COLLABORATION IN MATHEMATICAL PROBLEM SOLVING

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Abstract
The research literature on collaborative group work in problem solving contains generally positive results in terms of both the processes involved and the cognitive outcomes achieved. The project reported in this paper sought to look at several aspects of collaborative problem solving in relation to higher order functioning in the chance and data part of the mathematics curriculum. Previous work with students in Grades 6 and 9 suggested a hypothesis that students working in groups may produce higher level outcomes in the process of finding and justifying associations in data sets than those working individually. In the current study, students in Grades 3, 6 and 9 worked in groups on two chance and data tasks: one related to fair dice and the other related to associations among variables presented on data cards. Whereas other reports from this project have focused on the mathematical achievement of students completing tasks, this one will look specifically at students' levels and types of collaboration in relation to issues raised in the literature. The particular focus will be on determining what changes in cognitive functioning take place and what collaborative behaviours contribute to this change.

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Collaboration, and mathematical collaboration in particular, has been the subject of numerous studies and debates in educational research. Many authors (e.g., Lindquist, 1989; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990; Phelps & Damon, 1989; Davidson, 1985) have found some evidence that it can be a successful approach to mathematical learning. They highlight the benefits that come from providing students with opportunities for verbalisation, allowing students to resolve conflicts in understanding, and giving students increased responsibility. Furthermore, it allows a diversity of problem solving techniques and has great scope for accommodating students' needs and interests. In contrast, Stacey (1992) gives some examples of collaborative problem solving episodes which show that there can be a tendency among groups to choose ideas and approaches which are easily accessible, but not necessarily appropriate or correct, thus showing that a collaborative environment need not lead to successful conceptual development.

The main difficulty associated with trying to draw conclusions about the value of the technique is that there are so many factors involved. Slavin (1989-1990) emphasises the fact that while research has undoubtedly found that cooperative learning can increase student learning, what remains controversial is the identification of the specific conditions required to achieve such improvement. Various authors (Johnson & Johnson, 1985; Noddings, 1989; Good, Mulryan and McCaslin, 1992) propose a number of important variables, such as the nature and purpose of the task, the learning strategies employed, the occurrence of academic disagreement among group members, the time spent on task, the
support and feedback of peers in the group, the ability levels of group members, the composition of the group, the role of the teacher in preparing students for small group activities, classroom management, the age of the students, and the role of explanations given and received within the group, but acknowledge the difficulty in isolating these factors for study. In one report (Good, et al., 1992, p.190) it is emphasised that the way that students experience group work is so varied that it is virtually impossible to predict the outcomes of the collaborative event.

It should be pointed out that even the very definition of collaboration is a subject of debate. Gillies and Ashman (1995) distinguish between three types of peer learning: peer tutoring, peer collaboration and cooperative learning. According to their definitions, peer tutoring involves one person in the role of tutor teaching a novice, peer collaboration involves two individuals working together to achieve a task neither could do previously with neither participant assigned any roles, while cooperative learning involves a small group (often three to six individuals) working together on a problem. This last type of group work is characterised by greater structure in terms of task, organisation, expectation, reward and authority, and is a particular behavioural approach to classroom teaching which must be sanctioned by the teacher. The groups and peer learning approaches considered in this study do not fall clearly into one of these categories, being best described as peer collaboration for students in groups of three: there are no clearly defined roles imposed on the students, but there is the expectation that they will work together to solve the problem.

Although it has been suggested that certain statistics oriented activities be conducted in a collaborative setting (see, e.g., Russell & Corwin, 1989), there appear to have been only a few studies of whether or not there are benefits to the learner when encountering statistical and probabilistic ideas in that setting. There are reports of some successes but the data are by no means complete or conclusive (Lajoie & Lavigne, 1994; Lajoie, Jacobs & Lavigne, 1995). As part of a study which investigated how higher order thinking in statistics can be recognised and evaluated, Watson, Collis, Callingham, and Moritz (1995) speculated that students working in groups while interpreting data seemed to produce higher level outcomes than individuals. This led to the authors (Chick & Watson, 1997) carrying out a detailed study of a single group of three Grade 6 boys working on the data analysis protocol described here. The aim of that study was not so much to confirm the hypothesis about the outcomes of group work versus individual work but to ascertain what takes place as part of collaborative activity and how that affects cognition.

This study extends the investigation of some of these issues by observing groups of three students working on two kinds of mathematical activity, specifically from the chance and data (probability and statistics) part of the curriculum. Of relevance to the type of statistical activities encountered by half the groups in the current study is the article by Lehrer and Romberg (1996) which examines the choices children make about how to transform observed phenomena into data and construct suitable representations from which they can make meaningful statistical inferences. It should be pointed out that group work per se is not considered by Lehrer and Romberg. Some of the phenomena which occur in a collaborative environment and which affect the performance of the group will be identified and the interactions between them will be discussed. Of interest is the occurrence of cognitive change; this has two aspects, namely (a) collaboration which leads to a better outcome either for the group or for an individual within the group in comparison with that expected in the absence of a collaborative environment; and (b) occasions where the group or individuals within the group perform less well than might be expected because they are working collaboratively (in support of Stacey's identification (1992) that "two heads may not be better than one").
In addition, other features of collaboration will be identified which appear to have a direct influence on these two outcomes. These include the following.

1. General group interaction phenomena, for example, of a social rather than mathematical nature.
2. External factors which influence collaboration, such as the nature of the actual task, the role played by the interviewer/teacher, and the composition of the group (e.g., gender composition, friendships, physical placement of individuals).
3. Occurrence in a group context of phenomena seen in interviews of individuals.

Procedure
The research involved conducting activities with eight groups of three students: two groups from Grade 3 (a rural school), four groups from Grade 6 (two from a semi-rural school and two from a suburban school), and two from Grade 9 (a suburban boys’ school). Each group worked on one of two protocols involving either dice or data cards, and described below - in the presence of the Interviewer in a room removed from the regular classroom. A second Interviewer was present in the case of the Grade 6 girls dice group. The students in each group were all of the same gender, with the exception of the Grade 3 data cards group, which comprised two boys and a girl. The activity and interviewing lasted for about 45 minutes, although the Grade 6 boys data cards group participated in an additional session two days after the first.

Dice protocol
The purpose of this activity was to investigate children’s understanding of “fairness.” The activity began with a dice game involving two members of the group. Each player had a die and a playing board, marked with six columns numbered 1 to 6, corresponding to the numbers on the dice. Although the dice looked identical (apart from colour) one had been tampered with so that it was biased in favour of 2. The dice were rolled simultaneously and each player placed a block in the appropriate column of his or her board corresponding to the number rolled. The winner was the first to fill one of the columns on the board. This game was repeated two or three times with other pairings of players. In nearly all cases the bias of the loaded die was apparent to independent observers.

At the conclusion of the game the students studied and offered interpretations of a number of frequency graphs for various simulated dice, some of which were biased and some of which were fair. The students first examined six different graphs showing the results from six dice, each rolled 60 times. Each graph had a different colour and hence the colour of the graph could be used to distinguish the die which produced it. The process was then repeated with graphs which showed the results after 360, 600 and, finally, 12000 throws, for the same six dice each time. Examples of some of these graphs are shown in Figure 1. In some interviews the students were shown the (spurious) graph of a red die in which all six outcomes occurred with exactly the same frequency. The purpose of presenting the graphs was to find out what understanding the students had of the concept of fairness and how they thought that was revealed in the graphical representation. See Lidster, Watson, Collis, and Pereira-Mendoza (1996) for further discussion of the protocol, and Lidster, Pereira-Mendoza, Watson and Collis (1995) for a complete set of the graphs of the simulated dice.
Figure 1: Three sample graphs from the dice protocol, showing frequency outcomes for three of the six dice after 360 throws.

Of interest to our study of collaboration was the fact that the dice protocol had a fairly narrow focus, namely the observation of children's understanding of fairness. This made the questions and activities seem relatively "closed." The sessions proceeded along the lines of a standard interview, with the Interviewer having a large influence over the direction of the discussion, governing, to a certain extent, who participated and therefore the nature of the collaboration.

Data cards protocol
The group was given a set of 16 cards, each containing information such as age, weight, and fast food consumption for an individual. An example of one of the cards is shown in Figure 2. At the beginning of the session the students were shown the data cards and allowed time to examine them. The Interviewer then asked them to find and represent any interesting connections among the characteristics of the individuals represented on the cards and justify these for the Interviewer. The data provided were chosen to be consistent with responses the students might expect to receive if they had queried sixteen people within their own community. The sixteen people ranged from 8 to 18 years, their favourite activities were plausible considering their age, and their weights for age were within the normal range as published by the National Health and Medical Research Council (1957). In some cases the students completed cards with their own personal data, to give a total of 19 cards in the set.

<table>
<thead>
<tr>
<th>Name:</th>
<th>Jennifer Rado</th>
<th>Age:</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Favourite activity:</td>
<td>Board games</td>
<td>Eye colour:</td>
<td>Green</td>
</tr>
<tr>
<td>Fast food meals per week:</td>
<td>4</td>
<td>Weight (kg):</td>
<td>33</td>
</tr>
</tbody>
</table>
for the students to demonstrate their knowledge or understanding of a variety of graphical forms they were invited to contribute their own ideas and, in some instances, were also shown prepared examples of data representations such as tables, pie charts, line, scatter and column graphs. When this occurred they were asked if they had used any of these before. They were then given the opportunity to use one or more of them to relate variables from the data. Further description of the protocol can be found in Watson, Collis, Callingham and Moritz (1995), with examples of responses largely from individual students in Lidster, Chick and Watson (1997).

In contrast to the dice protocol the data cards protocol was inherently more "open ended" in the nature of the questions and activities. The Interviewer was able to leave the students to direct themselves more often than was the case in the dice activity, and the activity seemed more conducive to collaboration.

Recording the sessions
Each session was video-taped, then transcribed and annotated. The purpose of the annotation was to try to identify exactly what was said by group members and to illustrate the interactions and activity occurring during the collaborative process. Occasionally the annotator (at times a transcriber, but more usually one of the authors) has speculated about the exact nature of the observed behaviour. Since such conjecturing may be inaccurate question marks or other comments have been included in the transcript to indicate when such speculations have been made. In the excerpts from the transcripts below the annotator's comments are within square brackets; a [...] in the transcript means that material has been edited from it; an ellipsis with no enclosing brackets indicates the kind of pause or hiatus common in conversation. Students within a group are identified in the transcripts as S1 to S3 (occasionally with B or G instead of S to distinguish gender); however, note that for some of the sessions the placement of the video camera made it difficult to identify individual speakers. In this case just S is used, with the understanding that if two S’s appear in succession in the transcript with no other distinguished speaker between them then the authors are confident that the two are different students.

