done is on dragons...I had sixteen individual activities that are displayed for them, and they will be all sorts of things. They cover all parts of the curriculum, so there'll be bits on science, there'll be bits on maths, there'll be bits on everything, so they have their own contract and they work through the activities themselves, they chose what activities they're going to do...we still do classwork, we don't just do contract work"*).

And it wasn't just in the early years of primary school that teachers chose work that mirrored important or interesting activities outside the school. One sixth grade teacher taught multiplication of decimals by having her students play the stockmarket (*"the starter is the stock exchange, and it [the book] shows how the stock exchange works. So I get today's newspaper and I take it in and we develop a share portfolio. We make a list of the shares that they know the company, like Cadburys and Just Jeans and Weetbix and Qantas, so they know what the company is. So we list the value of the shares and I give them a \$10,000 budget, and they work out the shares they're going to use. So they're multiplying decimals, so they're learning the concept, learning something else along the way, they've got interest, and about every six to eight weeks we look at the newspaper and see how the shares are doing and they can see if they've made a profit or a loss ...they love it, they work it out accurately because they want to know whether they've made a profit or a loss"*).

Primary teachers also stressed the links between language and mathematics (eg, *"my classroom is an intensive reading classroom and we read the sums because it's just like reading, so it's three plus six, it is reading, it's just reading numbers instead of words", "we use a variety of language, like we don't use subtraction only, we'll use subtraction, take-away, minus, less than, so that you're using different language all the time, so literacy is really crossed over"*).

Most of the secondary teachers seemed to view mathematics as more self-contained, a set of logical relationships that existed in abstract form almost divorced from the everyday lives of students. Mathematics was conducted for the most part in classrooms where teachers gave examples on the blackboard and students then attempted problems in the textbook (eg, *"we try to establish the idea that when you walk into a maths classroom you're here to work...maths is very prescriptive", "they get immediate response to their work, they do their work then at the end of the lesson they get a mark and they know whether they've got it right or wrong, they don't have to wait for a week or a month for feedback"*). However, teachers did indicate when they were asked about strategies they might use but did not use, that more manipulation of objects or measuring of area would be desirable. They did not use manipulation because they did not have the time in an already full syllabus, they did not have secure home rooms where they could keep equipment, and students tended to

misbehave because they were out of routine. On the whole, however, the interviewer sensed that teachers felt comfortable with the traditional chalk-and-talk approach (eg, *"We did have a mathematics laboratory at one stage which was a special room set out with these sort of problems and each Year 7 class used to have one period a week and that was good...it worked well but then we lost all the bits and pieces and then no money to replace them...it wasn't destroyed or maliciously lost, it was just lost, not collected properly...but the kids used to enjoy that"*).

Motivational beliefs and practices

Table 2 shows primary and secondary teachers' beliefs about motivation and the ways they put those beliefs into action. Both primary and secondary teachers believe that the experience of success gives students the motivation to keep working. In the primary school this belief is seen in an emphasis on individual improvement (eg, *"we do a multiplication grid and they do it on a time system, but they try to beat their own time...they're too concerned about being their own time keeper and their own score to care about anyone else"*). One Year 6 teacher allowed students to re-take tests until they passed them (*"I've got three kids at the moment who are taking the test for the third time, and they're interested, they're working out where they're going wrong, and they'll come up after lunch and work on the blackboard together, because you've got to give them the security of success...they get really excited about doing assessment tasks because they know if they don't get it right this time they can do it again"*).

In the secondary school, belief in the motivational value of success is most obvious in teachers' endorsement of streaming classes by ability or previous achievement in mathematics. This way, students can be given work they can do successfully. The interviewer asked teachers about the effect of streaming on the lowest stream: did students in these classes give up trying because they were labelled publicly as low achievers? The general response was that though this did happen sometimes, but not always, the benefits of enhanced motivation for most students outweighed the problems for a smaller number of students (eg, *"if you stream them the top class does perfectly well and the second class does perfectly well and the third class does perfectly well. It's not until you get to whatever is the lower of the lowest stream classes that your problem really comes in, and that's not necessarily the case...it could be a lack of self-esteem, it could be the kid's too lazy, it could be that the kid doesn't care, it could be that the kid is quite talented but doesn't want to be bothered, so you finish up with a group of kids at the bottom who are an extremely difficult problem because they come from all sorts of backgrounds"*). It was interesting to hear secondary teachers argue that streaming was necessary in mathematics but not in other subjects because of the structured logical nature of mathematics, that is, mathematics stood apart from other school subjects.