The groups
The eight groups are distinguished by labelling which identifies Grade - gender - school type - protocol. The composition of the groups was determined by the students' teachers, who were asked to supply three students who would work well together on a mathematical task. The groups are listed below, together with a description of the distinctive characteristics apparent in each group as well as a list of the extracts which feature that group in the Results section. It should be noted that most of the descriptions are based on our observations of the groups only, rather than information from a more objective source, such as the students' teachers.

GRADE 3 - GIRLS - RURAL - DICE [EXTRACTS 6, 11, 14, 15]
This group of girls appeared to be comfortable with one another, although their bluntness when pointing out perceived misconceptions was probably typical of the social skills of the age group. S3 seemed to have a dominant personality, while S1 worked more quietly in the background and seemed less inclined to express her opinions.

GRADE 3 - MIXED - RURAL - DATA CARDS [EXTRACTS 12, 24]
This group differed from all the others in comprising both genders: specifically, two boys and one girl. They tended to follow their own ideas rather than work together, perhaps because of the egocentrism typical of the age, and also because the Interviewer suggested particular task allocations in order for them to accomplish
something substantial during the session (this is discussed further in Extract 24).
Judging from the interactions the three of them were not particularly close friends
outside the group, although the boys, one of whom was sitting next to the girl with the
other on the opposite side of the table, were both distracted by the same sorts of
things towards the end of the interview. The girl, on the other hand, seemed able to
concentrate for longer periods and was happy to work on her own. B2 participated
very little.

GRADE 6 - BOYS - SEMI-RURAL - DICE [EXTRACTS 2, 23, 26, 27]
These boys seemed comfortable with each other, but the extent of their collaboration
appeared to be affected by the closed nature of the dice protocol and the
Interviewer's questions, so that their comments were brief and, to a large extent,
directed to the Interviewer rather than to each other for discussion. In the case of this
group the position of the video camera and the students made it difficult to
distinguish individual participants.

GRADE 6 - GIRLS - SEMI-RURAL - DATA CARDS [EXTRACTS 5, 19, 20]
Although the girls seemed to be friends and collaborated quite well initially, their
decision to divide the data cards among them affected the extent of their
collaboration later in the interview. The transcriber suggested that they were also
reserved about interacting in front of the Interviewer and this, together with the fact
that the task was split up, resulted in responses which were often mumbled,
indicating indecision and, perhaps, a lack of cooperation. They also seemed to lose
interest in the exercise once they obtained a result that looked presentable or
acceptable in some sense; as if in producing some output - namely a graph - they felt
that they had satisfied the Interviewer's requirements. Note that because of the
placement of the video camera it is difficult to identify who contributed which
comment. In the transcripts a new G indicates (probably) a different person from the
previous G; a pair (or more) of distinguished G's, such as G*, indicates that on both
occasions the speaker is definitely the same. All of this notwithstanding, there are
only three children in the group. The authors' impression is that all three are
contributing, although one seems a little more vociferous than the others. She is
labelled G1 when she can be recognised, but she may also be one or more of the
G's. G3 is another who can be identified specifically from time to time. G2 seems to
contribute the least, but there are possible reasons for this as indicated at the end of
Extract 20.

GRADE 6 - GIRLS - SUBURBAN - DICE [EXTRACTS 3, 7, 10, 13]
The girls in this group worked well together and their interactions were comfortable.
This group was one of the more collaborative of those who undertook the dice
protocol. There was a second Interviewer present for this interview, denoted by I2 in
the excerpts. There was also a slight variation on the protocol for this group: the girls
were required to indicate how confident they were about their judgements of fairness
or unfairness for the various dice shown in the graphs, by indicating a position on a
confidence continuum.

GRADE 6 - BOYS - SUBURBAN - DATA CARDS [EXTRACTS 9, 18, 21]
This group was the focus of the study reported in Chick and Watson (1997) which
provided the impetus for the current work and in which many of the phenomena
described here were illustrated. These boys were obviously good friends with a
comfortable rapport and gave the impression of having the most collaborative
interactions of all the groups studied. They listened to each other, they brainstormed
effectively, tasks were delegated and accepted without fuss, leadership shifted easily
as required, and they respected each others' ideas, even when pointing out that they
were wrong.
GRADE 9 - BOYS - SUBURBAN - DICE [EXTRACTS 16, 17]
These students seemed comfortable together, but again the nature of the dice protocol may have interfered with opportunities for extensive collaboration. The discussion section of this paper contains a description of one of the students' perceptions of the group work experienced by him in this group.

GRADE 9 - BOYS - SUBURBAN - DATA CARDS [EXTRACTS 1, 4, 8, 22, 25]
These boys worked together cooperatively, giving the impression that they probably formed a friendship group outside the confines of the interview. As will be seen later, S3 may have experienced some disadvantages which restricted his contribution because of his physical location at the short end of the table.

Results
The results will be divided into two sections. The first considers cognitive change which arises from collaboration, defining and describing the phenomena "lifting" and "falling." These terms refer to collaboration which improves the cognitive functioning of group members and that which has a negative effect on cognitive levels, respectively. The second section presents some of the factors which influence collaboration, and discusses the role they play in determining positive and negative outcomes. For example, the influence of conflict within a group will be considered. In addition, a number of phenomenon which influence cognition and which can arise in an individual context but which may also affect collaborative behaviour and outcomes, will be investigated. Each phenomenon will be accompanied by at least one illustrative excerpt from the group interviews. It should be pointed out, however, that such an extract may, in fact, demonstrate a number of phenomena since they rarely occur independently. This means that some phenomena will appear in some extracts before they have been described.

Each phenomenon description begins with a heading which gives the name of the phenomenon and lists the extracts in which it can be observed. The extract numbers listed before the semicolon are found with the phenomenon currently being discussed; the ones which follow the semicolon are found elsewhere in the report. The extracts themselves are in numerical order, and are labelled with a heading which identifies the number of the extract, the group involved and the phenomena which are evident therein. The order of the phenomena listed in the heading is the same as the order in which they are described in the paper, so that the reader can determine whether to refer forwards or backwards from the currently described phenomenon to find descriptions of the others in the list. Thus, for example, Extract 14 appears in the section on conflict, and its ordered list of phenomena includes "Falling" (described earlier in the report), "Conflict" (which is the current focus), and "Egocentrism" (which will be described later in the report). A question mark in either of the heading types implies that the evidence for the presence of a particular phenomenon is not strong. In general, each extract contains an example of at least one of the following: (a) collaboration which leads to "better" outcomes than individual work, as encapsulated in the idea of "lifting"; (b) collaboration which has a negative effect on levels of cognition, denoted by "falling"; (c) factors which affect the nature and success of collaboration; (d) occurrence in a group context of a phenomenon which is not specifically concerned with collaboration, although it may influence it, and which may also arise in interviews with individuals.

Lifting and falling
In any educational technique one of the primary concerns is producing cognitive change. One of the reasons that group work is espoused is because better outcomes can be achieved by students working in a group in comparison to working as
individuals. On the other hand, it may be that the level of understanding and performance of the group may be reduced as a result of collaboration. The terms "lifting" and "falling" respectively will be used to refer to these two types of cognitive outcomes when they arise from collaboration.

**Lifting [Extracts 1, 2, 3; 7, 9, 25]**
The term "lifting" will be used when collaboration has cognitive benefits, raising the cognitive level of members of the group. This can arise because an individual is already functioning at a higher level and is able to lift the others in the group, or it may arise without an obvious source, for example when the group brainstorms or wrestles with an idea until all gain mastery and comprehension.

**Extract 1 [Grade 9 - Boys - Suburban - Data Cards] [Lifting, Charismatic Intellectual, Other Social Factors].**
As can be seen in this excerpt, which followed the boys’ initial consideration of eye colour frequency (see Extract 4), the Interviewer hinted that it might be interesting to look at the relationship between two variables and the boys chose to investigate the relationship between age and weight. Collaboration took place when they decided on the best way of representing the graph. The graph that they produced was nearly equivalent to a scatter graph, but with bars instead of points, and with their own method of dealing with age values which occur more than once. The decisions made by S1 and S2 regarding scale, choice of axes and dealing with double values help the group to attain a satisfactory outcome. It is difficult to know to what extent the same level of overall achievement could have been accomplished by the individuals on their own.

I

[...] Is there anything within that you can look at in terms of age and ... ?
Something else significant you could look at?

S2

What did we do before?

S1

Eye colour. [Pause] Do age and weight.

I

Want to graph it?

S2

Age and weight. Or just weight?

S1

Age and weight, so we could do two 16 year olds, two 18 year olds and stuff ...

S2

So what are we going for? ... Like across there we could put age going that way [horizontally], and that way [vertically] fat to skinniest ... unless you put weight down here going that way.

I

Do you want to use graph paper to actually draw a graph this time?

S2

Might as well.

S1

Can't draw it! [Nevertheless, S1 takes on the task.]

S2

Colour it in.

S1

How many 8 year olds?
S3  One ... no, two.
S1  How's the age thing going? Up the side?
S2  So you have like two 8 year olds and the skinniest to fattest there.
S1  Weights going up there ... age across the bottom? [S1 looking to S2 for confirmation as he works on setting up the graph.]
S2  Weight's going that way and ...
S1  ... age is up there. What's the lowest age?

[S2's decision here changes S1's approach: they put age on the y-axis and weight along the horizontal axis.]

S3  8.
S1  We'll start at 8. [Draws them up the side.] That's about it. And the weight was ... here we have ... Who's the lightest?
S2  Rosemary Black.
S1  If we go in tens say, 10 ... [starts to label the horizontal axis in tens] ... that makes it heaps too small. Do it in 5s. [S1 decides on an appropriate scale for the weights.] ... 57 there ...
S2  66. There's a 74 .
S1  OK, 85. [His upper limit on the horizontal axis.]
S2  Righty-o. So the first one's an 8 year old.
S1  How much?
S2  Umm ... 24. There's two of them. How will we do that? [They need to resolve the dilemma of how to deal with two cards having the same age.]
S1  Divide the squares up, like two so you got your line through the middle for the first one. How big is the first one? [S1 decides that they will make room for two columns with each age group spacing.]
S2  24. [S1 doing the drawing and dividing up the page, S3 handing cards to S2.]
S2  Next one is 30.
S1  Just 30. This is going to be hard. I don't think it's going to work out.
S2/3  Probably not.
S1 Should have pencilled it in then I could rub it out.
S2 Next one ... 9. How many 9's are there?
S3 Only two.
S1 Two 9's ... that's good.

[They continue to work together cooperatively to make the graph.]

Note that S3 contributes only rarely, perhaps because of his position at the short end of the table, and possibly also because S2 may play a particularly important leadership role for the group, aided, in this case, by S1. Both of these phenomena will be discussed in more detail in later sections, because they are circumstances which appear to influence collaborative outcomes (see Extracts 8 and 25).

**Extract 2 [Grade 6 - boys - semirural - dice] [Lifting?].**
As highlighted by Chick and Watson (1997) it can be difficult to ascertain whether and to what extent lifting has taken place. This is particularly true when one student agrees with another's correct assertion. The problem is that an observer may not be able to determine whether or not, in agreeing, cognitive change actually took place for the second student: he or she may merely be agreeing to avoid having to think about the issue or because the assertion was already known to be true. Similar comments apply to the phenomenon of "falling."

This example highlights this difficulty. An opinion - correct - is expressed by one of the students and the others in the group agree with him. A study of the events leading up to this exchange gives no indication of their level of cognition regarding the question prior to this display of understanding, although judging from the enthusiasm of the responses it seems safe to say that they are sure of their understanding, and are not agreeing out of laziness or faith in their colleague's opinions. The students were undertaking the dice activity and had played a number of games involving the blue biased die. After two of the dice games, S3, who lost the first game and so sat out the second, had the opportunity to play again. The person who was Player 1, with the blue die, won both of those earlier games.

S3 Can I swap sides? [to make him player 1, with the blue die which won the two games held before this]

S? And dices?! [Consensus laughter about understanding why he wants this.]

For this group there was a slight variation on the protocol: instead of each player having a board marked with the numbers 1 to 6, the two players shared a single board; player 1, with the blue die, had the columns 1, 2 or 3, while the other player had the remaining three numbers. Following the above exchange the Interviewer gave S3 the choice of which numbers he wanted. He readily chose the numbers which were likely to win, with the other students indicating that they were not surprised at his choice.

**Extract 3 [Grade 6 - girls - suburban - dice] [Lifting, Falling?, Tenacity of ideas, Previous experience].**
As this group of girls wrestled with the idea of fairness and bias, some of them regarded a die as being fair if the shape of its graph is maintained regardless of how many trials are conducted. They examined the graphs of various dice for 60 and then 360 throws, from which some discussion ensued about fairness. Later they looked at the graphs of the outcomes for 600 rolls, first referring to the fair blue die (where S2 showed that there is some internal conflict in her concept of fair - this will be further highlighted on another occasion) and then considering the light green die, which was biased heavily in favour of 4. S3 took the opportunity to explain her understanding of fair and did so quite well, but it was not immediately clear that she was able to "lift" the others. Note the changes in understanding for S2.

S2  [In reference to the graphs - 600 throws and the earlier 360 - of the fair blue die] It seems to be the same again but it's changed Cos this one and this one ... they're still leading again so ...

S3  They are all round about the same height really.

S2  They are only about 4 or 5 or 6 different really. [Here S2 hints at a better concept of fair.]

I  Is that fair?

S2  I reckon. [...]

S3  Like if this one was about here [indicates how to make it look unfair by lengthening a column] and there were about 10 different from the second it would be unfair but it's not. [Compares heights of columns.]

[...] [They then consider the light green graph.]

S2  It's the same again. It's the same. You've rolled it a different amount ... like double each time ...

S3  Each time ...

S1  But here ...

S3  ... and four is still leading ...

S2  ... and it's always been the same one leading ...

S3  Except for ... oh yes. [Potential argument dies and she agrees about the 4 having always been well ahead.]

S2  ... 4 has been leading all the time and I have to say now that it is quite fair Cos you have the same result each time basically.

[Here she is comparing graphs not dice, so her observed understanding of "fair" deteriorates.]
So the dice is a “fair dice”?

Yeah, I'd say that.

Well, not really because ...

Well, I think 4 ...

If you were playing a game and you knew that for a fact 4 would probably win each time ...

Oh yeah! [Not clear]

... and you rolled it 300 times or something like that then and the other people chose like 1 or something then you'd know that you had won because 4 ...

But in here in the rest of the game, 2 seems to be coming up a lot and 4 doesn't seem to be coming up a lot really at all. [She is referring back to the results of their earlier dice games, in which the 2 does do better.]

Because like with 4 there's 3 and 4 there and then 3 and 3 and 5 or something. [Seems to be referring to something off camera, perhaps the horse-race block graphs ... but it is not clear what she means by this.]

So it seems to be weird ... that it's not ... because we've done ...

But we haven't really played ... that wasn't really [played/rolled?] that many times ...

We said the game was to have a fair dice. Ignore the game for a second ... is the dice fair?

Yeah it is. [Has she been negatively influenced by S2’s opinion?]

Well ...

But the girls did make an interesting point then that this one was not the same as these because there wasn’t that many rolls compared to that one - which was interesting.

Yeah.

So what was the decision on the light green one?

I'd say unfair.

Yeah, I'd say unfair as well.

[Despite what she'd said earlier. Perhaps S3 has lifted her!]
S3 I'd be pretty certain ...

S1 I've sort of been half and half.

I What would you need to convince you?

S1 [laugher] I don't know! ... To convince me I'd probably need these to come up [waves hand across the lower columns] or 4 goes down a bit around there.

[She indicates that she wants a shorter 4 column. Interestingly, this is quite a reasonable description of what a "fair" graph should look like.]

Towards the end, however, S2 did seem to become fully converted. The following refers to a graph of 12000 rolls for the light green die (biased in favour of 4). In particular, S2 appeared to be lifted by S3's earlier description of what would happen in a game played using that die.

S3 Unfair, really, because all these ... if you took out 4 they'd be fair because they are quite like the same except 4 ... it shoots up ...

S1 It's the same again I think ... it's half and half ...

S2 It's half fair and half unfair Cos ...

S3 It's just 4 that stands out ...

S2 It's like and there ...

S3 If you cut it off like say here it would be just about the same as the others. [Indicates having a shorter 4 column.]

I2 Can a dice be both fair and unfair? The same dice?

S2 Well I'd say it wasn't really fair ... It's the same again as the other ones have been ...

Ss Yeah.

S2 It's four that's winning. Like what we said before if it was in a game and someone had ... like keep rolling up 4 if you used the dice it would be really unfair for the other people playing ...

[Note the use of the word "we" here: S2 has allowed the group to take possession of - and credit for (?!?) - S3's idea.]

The experience that S3 has had in the past playing games involving dice gave her a context in which to frame her understanding of fairness, and it appears that she has used that framework to lift S2's understanding.

FALLING [EXTRACTS 4, 5, 6, 7; 3?, 10?, 17?, 23, 24]
It may be that collaborative work fails to produce an increase in cognitive functioning; moreover, it may reduce it. The term "falling" is used when collaboration is detrimental to cognition. There are two levels of falling, firstly "picking the easiest ideas," which may only be detrimental in the sense that the group misses an opportunity to be lifted; and "pulling down," in which the level of cognitive functioning is actually reduced, so that members come to believe something that is incorrect or unsatisfactory. These will be discussed in turn.

As mentioned in the introduction, Stacey (1992) reveals that "two heads are not necessarily better than one" because groups can have a tendency to discard more complicated ideas for simpler ones. This will be referred to as "picking the easiest ideas." Although this will not always lead to errors or misunderstanding, it may prevent the group from extending its performance to a higher level of cognition, which is, in itself, an undesirable outcome. It is important to note that, for a group, picking the easiest ideas can lead to rapid, readily attainable outcomes, and that this may influence decisions: the group may choose not to tackle something perceived as difficult or time-consuming if there are "easier" options available whose shortcomings are not readily apparent. This is apparent in Extract 5.

A more active form of falling is "pulling down." In this case the correct understanding held by a student or students in the group is changed to erroneous belief because of convincing but incorrect arguments presented during collaboration. For example, a persuasive group member might give cogent reasons for a particular interpretation of the problem in such a way that other group members accept its plausibility. This is particularly likely in the case of a group member with considerable charisma and/or leadership ability but who is operating at a cognitive level insufficient for the task at hand.

Extract 4 [Grade 9 - boys - suburban - data cards] [Falling - Picking the easiest ideas, Social/organisational collaboration]. This example is rather simplistic and does not reflect that group's overall performance, but it serves as a useful illustration of "picking the easiest ideas."

As the extract shows, for a first representation of information from the data cards the boys considered eye colour and used the actual cards themselves to produce a frequency histogram. They recognised - and indeed S2 insisted - that what they produced at the end was, in fact, a graph, despite the Interviewer probing for a pencil and paper representation. While it is not appropriate, perhaps, to criticise this, it is proposed that it is the choice of eye colour that makes this a possible example of "picking the easiest ideas": it was an unsophisticated choice that was considered by all age groups including the Grade 3 students. It should be pointed out that these Grade 9 boys did go on to consider more complicated relationships.

I [...] Would you like to arrange them in some way? Look at one or two aspects of the information on the cards?

S3 Do all the eye colours.

S1 That's what I was thinking. I'll do the blues.

S2 I'll do browns.

S3 I'll do greens. [Comfortably/co-operatively sort cards between
them and then lay them out in columns in front of them. They seem very laid back and are not exerting themselves hugely!

I  Is that a way of representing it?

Ss  Yeah.

I  If someone came into the room they could look at it and say ...

S2  If it had a heading or something ... eye colours or something.

I  Do you want it represented as a graph?

S2  This is a type of graph.

Observe that their ability to assign tasks among themselves contributes to the rapidly attained outcome. It is not cognitive ability alone that produces successful collaboration.

**Extract 5** [Grade 6 - Girls - Semirural - Data Cards] [Falling - Picking the easiest ideas, Misunderstanding, The big picture, Task factors].