Most primary and secondary teachers agreed that students were more likely to work if they felt the teachers cared about them as human beings. This meant teachers had to show warmth towards students and interest in their lives outside classroom (eg, ask them about a new sibling or how their football team performed on Saturday). One group of secondary teachers argued that this human bond was the basis of all effective teaching (*"I think trust is the keyword...to win the child's trust in the classroom, that's the whole of the class too, the individual and the whole class...then you can more or less teach any way you want"*), *"we can all learn strategies, we can pick up a book which will give you ways of planning your classroom, ways of setting up your lessons and how to do that. That's fairly easily taught but until you've got the background behind that, a lot of other things don't come into play...a lot of kids don't get enough emotional support"*). One secondary teacher made a distinction between her behaviour in high streamed classes where she was more formal and distant (*"if I get too friendly with someone then I find it hard to reprimand them later"*) and low streamed

classes (*"there are more behavioural problems in the lower classes and I find that a more friendly approach or an encouraging approach works better with getting them back on-task, whereas a higher ability class will get back to work"*).

Some primary teachers argued that giving students choice in what they did generated in students a sense of ownership and this in turn enhanced their motivation to learn. In practice, this meant allowing students to choose the topic for the term, to draw up school rules, to provide contracts where students could choose from a range of activities, and having students write their own school reports. One secondary teacher mentioned that he tried to encourage self discipline in students (*"If a kid gets an 80 TER [Tertiary Entrance Rank], you say that's great but I don't get more money for that. If a kid gets 30, you say well, I know you're disappointed, I'm disappointed, but let's face it, I still get paid. It's your future that's at stake, not mine. It's your decision how hard you work"*). However, but there was no mention of any strategies like those mentioned above that in effect coerced students into taking more responsibility for their work

Both primary and secondary teachers saw collaboration and peer tutoring among students as a way of enhancing motivation because students enjoy being with their peers. In the primary school, this generally was reflected in formal or semi-formal group work. In secondary classrooms, collaboration meant more of a "quiet buzz" as students discussed work with their neighbours. In one secondary group, they agreed that female students worked well in groups while male students often were too competitive for the group to work effectively.

Some teachers, both primary and secondary, thought carefully about what they said to students who were becoming discouraged because they were having trouble with mathematics. Their aim was to keep students trying (eg, secondary: *"sometimes I set an exam that I think is pretty easy, that most should get 60% right. Then I look at the marks and see that so-and-so got 44 and so-and-so got 47 and I know that they'll be very disappointed, and so I change what I was going say...I say, I think that was a pretty difficult test, and that anyone who got more than 40 has passed in my book"*; primary: *"by the time they get to Year 6 they've already decided particularly in maths whether they're good at maths or not. They'll come into the classroom and say: I'm not good at maths. And I say: I know all you guys and I know that every one of you is capable of learning stuff because I know they all are, brainwise they're all capable of learning everything we cover, and I say: if there's anything in maths you don't understand then it's highly likely that we haven't worded it the right way to tell you how to do that yet. It makes a big difference to them"*).

Some primary teachers also indicated that students were keen to learn mathematics when they could see how it linked with their lives outside school. The examples of playing the stockmarket and learning addition using people getting on a bus given earlier are good examples of this. No secondary teacher made mention of motivation coming from connections with life outside school.

Finally, teachers agreed that enthusiasm was infectious: that if teachers appeared excited about what they were doing, then students would show more interest in the work. It was interesting to note that some teachers saw enthusiasm working at the whole school level as well as at the individual level. Staff at one primary school spoke of their strong school philosophy that energised the whole school (*"we're trying to produce a learning community, not just some isolated good learning...you've got to have a philosophical base, this is what we believe in, and our Principal always does this with staff every first day, we always write down what we believe in", "everybody's very positive here and everybody's very involved and we share the same philosophies", "at our school concert, every single child is involved"*). On the other hand, one secondary teacher described an unhappy, divided school with

tensions among the staff, particularly between teachers and the executive (*"the system gets you down...there's a huge level of frustration with the two at the top...it's all cv driven...we say I bet the computer's running hot tonight [adding another achievement to the cv]...it's very debilitating for the staff... there are two or three people on staff who carry the school, they could run rings around those two"*).