As mentioned in the explanation of the phenomenon of "Picking the easiest ideas," the decision to undertake a task because it is simpler to deal with in comparison with the alternatives (regardless of its correctness) can lead to the rapid achievement of a well-defined output. In this group the first twenty or so minutes of the interview were quite interesting from a collaborative and mathematical point of view, with considerable interaction and hypothesising. The students, however, then became focussed on a particular aspect and worked individually, having divided the cards among them according to number of fast food meals consumed (in the categories "lots," "some" and "none"). Each girl laboriously carried out the same tasks as each of the others for her own subset of cards, and, as will be seen in Extracts 19 and 20, there was no synthesis at the end. [The Procedure section in which the groups are described explains the peculiarities associated with identifying the students for the transcript.]

I  What interesting things do you think you could find out using all that information that you've got there?

G  How many have blue eyes and stuff.

G1  How much they weigh for the fast food meals. [Interviewer starts to speak, but breaks off] Cos she weighs less than me and she only has one fast food meal.

G*  And this one weighs 45 and has 5.

G1  But does he do active things? [Ends unclearly]

G*  He does football, so ...

G  She weighs 66 and she does reading. [Further examination of
the cards and discussion ensues.]

I You've obviously got some ideas there. How could you show somebody else what you've found out?

G That, umm, the more fast foods the more you weigh. Or if you do something like reading you weigh more.

G Because if you do something like football you lose it again.

I Would you like to organise those somehow so you can show those sorts of things? [General examination of the cards and the consumption of fast foods shown.]

G1 Put all the ones that don't eat any fast foods here.

G OK. [They examine the remaining cards and eventually settle on making a pile for those with 1 to 4 fast food meals, and a pile for the remaining more frequent consumers.]

G3 [With the group of fast food abstainers] They're all around 28, 30 [their weight]. It goes 8, 10, 12 for those [putting them in age order].

G They're pretty small.

G1 [Examines the activities listed on G3's cards] Netball, football, netball.

G And they all do activities, too.

G1 [Sorting her cards - the "lots" group - into age order] Most of these are old. [Reads out their weights:] 30, 45, 60, 66, 74.

[Meanwhile the other two girls are sorting the "some" group into age order as well.]

G1 [To Interviewer while others work] This guy eats [not sure about the word "eats", but it is not "weighs"] heaps. He watches TV and eats 12.

I What does that tell you?

G1 That he doesn't do much and he weighs a lot. [Looks at her other cards] So does this person and so does this person. Most of them watch TV. Maybe fast food makes you lazy. [Laughs. The cards are now in three groups, one pile in front of each girl. Each group is determined by fast food consumption ("none," "1-4" and "5 or more"), and has been further ordered by age.]

I What sorts of things have you found out so far?
These ones weigh a lot Cos they watch telly and, umm, they eat a lot of fast foods.

[Referring to her "none" pile] And there's less than any of them that don't eat any fast food meals per week [referring to the fact that her pile is the smallest] ... and they all weigh around the 30 kilos ... so they're pretty light and they do activities as well.

[The exact details of the next bit are difficult, but I think the girl with the "1-4" group is asked for some specific observations about her group, but it seems that they're too miscellaneous for her to do any more than indicate there's nothing she can see. Refer also to Extract 20 for a possible reason for her difficulty. Note that up to this point the ideas have been quite sophisticated.]

Okay, how would you represent that information, then?

Graphs.

What sort of graphs?

Bar graphs [other suggestions difficult to hear].

Would you like to have a go at doing something like that?

Okay [but enthusiasm seems reduced]

[...] You could use some of the materials here to make it without having to write if you wanted to.

Yeah.

What ages are you going to have the blues to be?

[Points to the cards in her set of "5 or more" fast foods] These, these, these and these.

[Realises that G3 has something else in mind for the use of the blue blocks] Oh, okay, do you want to do it that way?

What way? [Perhaps G3 is not sure what she has in mind! This misunderstanding may contribute to the decrease of cognitive functioning.]
G1 Or you could have it in these groups [points to her whole set]

G3 Okay, so you put them in how many fast food groups.

G1 Okay. [This extract is continued in Extract 19.]

They then proceeded to use two attributes to illustrate just one variable: they built towers to show exactly how many fast foods were consumed, and the towers had different colours depending on the fast food consumption group they belonged to. Thus a high level idea was lost thanks to the "two heads worse than one" syndrome: each girl kept her own set of data and, as will be seen in Extract 19, there was no attempt to recombine all the results. G3, who had the "none" group, had no towers to build, and acted as scribe but it was not clear what she wrote. They then built separate towers to represent the ages with each girl using a different colour from the fast food colour and it was also a different colour from that used by her companions - again indicating that the data were split irreconcilably into three groups.

The age towers were placed between the fast food towers. G3 built her age towers as well, and left gaps between them to show the lack of fast food consumption. G1 suggested she place counters in the gaps to represent zero. We note that a concrete materials representation (e.g., a graph built out of blocks) is just as legitimate as a drawn representation and that it is not necessarily the case that one indicates performance at a higher level than the other.

The above exchanges highlight that they were thinking about relationships, but they failed to develop techniques for showing them.

Whether this is because they genuinely did not have the skills, or because they lost sight of those relationships in the minutiae of producing some sort of output is difficult to determine. It must be highlighted that it is not always possible to tell students' cognitive levels of functioning by observing the outcomes of their activity. The decline of functioning associated with losing sight of overall task is further discussed in the section on "The big picture."

They appeared to lose interest in the exercise once they had accomplished some sort of outcome that looked pretty or presentable or acceptable in some sense. It may be that they were thinking "It looks like a graph so it must be what the Interviewer wants so once I've done something like that I don't have to worry any more." This is just a conjecture on the part of one of the authors; nevertheless the enthusiasm, interest and - perhaps - ability definitely waned as the interview progressed.

It is of interest to contrast the attainment of this group with that of the Grade 6 - boys - suburban - data cards group as described in Chick and Watson (1997). [Although the boys' group contributes extracts to the present study, their overall cognitive levels are more apparent in the other study.] The girls achieved a well-defined output much faster than the boys, but they only considered straightforward information (see also Extracts 19 and 20) and seemed to miss some of the higher level issues. Admittedly, some of the higher level ideas were present in their discussions, as evidenced above, but they avoided the issue of dealing with these ideas in their representations. The boys, in contrast, seemed to want to deal with these more difficult ideas, but had trouble with the depiction, which may be why their output appeared less "well-defined."
Towards the end of the dice protocol the Interviewer gave these Grade 3 girls the chance to draw the sort of graph which they thought would arise if they had a fair die and then repeated the process for an unfair die. Before starting, she also showed them the graph of the impossibly fair red die, whose columns were all the same height. Although the students saw nothing particularly remarkable about it, it is conceivable that it influenced what they drew for their fair graphs later on, especially as it remained on the table in front of them. The interpretation of fairness based on having columns of equal height has been discussed in Lidster, et al. (1996).

I: Now, I'm going to get you to draw me a graph, each of you, of a dice that you think is a fair dice ... could you do that?

S2: What do you mean?

I: Well, a graph like one of these but one where you think "Yes, that's fair."

S2: So ... Can we have these dice down there or do we just make it up ...

I: Just make it up out of your head. Could you do that? [...] 

I: First of all, what do you think of that one there? [Interviewer shows them the graph of the implausible red dice, which has all its columns identical in height.]

S1: That one is fair.

S2: Fair.

S3: Very fair.

S1: Very fair because they're all the same ...

S3: I reckon that one is the fairest one out of those.

I: Does that happen? Does that happen when you throw a dice?

S?: I don't know.

S2: It could. [Students prepare to draw their own graphs]

I: And if you could draw a graph of the dice that you think is fair.

[...]

S3: But I can't do a graph that's unfair ...Cos they're all fair.
I You don't think there are any ... that's all right, so you don't have to draw one.

S3 I'll try to.

S2 This one's fair, I think.

S3 How far do you have to go? Do you go all the way to the top? [She is wondering how tall to make her columns.]

I Oh, you can do it as big as you like. [Students continue drawing graphs.]

S3 {S2}'s doing the same thing as me. [S2 has six exactly even columns at first.]

S2 There, I done it. [Shows her work to the Interviewer: her graph has columns of equal height although they have not all been coloured in.] No, I haven't finished yet. [She returns to colouring in the columns and as she does so she makes the fourth column one unit and then two units taller.] [...] 

S2 I'm not doing the same as you {S3}.

S3 And I'm not doing the same as you.

S2 I'm doing mine different. [She now has one column uneven.] [...] 

[S2 adjusts her uneven column some more; the five other columns are even.] [...] 

S3 You're copying.

S1 That's an unfair dice. [S1 points to S2's graph, with its one uneven column. S1's is all even. Observe the effect that this comment seems to have on S2 in what follows.]

S2 No it's not, you don't know what I'm going to do yet, though, do you?

[...]

S1 Finished my fair dice. [All the columns are the same height.]

I Good girl, ok, so that's your fair one {S1}. Would you like to draw one that you think might be unfair? [...] [Shortly after this S2 starts readjusting her graph so that all its columns are equally high again.]

S1 I missed out three sides and this is my unfair dice. [She leaves 3 spaces on the graph paper before starting the next graph.]
Right ok. […]

S2 I've finished. [S2 has now brought her 5 other columns up to the other one so that all are equally high.]

It is not clear why S2 suddenly changed her graph to have one column taller than the others after commencing with even columns, but it is highly likely that S1's critical comment about it being an unfair die influenced her to change back to the equal height graph. In addition, the Interviewer's praising response to the equal height graph that S1 produced may have been regarded as praise for a correct answer, rather than praise for having completed the task (which is probably what was intended), and could have provoked S2 to change her response. Thus a misinterpretation of what was said by an authority figure could have had deleterious consequences for one of the group. Meanwhile S3, who had been working more slowly and who allowed herself to be sidetracked from time to time, by choosing the colours of her textas for example, also constructed a graph with equal columns. She may well have been influenced by what the others had done because she could see their work, although she neither studies it closely nor comments on what they have done. After the extract above she commented on how slowly she had been going, adding "I'm trying to get mine to look pretty, not messy."

Having all the columns the same height is certainly neat, so it may even be that this feeling guided what she did to some extent!

Extract 11 follows almost immediately from the above excerpt.

**Extract 7 [Grade 6 - girls - suburban - dice] [Lifting, Falling - pulling down, Tenacity of ideas].**

In this group, as hinted in Extract 3, S2 seemed to have the idea that fairness has to do with a given die's frequency graph having the same shape regardless of the total number of rolls. The earliest indication of this is in the first of the following quotes, although S2's train of thought therein is a little muddled. That particular excerpt arose just after the Interviewer first showed the graphs with 360 throws. Note that the graphs for the 60 throws were still in front of the students. Later it seemed that S1 also latched on to this idea, although S3 maintained the correct view of fairness. These events occurred prior to Extract 3.

S2 I would say that it's pretty fair because ... I suppose it would depend on what game you're playing and see how ... like what you have to roll and everything ... and with this one if you had toCos ... how many times you've rolled it in this one ... if you like ... what's the word ... they seem pretty fair because this one you've rolled it more times than that one and it seems the same way that they are still fair.

Almost immediately after this they looked at another graph.

I How about the light green one?

S1 Seems to be the same as before ... [Their pointing highlights that 4 is still well ahead.]

S2 It's the same with this oneCos they are still basically the same...
S1  No, that's not really true. It seems ... number 2 when you roll it more times you've got bigger average... it goes up. [Agreement in the background.]

S3  Cos 2's the third tallest here [on the 360 graph], where before [on the 60 graph] it was the smallest ... and 6 and 5 have swapped places really Cos 6 used to be smaller than 5.

S2  But they're still basically the same.

S1  Basically the same ...

S3  They are still like kind of unfair because with 4 that's nearly double. [S3 has the right idea about fairness here.]

S2  If you just took a blink at that one and a blink at that one, you'd think they were basically the same ... the way you looked at them ...

[S2 clings to the consistency between graphs idea of fairness.]

I  You mean the top and the bottom? [Referring to the two graphs, i.e. 60 and 360]

S2  Yeah. If you took a blink at that one and then a blink at that one I would think they were still the same.

I  And so your decision about them?

S3  Fair ... unfair ...Cos with 4 ...

S2  I'd say unsure ... sort of unsure ...

S3  I'd say about there.

S1  It's sort of unfair but it's sort of fair. [S1 fence-sits, perhaps influenced by both S2 and S3.]

S2  It's fair the way that they are similar [graphs].

S3  Yeah. [S3 seems to fall, but it may be that she is agreeing with the observation that the graphs are similar, and not necessarily agreeing with S2's apparent interpretation that this implies fairness.]

S1  No but like ... but look at that one [the 4 column] it's sort of fair ... more than half this is than that and it's ... if you sort of took this one [the 4] off then they would be basically fair Cos they are around sort of that amount. [The latter half of this suggests that S1 has a reasonable partial understanding of fairness, and this
seems to support S3:]

S3 It's just it looks like 4 is like kind of ... all the others if you took 4 away then it would look quite fair, except with 4 it kind of looks ...

S2 ... it looks different ...

S3 It stands out ... [Dealing with the dominant 4 column is proving problematic.]

S2 I'm pretty ... how do you ...

S3 I'm about here ...

S2 I'm unsure that it isn't fair.

S1 ... that it isn't fair.

S2 Like I'm sort of half way between both of them. [They are trying to indicate the confidence levels that they have in their judgement about fairness.]

Cognition seems to rise and fall in this extract as the girls struggle to gain an understanding of fairness. The final few phrases occur as they try to indicate their confidence about their judgement of fairness. It is conceded, however, that trying to indicate a certainty level about a probabilistic idea such as fairness is, perhaps, likely to be confusing for students of this age.

Factors which influence collaborative outcomes
CHARISMATIC INTELLECTUAL [EXTRACTS 8, 9; 1, 12, 18, 22, 24, 25]

It has been found (see, for example, King, cited in Good et al., 1992) that high-achieving students can dominate group thinking and play leadership roles, while low achievers tend to be more passive in collaborative situations. The existence of a "charismatic intellectual" - that is, a group member with both cognitive and leadership ability - can be a key factor in a group's performance on collaborative tasks. If only one of the two ingredients is present then group success may be more limited: good leadership may not be productive as far as cognition is concerned, while intellect only may lead to the production of useful ideas but the group may not make use of them. There was no a priori information about the students' ability levels in the groups considered for this study, but to a certain extent judgements can be made about this phenomenon based on individuals' performances on the particular activity concerned.

The example below provides some evidence for S2 being the charismatic intellectual for the Grade 9 - boys - suburban - data cards group. A reading of Extracts 9, 18 and the excerpts in Chick and Watson (1997) suggests that S1 in the Grade 6 - boys - suburban - data cards group has those attributes, although there is a remarkable sense of equality in that group; while in the Grade 6 - girls - suburban - dice group S3 probably has the role, with enough charisma (or, at least, there is enough rapport within the group) to lift the others on a couple of occasions. In contrast, Extract 24 gives an illustration of the difficulties which can arise if only one of the ingredients is
present: the girl seems to have the cognitive ability to assist the group, but is unable to lead the two boys to a higher level. There may be other factors influencing this outcome as will be discussed later under the heading "Other social factors influencing collaboration."

**EXTRACT 8 [GRADE 9 - BOYS - SUBURBAN - DATA CARDS] [CHARISMATIC INTELLECTUAL, SOCIAL/ORGANISATIONAL COLLABORATION].**

After the boys have named some different methods of representing information (without discussing what is meant by each term and how it might be used) the Interviewer encourages them to consider an alternative way of representing their information. In the following excerpt S2 provides considerable productive direction for the group, suggesting he may be the charismatic intellectual for this group. Additional evidence in favour of this assessment of S2's role is provided by Extracts 22 and 25. Observe that, as in Extract 4, organisational ability again stands them in good stead.

I  

[...] Would you like to choose another way of representing the information you've already put there?

S1  

Should we choose another aspect?

I  

Would you like to discuss that and work out what sort of things you'd like to look at.

S2  

We could do weight between them, 25 and so on ...

S1  

We could do sport ... or how much fast food they ate in a week.

S2  

There's a lot of difference ...

S1  

... variation ...

S2  

... between the fast food things. Might as well do age.

S1/2  

Age I think. [Easy agreement - they just do it; although in what follows S2 seems to direct proceedings.]

S2  

... do 0 to 8 [Indicates towards S3]. 9 - 15 [self]

S1  

I'll go from 15 up. [All accept this as the way to go, and they proceed to sort the cards]

S1  

Oh there's someone here with the same name as me!

S2  

Where!

S1  

There! [S2 shakes head, humorously indicating "So what?!"; pleasant banter as they sort the cards into three columns determined by age groups.]

I  

OK, what have you got there?
S3  Age groups.

I  So that ... Show how many in each age group.

S2  From 0-8 ... How come you've got a 9 in there?! [S3 has accumulated the two 9 year olds, giving him four cards instead of two. S2 checks S3's column and realises how few cards are involved.] 0 to ... might as well not have that column!

S1  No-one below 9.

S3  8.

S2  Make that 0 to 10. Then I go from ... 10 to 14 .

S1  Yup, then 15 up.

**EXTRACT 9 [GRADE 6 - BOYS - SUBURBAN - DATA CARDS] [LIFTING, CHARISMATIC INTELLECTUAL].**

This is but one of several episodes in which S1 has a positive dominant role which provides constructive guidance for the group. Note, too, that charismatic intellectuals are not perfect: S1 receives correction from S3, so that, in fact, it is S3 who lifts S1’s performance at that point.

S1  Oh yes ... do you still have the calculator so we can work out the average? So {S3}, are these all the age groups?

S2  Yup ... starts with 8.

S1  Well, we have to do the average of each age group and how much they eat. [General conversation follows.] Where's 8s? These are the youngest people.

S3  What am I doing?

S1  You're writing down ...

S2  the average ...

S1  So put down all the ages 8-18 ...

S3  Oh, I forgot something ... how much is each ... ?

S1  About 5.5 is the first one ... the average for 8 year olds is 5.5 fast foods a week. [...] 

S1  Put 5.5? [to S3] That's from the information we've got. [S2 on the side is having some difficulty with his graph. The Interviewer and S1 suggest he turns it round and starts again which he does.]
S1 What's the next one ... 4.5 for 9 year old. Zero for a 10 year old. 1 is the average for 11 year old. 12 year old ... 2.5. 3 for 13 year old. 10 for 14 year old.

S3 Ten!!!

S1 Out of the information we've got. 15 ... 2. 2 for 16. 8.5 ...

S3 4.5.

S1 How can that be right?

S3 4.5 ... Cos 8 plus 1 divided by 2...

S1 Yes 4.5 ... Next one's 4, 12 ...

S3 This is zero ... I think so ...

S1 8. Going alright? [to S2]

S2 Yes, it's neater.

S1 Let's see ... we have to ... out of the information we've been given ... that's the average for each age.

S2 Shall I top it off with the pen ... to make the bars ... bit neater.

S1 Yup ... so we can get a page out of here ... [gets graph paper] There you are {S3} [who is the scribe] Now put all the ages down the bottom ... every inch, line inch ...

S3 Every second one?

S1 Yeah. 8, 9 ... up to 18. [S3 writes in numbers along bottom of graph]. Now up the side put 1 to ...

S3 10 ... [S3 and S2 working away at making the graphs] I just went up to 11 ...

S1 OK ... for an 8 year old ... 5.5 is in the middle on that one there ...

S3 OK so ...

S1 Well you read them out and I'll put them in ... 5.5 ... [S1 takes over drawing the graph.]

Although the focus of this study has been on collaboration, it should be pointed out that, in terms of mathematical cognition, S1 was one of the few students in any of the
groups observed to highlight that the results obtained by the group only applied to the data available: he emphasised this three times in the above exchanges.

**Doubt [Extracts 10, 11]**
Doubt often precedes cognitive change. New evidence or arguments may cause learners to reevaluate their current understanding, with well-argued correct reasoning capable of turning doubt into positive cognitive change. There is, however, no guarantee that favourable outcomes will eventuate from doubt. For example, students who lack confidence in their conceptual framework may be negatively influenced by a more confident student and, as a result of stronger conflicting opinions, they may come to doubt their original understanding, even if it was correct. Alternatively, some students may be too timid to put forward their thoughts for the group to consider because of their lack of certainty about those ideas.