Pedagogical beliefs and practices

Both primary and secondary teachers agreed that students learned if they were given work at an appropriate level of difficulty for them. At the primary level this belief sometimes translated into contract work, where students could choose "standard" work or work at a more advanced level. At the secondary level, this belief translated into teachers' support for streamed classes. On a related issue, there was agreement that teachers have to ascertain students' present understanding so that they can pitch new work at the right level. Both primary and secondary teachers mentioned this strategy (eg, secondary: *"you as a teacher has to know exactly what they know...your job becomes doubly difficult to get the pre-requisites covered before you start the new work...the kids are usually unhappy if they're sitting there and you ask them something which they don't understand"*; primary: *"we assess all the time so you can find out if the child is at the stage you think he's at", "you have to have evidence to support your findings"*).

Primary teachers argued that students understood mathematical concepts much better if they could see the concepts demonstrated in physical ways, often referred to as hands-on learning (eg, *"we can do a subtraction sum and represent it in a physical sense", "we can actually represent it in a concrete form...kids go into high school with better knowledge because it's accurate or it's properly formed, where before they had all these little building blocks missing, so that when they put the harder maths on it, it just collapsed and they said, oh, I can't do this", "once we used to do subtraction as borrowing and paying back, whereas now we trade because you can represent it, we can do a subtraction sum and represent it in a physical sense"*). One primary teacher also commented that some young children had trouble with visual discrimination, and so could not distinguish numbers written on a blackboard.

Secondary teachers agreed that manipulating physical objects would help students to understand mathematical concepts, but argued that it was difficult to use this strategy in high schools (eg, *"I know things like hands-on games and models and things like that are really the trend at the moment for learning, but with a lot of the discipline problems we have now, and a lot of the time it's just not possible", "I think the biggest constraint is the number of kids we have in the classroom, if you've got thirty in a class you're really limited to what you can do, you can't even move on the floor let alone get kids to move around and do things, get up and wander round and have a go at this and have a go at that, try this and try that", "if you're doing a geometry lesson on solids you'd like to be able to give all thirty kids a cube, a sphere, a cone and get them to investigate and find different things about them, but for a start we haven't got thirty of them, and some of the lower ability kids they can't handle the change of routine...probably that's our fault, maybe we should persevere and train them, and maybe we should do it more often and get them used to a change of routine, but with the lack of resources you would have to have six kids sharing one cube, and one cone, and one pyramid and the lower ability kids they don't share too well"*).

Some primary teachers said they related school work to students' everyday lives because this heightened students' interest and so they gave closer attention to the work (eg, *"it's not just your times tables these days in the teaching of mathematics, it's not just sums or algorithms of the four operations, they are part and parcel of the whole problem solving process, so it's trying to make maths real life problem solving relating to the kids"*

experience...they've got to be involved in their own learning"). Primary teachers also tried to give a game-like feel to some mathematics so that students would not get bored with the repetition of basic concepts (eg, "they don't realise, but they're constantly practising this one set of facts, and after a while most of them can't help but improve").

Primary teachers also mentioned the importance of moving slowly through the syllabus so that students developed a proper understanding of concepts (eg, "*they [syllabus developers] actually slowed down where we did a lot of the things because the concepts weren't understood...people learnt them and could do them off by heart, but when it came to algebra and stuff like that, they couldn't make sense of it*"). In contrast, secondary teachers complained that the demands of the syllabus meant they did not have time for the hands-on activities that would help students learn. Finally, a number of primary teachers and one secondary teacher pointed to the link between mathematics and language (eg, secondary: "*sometimes kids get problems wrong not because they can't do the maths, but because they don't understand the question. One of my friends told me a classic example. She was introducing the topic of volume and going on about it when one of the kids said, but Miss, isn't volume the knob you turn on the TV? So for them it had nothing to do with silos*"; primary: "*the language is the most difficult for children, that's where they go down in tests because they don't understand the language of maths, that's what we talk about all the time*").

Beliefs about why students fail

Primary and secondary teachers agree on a number of reasons for students' poor performance in mathematics. These include lack of ability, parents who show little interest in what their children do at school, parents who do not instill self-discipline in their children, students who do not value school work, students from unhappy homes who had psychological problems, and poor teaching. Teachers recognise that low achieving students can be their own worst enemy, and are not helped by cruel barbs from other students (eg, primary: "*they create a lot of their own problems...they say I'm not going to try to do this, they give up*", "*there are some kids who can be a bit cruel at times and use the lack of academic ability as a taunt, and these are often the kids that cause the behaviour problems within the schools*").