**Extract 10 [Grade 6 - girls - suburban - dice] [Falling - pulling down, doubt].**
In studying one of the graphs, which showed the results of a biased die after it was rolled 360 times (a die which had looked reasonably fair after only 60 rolls), S3 seems confident initially in her assessment that it is unfair, but loses that confidence by the end of the sequence, perhaps influenced by S1's reasoning.

S2  It looks like it's changed a lot really.
Ss  Yeah.
S2  ... quite a bit really ...
S3  Now it looks kind of unfair because there are some quite tall ones and some quite small ones ...
S2  Like this one again ...
S  Yeah.
S1  But I think it's really pretty fair because from there to there it's a tiny bit more than half. [Points out that one of the columns is about twice one of the others. I am not sure, however, of what she might be thinking in interpreting things this way.]
S3  Probably if it was set out where you do the largest [height order] it would look probably more fair.
S2  Look different really.
I  Fair or unfair?
S2  I don't know! [In a funny voice, marking the humour of it.]
S3  I'm unsure ... I don't know. [They all join in with a chorus of "don't know".]

**Extract 11 [Grade 3 - girls - rural - dice] [Doubt, conflict, previous experience, social collaboration].**
The students were drawing their own graphs to show what they thought fair and unfair dice would do, starting with a fair die. S3, who had been lagging behind the others (see Extract 6 which precedes this almost directly), noticed some patterns in the graph she had been drawing, and this led to some debate about the existence of certain kinds of dice. There is considerable doubt expressed by S3 and S1, whose previous experiences of dice have perhaps included only six-sided examples.

S3 Hey, I can see a pattern here.

I Can you?

S3 Yep.

S1 [Student 1 leans across and points out on Student 3’s graph] Purple, green, purple. Plus you've got a pattern there, purple green purple.

S3 Yeah, and another pattern.

I Right.

S3 Two, four, six.

I Yes.

S3 And if you had two, four, six, eight and if you went up to like ten you'd go two, four, six, eight, ten.

I Mm, mm, except [...] this dice doesn't go up to ten does it?

S2 I've seen the dice goes up to ten though.

I Have you?

S2 Mmm, up at our classroom.

S3 You don't have a dice that goes up to ten. [Sounds horrified at the thought.]

S2 Yes we do. [Adamantly.]

S3 No we don't.

S1 Where?

S2 In the basket.

S1 How come nobody uses that dice?

S3 You'll have to show me when we go back up.
S2 Yeah.

S3 And I'll find out if you are telling the truth. [She's still not really convinced it exists!]

**Conflict [Extracts 12, 13, 14; 6, 11, 16, 17, 18, 27]**
Conflict often has an important role in bringing about cognitive change within groups. Social conflict runs the risk of diminishing the group's efficiency and performance, because it may distract the group from the task. Cognitive conflict, on the other hand, often brings about a reevaluation of current understanding as the group struggles with the problem in an effort to understand it and to solve it. If the conflict is not resolved, group members may be left with different and possibly contradictory ideas and the group may be unable to obtain a consensus understanding of the difficulty or complete the task. On the other hand, if the conflict is resolved the outcome may be either lifting or falling. This depends on how convincing the arguments presented are, and may even be determined by who puts them.

**Extract 12 [GRADE 3 - MIXED - RURAL - DATA CARDS] [CHARISMATIC INTELLECTUAL, CONFLICT, TENACITY OF IDEAS, OTHER SOCIAL FACTORS, EGOCENTRISM].**
The next example is from the group with one girl and two boys working with the data cards. It gives an indication of what can happen when two ideas clash, particularly at this age level when the social skills for dealing with conflict are still developing.

[All three of them are initially working on sorting the cards into age order. Then B1 becomes interested in the "activities" category and is looking for who else plays netball and watches TV. This comes to a head:]

G Is there any more ... no, there's none under 8.

B1 Who else likes netball? [Puts older, previously sorted (with respect to age) netballer next to the 8 year old netballer. Moves on to the next unsorted card.] TV ...

G Keep working with age. [The boys don't seem to have heard her. G starts assigning coloured blocks to each card depending on the number of fast food meals. She starts with 0, represented by black.]

B1 Here's another TV-goer. [Places TV card and takes another card from unallocated pile.] Boardgames. Who else likes boardgames? [Searches the laid out cards.] Here's boardgames. [Places his card, which is a 9 year old boardgamer, next to the 11 year old boardgamer.]

G Is that 9? [Takes the 9 year old boardgamer and places it with the other 9 year old card.]

B1 No ...

G And you put the blocks on for those ... [referring to blocks representing number of fast foods]
B1 I wasn't doing that ... because he likes boardgames, so ... [Moves card back with the 11 year old.]

G You put them with the age and then you put a different colour and then you put it with the boardgames ... because it's boardgames. [Tries desperately to cope with the two variables.] Get a colour ... not black.

B1 Blue. [B1 places a blue block on each of the two boardgamers’ cards which are now separated having been sorted according to age at G’s insistence. Later the Interviewer suggests that they concentrate on fast food consumption first, and so the blue blocks (representing boardgames) are replaced.]

In this situation G realised the necessity of keeping control of one thing at a time; however, B1 still clung tenaciously to his idea of dealing with the activities. G attempted to explain how to resolve the situation so that the situation concluded with compromise of sorts, and, in the end, it is the Interviewer who further resolved the conflict by suggesting that they deal with just the one variable. The course of the conflict is interesting: each student started with a disparate idea but when both needed a particular card they became aware that (a) they could no longer deal with the separate issues on their own and (b) they would not easily be able to deal with them together. There was a clash as B1 wanted to continue dealing with activities, which was only resolved when G proposed a solution. It is perhaps relevant to note that this particular exchange was somewhat more strained than those of any of the other groups or, indeed, the rest of this interview, and when B1 said "I wasn't doing that ..." there was a hint of genuine tension and a threat of social conflict, which G's suggestion fortunately diffused. The relationships in this group and other possible influences on the way the students collaborate are discussed further in the descriptions of the groups in the Procedure section and also in Extract 24.

**Extract 13 [Grade 6 - Girls - Suburban - Dice] [Conflict, Task factors, Egoencentrism].**

The students, working on interpreting the graphs in the dice protocol, were asked to review their interpretation of the dark green die’s graph after 60 rolls, because two of them had said it was fair and one had disagreed. The following exchange involved a couple of conflicting ideas as they tried to deal with the fact that the 4 column was markedly bigger than the others, although not by as much as occurred for the light green die, which they had all agreed was unfair.

This excerpt precedes the events quoted in Extracts 7 and 3.

I2 You agree on all of them except the [dark] green one [...] Two of you think it's fair, one unfair. What is there about that that makes it different?

S2 Me and {S3} agree because it's like ...

S3 Like number 4 compared to 1 is quite ...

S2 Number 1 is only about a quarter of number 4’s size so if you compare it to some like the other ones it's bigger.
S3 Like 6 and 1 are a bit like ... are one different, except 5 is more ... but if you compare them all ...

S1 Well I think this one ... we decided this one [the light green graph] is unfair and that's [indicates the dark green graph] sort of ... this biggest one [on the dark green graph] is quite lower than this one [the light green graph] and I reckon it's just fair but I'm just above the unsure and unfair. [She is indicating her assessment of her own confidence in her judgement about unfairness, but, as in Extract 7, the two “fuzzy” ideas - unfairness and certainty - may be a cause of considerable confusion.]

S3 I kind of think it's fair and not fair, because like there's two that are the same and these two are quite the same. [Comparison within the graph: there are a couple of pairs of columns which are the same height.]

S1 Like if this probably wasn't there [the 4 column] then I reckon that's nearly fair.

I What would you need to know to convince you? What more would you need to know?

S1 To see what the dice look like.

I Ok I can't show you the dice. If I can't show you the dice, is there any other information you could ask to help you decide?

S3 What the number ...

S2 You could do the test again yourself.

S1 Maybe a bit more than these were done.

S3 Are the these numbers the same as these ones?

I Yes they are still 1-6 and they've all been rolled 60 times. You said something about needing some more.

S1 Rolling the dice more so you could make a real different answer.

I Well, we just happened to have rolled it some more times […]

Although there are contradictory ideas there is little resolution of them among the girls. It is not clear that they are even aware that their opinions cannot all be true; in particular no comment is made following the highlighted comments of S3 and S1.
EXTRACT 14 [GRADE 3 - GIRLS - RURAL - DICE] [CONFLICT, EGOCENTRISM (NO TRANSFER OF IDEAS)].
For students as young as these Grade 3 girls it would be surprising to find clear and correctly formed ideas about fairness, and, as the following excerpt shows, the students had not yet developed correct ideas about the situation. Nevertheless, despite their youth and inexperience, they did seem to have some beliefs (they did not merely plead ignorance) and they attempted to present some reasons for those beliefs. Their powers of persuasion, however, were lacking or perhaps the other students were egocentrically holding on to their own ideas or were simply unconvinced by the arguments, because there does not appear to have been any transfer of ideas from one student to the others. This is despite the fact that conflict occurs with reasoned arguments being presented.

[The Interviewer shows them the chart with six graphs of 600 rolls.]

S3 Oh not this one!

S2 See four again. [Points to column on the light green graph.]

S3 Oh, that one is very high [pointing to same place on graph] ... fair, fair, fair, fair [points with pen to high areas]

I Why do you think they are fair?

S2 Because, by the looks of them ...

S3 I reckon, all the dices are fair, every single one of them. I reckon they're all fair.

S2 Yeah but {S3}'s just like looking at em and thinking, Oh, that's fair, that's fair, that's fair.' You don't have a reason for the fair!

S3 I do.

S2 What's the reason?

S3 Well, every single dice that I've thrown ... it's just ... it's the way that you throw the dice, that's the reason why I think they are all fair, it's just the way you roll the dice.

I Ok, so she has got a reason.

S2 Yeah, but ...

I That's alright, you don't have to agree with one another. What about, {S1} what do you think about these ones now?

S1 [ Shrugs]
S3 I reckon that one's the fairest one of all. [Points to light green.]

S2 Oh, that one - I don't ... I think that one's the worst.

S3 I think it's the best.

I Why do you think it's the best?

S3 Well that dice is obviously, probably the best dice, or I reckon it's the way the person threw that dice. Or I reckon they know how to throw the dice properly.