In addition, primary teachers indicated that they had trouble convincing parents to place their children into classes for students with intellectual disabilities. It also was more difficult these days to get students admitted to these classes. One teacher remarked that sometimes she had to work hard to find what ignited interest in a student (eg, "*if they have the ability to use all those avoidance techniques, they certainly have the ability to learn something and our job is to find the door that gets us in...sometimes it's like opening a walnut, you can get into one little crevice and you think, oh dear, that didn't get me where I wanted to go...You feel when you actually hit it, it's like you're talking to someone and they're suddenly really interested in what you're saying and you sort of feel the heat*").

One secondary teacher also argued that it was too easy for teachers to blame the students for poor performance, when poor teaching was the true cause ("*when you think about it, a student deals with six teachers at a time. Well two of those six may be top-drawer, but the other four may be pretty poor...it's cop-out for many teachers to say that well, when you look at where they come from, of course they won't achieve much, it's their fall-back position... often if you take an interest in kids, they warm to you and start to work because they like you*").

Discussion

This paper looks at differences between primary and secondary teachers in their beliefs about effective teaching strategies for mathematics. The most marked differences emerged at the epistemological level, that is teachers' conceptions of the nature of mathematics and its place within the school curriculum. Primary teachers tended to see mathematics as tied to students' everyday lives, and linked with other aspects of the curriculum. This conception of mathematics translated into classroom activities that mirrored outside-school activities. It also translated into activities incorporating aspects from different syllabus areas, held together by an overarching theme. In contrast, secondary teachers tended to see mathematics as self contained, and it was their role to guide students through its orderly, logical structure. This conception translated into fairly traditional lessons with teachers introducing a new concept, followed by students practising examples from the textbook. Though secondary teachers did acknowledge that students would benefit from physical manipulation of objects, they argued that impediments within high schools often prevented this from happening.

Looking at teachers' beliefs about motivation, it was interesting to see similar beliefs translate into different classroom strategies. At the primary level, the belief that the experience of success motivated students to keep working appeared in practice as a focus on individual improvement. Students were instructed to keep a record of their performance (eg, number correct, time to finish an activity) and to strive to do better next time, akin to the "personal best" approach in many sports. With teachers defining success as personal improvement, it was students' effort that largely determined whether or not they achieved success.

At the secondary level, the belief that success triggered motivation was linked with streaming classes by ability or prior achievement. As noted earlier, in NSW individual schools can choose their own grouping procedures for Years 7 and 8, but there is formal streaming into Advanced, Intermediate, and Standard mathematics classes for Years 9 and 10. It was by streaming, teachers argued, that students would be given work that they could do successfully. Though some teachers did point out that at times students were put into lower streams because of poor performance rather than lack of ability (they had been badly taught in previous years or they had little interest in mathematics), generally streams were equated by teachers with ability in mathematics. That is, ability level was the principal way of distinguishing among students. Indeed, in one mathematics staffroom in particular the interviewer was struck by the way teachers spoke quite unselfconsciously in terms of "higher ability classes" and "lower ability classes". Some of the primary teachers also used within-class mathematics groupings using the argument that it gave students the experience of success. However, these within-class groupings lacked the strong structural element of high school between-class groupings.

Primary and secondary teachers also agreed that students' motivation increased when they were allowed to work with their peers, but again there were differences in practice. Primary teachers used a lot of formal groupwork and peer mentoring, either mentoring within the one class, or having children from the higher years helping children in the lower years. Secondary teachers did not use much groupwork, or only used it with smaller senior classes. They said they found groupwork too disruptive. Students were taken out of routine and treated groupwork as a social occasion. Teachers preferred the "quiet buzz" of students discussing their work with their neighbours.

Looking at pedagogical beliefs and practices, primary and secondary teachers agreed that it was important that students were given tasks appropriate to their level of development. This was translated into contract work at the primary level, where students could choose to do

more advanced tasks or more standard tasks, whereas at the high school level this was translated into streamed mathematics classes. Primary teachers more so than secondary teachers mentioned the connection between interesting tasks and more intense attention on the task. One way to do this was to select tasks that mirrored activities that students would do in their lives outside schools. Similarly, they argued that hands-on activities generated interest that lead to closer attention to the task and that lead in turn to greater understanding of the task. There is a widening body of research on the role of interest in learning (eg, Hidi, 1990), and this finding of the ways in which teachers link interest, attention, and achievement warrants further investigation.

The data for the present study, though not extensive, does provide evidence that primary teachers' beliefs about mathematics and the strategies they use to put beliefs into practice are more in line with the mathematics reform movement than those of secondary teachers (eg, Battista, 1994). In particular, primary teachers say that they try to link mathematics to students' everyday lives. They also make strong connections between mathematics and language, pointing out that many problems with mathematics come from misunderstanding words not misunderstanding mathematical concepts.

The interview data did not make clear, however, the extent to which primary teachers have embraced the conception of mathematics as sense-making rather than mathematics as following set procedures. As a rule, primary teachers do not cover a lot of mathematics at the university level (having to cover six syllabus areas leaves little time for in-depth study of any of them) and they have been criticised for a lack of mathematical understanding. It seems likely that their broader epistemological beliefs about mathematics stem not so much from a sophisticated understanding of mathematics, but rather from a more wholistic approach to teaching at the primary level. That is, teachers see the student as a whole (not just his grasp of language or his grasp of science) because of daily contact for a whole school year. Because they teach across syllabus areas and often take a thematic approach, they can see how students' understanding of the world does not fit within discipline straightjackets. In addition, the primary mathematics syllabus encourages teachers to see mathematics as a problem-solving activity. In fact, a number of primary teachers commented on the marked change in the way mathematics was taught from the time they themselves were students, or for more experienced teachers, from their early days of teaching.

Secondary teachers' conception of mathematics seemed to have more in common with the following set procedures approach. Certainly, it seemed self-contained, separate from other syllabus areas, and not strongly connected to students' lives outside school. Occasionally, teachers spoke of giving some challenging problems to smaller groups of brighter students, but generally the routine seemed to be teachers working through problems on the blackboard followed by students attempting examples from a textbook. As with primary teachers, it seems that secondary teachers' approach to mathematics is determined to a large extent by the way high schools are organised. That is, schools tend to be large with subject-based staffrooms dotted around the school, so mathematics teachers spend most of their time with other mathematics teachers. Because they teach a number of classes, they do not have much individual contact with their students, and because they do not have control of an individual classroom they tend to move from one spartan classroom to the next with few physical "props" to help students learn. The large statewide examinations at Year 10 and Year 12 tend to be the focus of attention, and teachers are obliged to cover all aspects of the syllabus even if they suspect that students' understanding of an earlier aspect of the syllabus is weak.

The high school practice of streaming mathematics classes by ability or prior achievement, as noted earlier, inevitably focuses the attention of both students and teachers on ability or lack of it. This can lead to motivational problems for students who are working hard and still

not doing well. They are likely to attribute their poor performance to low ability, and because most adolescent students see level of ability as a fixed commodity, have little hope for future success and give up trying. The result is learned helplessness (Stipek, 1998). This is the response of one secondary teacher when the interviewer asked what teachers say to students who try but who still do not do well: *"That's very difficult, we do have those kids who try hard and don't achieve success. It's very hard and all we do is try to encourage them as much as we can, and usually they realise they don't have any mathematical ability, so we just try to encourage them and assure them that they're doing their best and we can't expect any more than their best."* The greater stress in primary schools on individual improvement is more desirable from a motivational perspective. The focus shifts from ability to effort expended, and effort is seen by most people as more under individual control than ability.

More generally, teachers' beliefs about why students do poorly in mathematics deserve further investigation. Primary and secondary teachers frequently mentioned students' home backgrounds as an important factor: parents were not interested in what their children did at school; parents did not discipline their children who then turned into behaviour problems at school; parents led transient lives so their children changed schools frequently; children modelled their parents' dislike of schools. One group of teachers replied jokingly that *"we've got the shallow end of the gene pool here."* These attributional beliefs often, no doubt, reflect reality - many parents do not endorse teachers' arguments about the benefits of succeeding at school. However, these beliefs highlight the social class divide between middle class teachers and working class students. As Secada and his colleagues (Secada et al., 1999) point out, research into teachers' beliefs about mathematics should extend further than epistemology and pedagogy. Beliefs about students' social class, ethnicity, and gender also play an important role in teachers' classroom behaviour. One secondary teacher, as noted earlier, commented that it was too easy for incompetent teachers to blame students for poor performance: *"it's a cop-out for many teachers to say that, well, when you look at where they come from, of course they won't achieve much...it's their fall-back position."*