S1 [Confidently] But the computer threw the dice.

S3 Obviously, the computer knows how to throw the dice properly.

I Ok.

S2 I don't think the computer can do that!

S3 I don't think the computer has got a good brain!

I What do you think? Which ones do you think are fair?

S2 I think that one is fair [she indicates a particular graph] because most of the things are the same size and I'm not sure about those two there and that one's left out and for that one I think that's in between because there's no other one across there, there's no other one across there, no other one across there and there's no other one across there, or there [looks up for confirmation of these facts] so that's sort of like in between more unfair than fair. [She has been comparing columns to see if there are any equal columns. If there are, she says the die is fair; if not, it's adjudged unfair.]

S1 But {S2}, what if there weren't any here and this and this dice would be unfair cause it wouldn't roll over [inaudible].

S3 There is a little [inaudible] I think.

S2 And this one I think [referring to a different graph] ... that one's ... um ... that's fair ... yep ... that one's fair.

I Why's that?

S2 Because, um ... actually in-between ... right ... because there's none here, there's none here, there's none there, there's none there and none there. [She again looks at each column in turn to see if there is at least one other column in the graph which is level with it.]
S1  But what about any of these?
S2  I think this one is okay because there's two there; that one is [inaudible]; that one's in between, there's just one right there, I think was good. That one no, no.
S3  Why not?
S2  Because I think that ...
S1  [S2] this one is nearly level with this one. [Referring to the pink graph, which has two columns out in front.]
S2  Yeah but I don't think this one's fair [referring to green, which has no pairs of columns of even height] because there's none here, none here, none there, none there, none there and none there. But with some of them there's like um, which one, just say that one ... see these ... there's two together the same size, so that's how I think.

Misunderstanding [Extracts 15; 5, 6?, 23]
Collaborative exchanges are often sprinkled with misunderstandings, when ideas are misheard, or are not satisfactorily expressed, or are misinterpreted by the hearer. The term "misunderstanding" will be used when one member of the group does not comprehend what is said by another. Whether or not the group notices the misunderstanding and the way in which they deal with it is another factor which influences outcomes. If recognition of the misunderstanding forces the group to clarify the issue then "lifting" may result, while on the other hand if the misunderstanding is not noticed or not dealt with then the individual may retain a lower level of cognition than the others.

Extract 5 contains perhaps the best example of misunderstanding which, in that case, had negative consequences. At the end of that excerpt G1 and G3, who were on the threshold of doing something quite productive relating two variables from the data cards, failed to resolve G3's misunderstanding about using colour for fast food group and height for age. In fact G1 then lost track of that idea, which she initially appeared to understand, and the opportunity for higher level outcomes was missed.

Extract 15 [Grade 3 - girls - rural - dice] [Misunderstanding]
It is debatable whether or not misunderstanding has actually occurred in this case, but the potential for a teacher's positive comments to be misconstrued, when they are intended to praise effort but not necessarily indicate successful performance, is probably very real.

I  Right. ... So if I showed you those dice and said "All right, let's have a look at the blue one", can you tell me whether you think the blue dice is a fair dice, an unfair dice or you're not real sure. So the blue dice at the end there, is that fair?
S  Fair. [all]
I  What about the pale green one?
S2 Unfair.

S1 Unfair.

S3 I reckon they're all fair.

I You think they're all fair. OK. {S1}, what do you think?

S1 All fair. [Why has S1 changed her mind? Did S3 influence her directly or was it because of the Interviewer's response of "Okay" to S3's idea? The Interviewer probably intended the "Okay" to be a "Thank you for your idea" rather than "Yes, that's correct", but it could be misconstrued easily.]

The possible misconstruction of an authority figure's comments also occurs in Extract 6.

**Tenacity of Ideas [Extracts 16, 17; 3, 7, 12]**

This phenomenon does not arise exclusively in the domain of collaboration per se, but may influence it.

It is well documented that students can retain preconceived erroneous ideas even after corrective intervention (see, for example, Anderson, Sheldon & Dubay, 1990); this is known as "persistence." A related, but different, phenomenon is "tenacity of ideas" in which a student clings to a particular idea and, in the group environment, may only relinquish it after the group acknowledges its existence and the perpetrator's role in it. Such students may exhibit "obsession" for the idea, bringing it up repeatedly until it is recognised, even some considerable time after it is first mentioned. This can be advantageous, because if the idea is worthwhile such tenacity can "lift" the group. On the other hand, if the idea is unsuitable and the student imposes it on the group in his or her tenaciousness then it can be detrimental to the group's outcomes. There is a second, more neutral, possibility for less satisfactory ideas and that is that the group does not take up the idea but the proposing student maintains belief in it. There are some good examples of tenacity of ideas in excerpts from the Grade 6 - boys - suburban - data cards group illustrated in Chick and Watson (1997).

**Extract 16 [Grade 9 - boys - suburban - dice] [Conflict?, Tenacity of ideas, Task factors, Egocentrism].**

The boys in this group had been interpreting the Interviewer's dice frequency graphs with 60 rolls per dice. The extract shows what happened as the Interviewer had them consider the graphs arising from 360 rolls per dice.

I What would you say about them now? [Some discussion takes place, not all of it decipherable.]

S3 It's happened twice in a row. [Indicates the biased light green graph which, as in the 60 rolls case, is biased in favour of 4.] ... The {chances are?} it's not fair.

I Was that the same one as last time?
Ss    Yeah ... I think so...
S1    And this one has evened out.

[He indicates the dark green graph, which seems a bit biased, but less so than it appeared in the case of only 60 rolls. Some discussion then ensues between them about various graphs - each offers individual and correct comments, so there is no chance to tell to what extent they really agree with each other because they may just be concentrating on their own remarks or they may see no need to state their agreement because it seems so obvious to them. In observing that the previously apparent bias now seems reduced one of them adds ...

S    It's gradually changing to the same place. [...] [Interviewer then asks them specifically about one of the fairer looking graphs.]
I    Is that what you'd expect if you'd thrown the dice a lot of times?
S2    Umm ... I suppose ...Cos it could change ... a bit ... if you throw the dice another 100 times it could be different again.

[They look at and discuss some of the other graphs ...]
S3?   When you roll it enough times it probably all just averages out about the same ... it's different stages [unclear]
I    So what would you need to be absolutely certain about these dice?
S?    Probably {more?} graphs ... 
I    Rolled how many times?
Ss    [Uncertain laughter.]
S3    How many more graphs ... or ? [Seems to be trying to understand the Interviewer's question.]
I    Like that one there is rolled 360 times ... those dice ... each ...
S?    Maybe that again.
I    So that again?
Ss    Yeah. [They turn to the purple graph which is a bit higgledy-piggledy.]
I    What about that one?
S?    It was fairly even [in the 60 rolls case] but now the 1 has gone
out in front.

S1  It's probably going to gradually become unfair ... and that one
[points to the light green biased graph] might probably become
fair and this might become unfair.

[It seems that he may see fairness as a time-dependent property. His interpretation
may be that fairness is not a property that a dice has for the duration of the
experiment, but a property which is not only revealed by the experiment but which
changes during it, even because of it ... perhaps!] […]  [They then turn their attention
to the graphs showing 600 rolls per dice. They quickly get to discussing the light
green die's graph which still clearly shows bias in favour of 4.]

S1  This one's fair except for the middle one. [They then refer back to
the 360 roll graph.]

S3  It's the same number again ... number 4.

S?  Yeah, number 4.

S1  Everything's ... like ... almost fair ... but number 4's just making it
... unfair.

I  So can a dice be fair and unfair at the same time?

S?  Yeah.

S3  Depends ... It could be just be unfair on one side ... but ... nah ...
then it'd be unfair.

S?  If one bit's unfair then it's ... probably ... all unfair.

S?  Yeah.

I  I'm just exploring your understanding of what fair is.

S3?  I reckon it could be just how it lands.

S2  Yeah. [unclear] You might just pick it up [unclear] and then it
might flip and come back down ... if you keep doing that it'll ...
then the 4 might keep coming up.

S?  Yeah.

I  Right.

S3  If it doesn't go to either side it's got more chance ... it's only got a
chance of four if you roll it straight. [Again he indicates with his
hands rolling the die end over end so that four of the sides are the
only possibilities. Note the tenacity of the idea that rolling
technique is important. He had discussed this idea earlier in the
interview.]

I  Right.

S3  That one's still ahead by around the same margin ... oh, no ... a
bit more. [Refers to the light green graph] I still reckon [?] that's
unfair. [...]

S1?  It could have something to do with gravity ...

S2?  ... or the surface you roll the thing on. Like if there was a slope it
could roll back [unclear]

S3  The chances are ... if it happened three times in a row and it's
getting further and further ahead, then it's weighted or something.

[Note again the tenacity of ideas - he had talked about "twice in a row" at the
beginning of this extract. They then look at the pink die; this is biased to the numbers
1 and 2, with 3 also being ahead on the 360 roll graph.]

I  What do you think about the pink dice ... is it a fair dice?

S3  [Not confidently] Yeah ... I think so ... Cos the chances of those
two ... like it's not as if only one that's really shot up ... Cos you
can't weight it to two sides can you?

The examples of tenacity are highlighted in bold type. [HTML translator's note:
Unfortunately the bolding did not survive the transfer to me.] Note that, as expounded
in Extract 26, it may be that the students did not believe there was a right or wrong
answer about where the bias comes from, and hence they may not have been
perturbed by seemingly contradictory opinions. Alternatively, they may have been
treating the interview as a "brainstorming session" at this point and, as a result, were
more interested in expressing possible ideas than in considering ideas proposed by
others.

EXTRACT 17 [GRADE 9 - BOYS - SUBURBAN - DICE] [FALLING - PULLING DOWN?,
CONFLICT, TENACITY OF IDEAS, PREVIOUS EXPERIENCE, TASK FACTORS].
The interview with the Grade 9 boys concluded with the consideration of the graph of
the impossible red die whose outcomes all have exactly the same frequencies.

I  What would you think of that dice?

S2  It was ... fair.

S1  Umm ... it's ... just like ... the same ... it's been rolled like the
same type of thing ... not differently ... umm ... the same surface
and everything ... like all the sides must be the same. [unclear]

I  Now {S3}'s been shaking his head ever since I turned it over.
S3 No way if you rolled it that many times would it come out all exactly the same unless you rolled it exactly a special way so it would come out exactly the same every time.