There was little evidence in the data that teachers' beliefs or practices were the product of professional preparation courses, though it should be noted that in the interview teachers were not asked the origin of their beliefs or practices. This fits with research that teaching decisions tend not to be the result of a conscious selection of a theory of learning and resulting teaching strategies (eg, Pajares, 1992). A number of primary teachers did indicate that their practices were the result of following syllabus guidelines. There was a notable exception to this theory-free approach. The staff of one primary school had deliberately adopted as its school philosophy Glasser's control theory (Glasser, 1985) and their teaching decisions were the direct result of this theory (eg, allowing students choice of activities, stressing personal responsibility for one's actions). According to the teachers interviewed, the whole staff enthusiastically embraced the school philosophy, and they were being asked to explain the school's success to other schools and to speak about it at teaching conferences.

This study has been small-scale and preliminary and its findings need replication. However, the findings do support previous research into the links between teachers' beliefs and their classroom practices. Primary teachers more so than secondary teachers appear to be attuned to the conceptualisation of mathematics advocated by the mathematics reform movement. However, this discrepancy in approach may be attributed to differences in the organisation and educational philosophy of primary schools and high schools rather than to a more sophisticated understanding of mathematics by primary teachers.

Table 1: Primary and secondary teachers' epistemological beliefs and teaching practices

Teaching practices Epistemological beliefs

Primary	
showing students practical applications of mathematics (eg, counting buckets of sand in sand pit, "playing" the stockmarket)	mathematics is linked to everyday life
making students manipulate everyday objects	* mathematics is linked to everyday life *students learn best through hands-on activities
combining activities from various syllabus areas under a single theme	mathematics is linked to other syllabus areas
using a variety of terms for concepts, eg, subtract, take-away, minus	language and mathematics are intertwined
allowing activities to "flow" during the school day, rather than strict compartments for English, maths, etc	school subjects aren't compartmentalised in everyday life
Secondary	
examples on the board followed by individual work on text book problems	mathematics is self-contained and orderly
establish a workmanlike atmosphere in the classroom	mathematics is prescriptive, more orderly and accountable than other high school subjects
acknowledge to the class that some mathematics is boring	what is to be studied in mathematics is set by a syllabus committee
students are given maths problems to solve	mathematics consists of answers that can be marked right or wrong immediately
streaming	mathematics requires building up concepts

	in a logical fashion
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Table 2: Primary and secondary teachers' motivational beliefs and practices

Teaching practices Motivational beliefs

Primary	
focus on individual improvement (eg, students time themselves as they practise basic maths facts)	students are motivated to continue working when they experience success
focus on self-competition	as above
students make some decisions about what they do in class	students are motivated to learn when they feel a sense of ownership of their work
students write their own school reports	as above
teachers show enthusiasm for their work	students model teachers' behaviour
teachers act as though they like students, will protect them from harm	students will not learn if they feel unsafe or not wanted
set up ability groupings in mathematics	<ul style="list-style-type: none"> * gives all students the chance to feel successful, and this enhances their motivation * students who experience failure when those around them are succeeding quickly give up trying * students can help each other - this collaboration enhances a sense of community - and this enhances students' motivation
encourage students to praise each others' work	<ul style="list-style-type: none"> * models desirable behaviour * enhances a sense of community and this enhances motivation
students re-take maths tests till they pass them	students feel that they will be successful eventually, and this enhances their

	motivation
explain to students how what they are learning in school relates to their lives outside school	students are motivated if they see how learning will help them in their lives
give students tasks that mirror things they do outside school	as above
developing a school philosophy among the staff	if teachers feel positive and involved in school activities, then their enthusiasm will "rub off" on students
tell students that if they don't understand work then it's the teacher's problem and she must find another way to explain the work	students' confidence in their ability to cope with the work rises
Secondary	
teachers develop warm relationships with students	students work hard because they know teachers care about them
streaming	gives students work they can do and this encourages them to keep on working
avoid highly competitive situations	some students thrive on competition but it is very upsetting for a lot of students
competitions involving different sorts of activities (eg, abstract, practical)	gives a greater number of students the chance to experience success and success motivates them to keep trying
teachers show enthusiasm for work	students "pick up on" teachers' enthusiasm
talk about the importance of self discipline in students	sense of ownership enhances students' motivation
focus on self-improvement	students feel successful and this enhances their motivation
use personal anecdotes in class	helps develop a personal relationship with students, and gives a guide for coping with difficult situations, eg, failure
friendly relationships among teachers	makes for happier teachers and this "rubs

	off" on students
don't become too friendly with students	it can be difficult to reprimand students if it's required later on
be more friendly with lower ability than with higher ability students	<ul style="list-style-type: none"> * lower ability students can't stay on-task for a whole lesson * lower ability students are interested in personal details * the hard-handed approach doesn't work with lower ability students
students helping each other solve problems	most students prefer to help each other than to compete against each other
poor school climate with a gulf between the executive and the teachers	makes for unhappy teachers and this "rubs off" on students

Table 3: Primary and secondary teachers' pedagogical beliefs and practices

Teaching practices Pedagogical beliefs

Primary	
contracts - students can make some decisions about what they do	students can choose work suitable to their level of development (eg, talented students can do advanced work)
hands-on learning	<ul style="list-style-type: none"> * they understand concepts more deeply than when talking about it * young students can have problems with visual discrimination - numbers and letters on the board can look very similar to them
reading mathematics problems	helps students see the link between language and mathematics
asking students to explain their answers	gives an insight into students' thinking

assessing mathematics	gives teachers evidence of students' progress to help them with teaching
mathematics problems that resemble real life problems	students pay close attention because they are interested in the result
have a variety of activities	students learn in different ways (visual, tactile, etc)
move slowly through curriculum	if teachers move too fast students learn algorithms with no real understanding
use mathematical games	mathematics requires a lot of repetition, so make it as fun as possible
revisit concepts over time	students must develop full understanding of concepts
streaming groups	allows teachers more individual time with struggling students
Secondary	
newer teachers with poor mathematics background struggle in classroom	it is impossible for them to teach the advanced high school subjects
few hands-on activities	<ul style="list-style-type: none"> * don't have time if the syllabus is to be covered * don't have home rooms to set up a lot of equipment * students are out of their routine and so muck up
streaming	<ul style="list-style-type: none"> * the curriculum requires that students are streamed in Years 9 and 10 * students can understand the work they are given
make sure the lower stream classes have fewer students than higher stream classes	allows teachers to give more attention to struggling students

allowing quiet talk between students (not formal group work)	students can help each other to learn
memorising basic number facts (tables)	impossible to move on to more difficult maths if can't do basic maths quickly
use group work sparingly	* students tend to go off-task * use only with small class groups because space is a problem
give more difficult problems to groups of high ability students	it works with higher ability groups because students want to find the answer
assessment	kids work when they know a test is coming up - it's the carrot - because they want to do well

Table 4: Primary and secondary teachers' beliefs about why students do poorly at mathematics

Primary
students who are not doing well tend to give up, refuse to try, and this makes the problem worse - this behaviour gets worse as they get older and they start to compare their work with those of other students
high achieving students can taunt low achievers and this makes the problems worse
students with psychological problems (eg, trauma experienced in the home) have trouble learning
sometimes teachers can't work out why students can't learn
it can be the problem of the teacher who hasn't worked out how to engage students (getting inside the walnut)
parents refuse to have their children placed in classes for students who have intellectual disabilities
students lack academic ability
teachers don't explain concepts clearly

parents indulge their children so they won't pay attention in class

parents don't take an interest in children's school work

students are transient and so miss a lot of school

it's more difficult these days to get students placed in classes for students with intellectual disabilities

there are children with attention deficit disorder who have difficulty concentrating in class

Secondary

students haven't been well taught in earlier years at school

students don't value school work

parents don't value their children's school work

students lack ability

the system allows students to progress through grades without passing subjects

maturational level - students mature at different rates - they may not be able to grasp concepts now but they could in a couple of years' time

poor teaching

teachers blame the students for poor performance when it's the teachers' fault

students have psychological problems because of unhappy home lives

teachers don't have a good mathematics background

students' poor behaviour in class means they don't pay attention to the work - discipline problems in schools are on the rise - it's part of wider societal problems

students lack self discipline - they're not prepared to work

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