I So you don't think that's possible?

S3 Oh, it's possible, but not very likely.

S1? Unless you've got magnets or something inside you can control.

S2? The dice would have to be perfect to get that if it was ... Cos normally you don't get exactly the same ...

S1 ... yeah, like you could have something inside it so you could control it.

I But you don't think that would happen?

S2 Yeah.

I I must admit that I did just draw that one. [...] There's one more question that I want to ask about graphs ... something that you said about "becoming fairer". Can you explain what you mean by "becoming fairer"?

Ss [all talk at once until one says "You go"]

S2? Like it might start out even and then as you roll it it could get ... gradually ... umm ...

S? ... it increases ...

S2? Yeah, if it started out unfairly your luck could change ... all the ones gradually the same on the end.

[In Extract 16 there seemed to be an idea of time-dependent fairness espoused by S1. Has this "pulled down" S2's understanding?]

I So if I took that pale green dice [the biased one] and started playing with it ... could it be called a fair dice ... if I played with it in a game like this.

S? Umm ...

S? Probably not with that [unclear ...] ahead.

S? Yeah. [...]  

I Can it be fair at the beginning and then unfair later on?
Ss  [gradual chorus, of three definite responses (but maybe only two voices), but then see below.] Yeah ... it could.

S2  It depends on the dice.

S3?  Not unless you change the dice. [...] 

I  If we went to the pink one which you decided at the end was unfair and yet here [with 60 rolls] you decided it was reasonably fair ... was the dice fair then or was it just that it looked fair?

S1?  It's probably because like ... you don't know what's going to come up, and when it comes up you go "Oh, alright, I'll take that" when it comes.

S?  Yeah.

S1?  You can't really know unless you've got an idea ... like you want to {spin?} this and you throw it this way. [Still strongly believes that throwing has a huge influence and that it is controllable - see Extract 16 where S3 expressed a similar idea.]

S2?  You mightn't really find out whether it's fair til after a couple of graphs to see what they're like ... so that you can tell whether it was fair or not.

S?  Sometimes you don't even know whether it's going to be fair or not ... like what is fair?!

Note that while there were contradictory opinions offered, virtually no conflict takes place. Although S3 had correct ideas about the likelihood of the outcome shown on the red graph and whether or not a die can change its fairness, there was little discussion among the boys to clarify whether or not he really was right and how the other alternative views fit in with his. As mentioned in earlier extracts this may be due, in part, to their beliefs about the dice protocol itself; that there is no right or wrong reason for the apparent bias.

**The Big Picture [Extracts 18, 19, 20; 5]**

The successful achievement of a particular task, whether tackled individually or in a group, is often predicated on the extent to which the student or students can keep in mind the overall goal - the so-called "big picture." For example, work may be commenced with an end point in mind, but, in needing to start some subsidiary tasks to achieve that goal, the focus shifts to the minutiae of the immediate subtask. The details of this may be sufficiently demanding to distract the group or individual from recalling and attaining the overall objective. This is exemplified in Extracts 19 and 20. In contrast, if the group can maintain perspective on the overall goal and situation then they are more likely to achieve higher level outcomes. This is illustrated in Extract 18.

**Extract 18 [Grade 6 - Boys - Suburban - Data Cards] [Charismatic Intellectual, Conflict, The big picture].**
The boys had added their own data to the data cards information, giving details about 19 people in total. In recording favourite activities, however, one of the boys had proposed two favourite activities or "hobbies," as the boys referred to them. This extract considers what happened as they started to collate and represent information about favourite activities.

S1 {S3}, you start sorting them into age groups and we'll write them down.

S2 Remember we should show how many people this is out of ... 19. That's out of 19 even though it equals up to 20 Cos I had 2 hobbies.

S1 I didn't put your 2 hobbies down. I only put your first hobby down cos otherwise it wouldn’t work.

S3 I put down his 2 hobbies ...

S1 Did you ... ? Oh ... [Slightly regretful, and uncertain how to deal with this.]

S2 Well, you can cross out computers there.

S1 Ok, there were no computers? [Reaches across and crosses it out].

Both S1 and S2 have sufficient perspective on the situation to deal with the doubling up: in fact, S1 thought that he had resolved it when the data were first collected earlier in the interview, while S2 remembers what he did and realises how to adjust his information to correct the problem. There is evidence here, too, of S1’s charismatic intellectual role, while the cognitive conflict is resolved without difficulty because of the boys’ ability to see the big picture.

**Extract 19 [Grade 6 - Girls - Semirural - Data Cards] [The Big Picture].**

As observed in Extract 5 these girls began with quite an enthusiastic examination of the data cards, hypothesising about relationships among various aspects of the data. After they had divided the cards up amongst themselves (according to fast food meals), however, the data remained in three groups (lots, some, none) for the remaining analysis even when the Interviewer suggested that they combine results to show what they have done; thus they were no longer looking at the complete set of data as a whole. This transcript follows on from Extract 5, and the girls’ situation is further clarified in Extract 20 which brings the interview to a close. Recall that for this group the identification of particular individuals for the transcripts is difficult.

I How are you going to put this all together to show somebody what it means?

G Umm ... didn't think of that.

I Does it have to be all on one graph?

G No.
EXTRACT 20 [GRADE 6 - GIRLS - SEMIRURAL - DATA CARDS] [THE BIG PICTURE, TASK FACTORS].

At the very end of the session, after each girl had put the block towers representing her age and fast food meals data onto a sheet and written each person's weight and favourite activity beside it, again without recombining the three groups, the Interviewer asked for an overall summary to give them the opportunity to synthesise their results.

I What do you think it shows?

G1 [Who has the "5 or more" fast foods group, who are also older and lazier (in general!)] Once you get older they all watch TV.

G3 [Who has the "zero" fast foods group] Umm, these people only weigh a little bit ... like this could be Ôcos they don't have any fast food meals and they do activities.

I And what else could you do with the information if we had some more time? Could you change it or rearrange it?

G You could make it look nice.

G You could look at eyes.

I Would you like to have a quick go at that?

[They look at eye colour, but again only within their separate groups of data. Moreover, G1 does not tally the information, but rather adds it to the list of detailsrecorded alongside each of the individual towers. Her actions seem to prompt G3 to do the same, although G3 did produce a bar chart at first (trouble is, she only had three people in her group!).]

Thus it is apparent that they had ideas about possible relationships, but just could not grasp how to show them in a coherent, complete, convincing and synthesised way. This may have been because of fatigue, since this came near the end of the 45 minute interview. The interview concludes as follows.

I Okay, just looking at all of the information that you have put down, what sort of things can you say about this whole group of people?

G Umm ... [pause, awkward silence] ... Err ... [further pause then some mumbled responses about the age distributions of their individual groups].

G2 [Who had the larger and more miscellaneous “1-4” group, and
who does not seem to have contributed much to proceedings perhaps because of this (e.g. because there were no obvious relationships in her data and she always took longer to represent them than the others because of their size - there were 11 individuals in her group, compared with 5 and 3 for G1 and G3 respectively).] They just step up a little bit in age.

The very fact that the cards were shared out among the girls may well have contributed to the less than satisfactory outcome, thus indicating that the task itself can interfere with performance.

**Previous Experience [Extracts 21; 3, 11, 17]**
The performance of a group is often enhanced by the previous experiences of its members. Suppose, for example, that the successful completion of a task requires certain abilities or knowledge. If a member of the group has already gained that understanding in another setting, and can recognise the need for and recall those skills for the current setting, then the group is much more likely to achieve the desired outcome. Moreover, when a student guides other members of the group to learn what he or she already knows, cognitive change - in particular, lifting - may take place. As indicated in Chick and Watson (1997) this previous experience may even be something as simple as the suggestion of terms that the proposer thinks are relevant to the task at hand. A student's lack of experience, on the other hand, can be a disadvantage, as alluded to in Extract 11.

**Extract 21 [Grade 6 - suburban - boys - data cards] [Previous experience].**
In the Grade 6 - boys - suburban - data cards group, S2 put forward a number of terms in the hope that they were appropriate. These examples are quoted in Chick and Watson (1997) and included the use of terms like "logic graph," "pie graph," "percentage" and "average." In contrast to these instances, in the example below it was S3 who dominated proceedings with a list of known graph types, although this list also included the proposal of a term which "sounds good" but was unhelpful.

I What sort of graphs are there?

S3 Pie graphs, column graph.

S1 Bar graph ... and what are those side graphs ... ?

S3 Dot graphs.

I A line graph...

S1 Umm

I Pictograph....

S3 A holograph.

I Not quite the same, what were you thinking of?

S3 I was thinking of joining a dot...... there would be a line and then a
dot of colour in the middle.

S1 I think I know what he means ... like a line and then a dot of colour and the colour corresponds to something down the bottom.

Social and Organisational Collaboration [Extracts 22, 23; 4, 8, 11]
This classification is used to refer to collaboration which is more general than that which directly produces cognitive change. In particular, it incorporates general social discourse, which may well be off-task (as an example, the girls in the Grade 3 - girls - rural - dice group had a lengthy discussion concerning the dentist in the middle of their interview not long after Extract 11), and organisational collaboration, which may be on-task but to do with the mechanics of task assignment rather than the cognition required to deal with the task. Of course, appropriate task delegation can lead to successful outcomes, although it must be emphasised that if tasks are allocated according to group members' strengths then individuals may miss opportunities for progress in other areas.

Extract 22 [Grade 9 - boys - suburban - data cards] [Charismatic intellectual, Social/organisational collaboration].
The group was working on producing a graph of age and weight, and both S1 and S2 had a turn at filling in some of the details. After a while S2 noticed S3's lack of involvement. Observe, however, that at this point there was not much left to do to finish the graph!

S2 [to S3] Do you want a go at this? I bet you do! [Finishes his part and passes it over; this is all handled in a very good-natured way.]

Extract 23 [Grade 6 - boys - semirural - dice] [Falling, Misunderstanding, Social collaboration, Egocentrism].
The boys in this group did not show extensive collaboration, perhaps because of the nature of the dice protocol, and this excerpt was probably the most collaborative of their whole interview. With regard to the issue of social collaboration, observe that their activities with the dice triggered an initially relevant discussion about another dice activity encountered elsewhere, but which drifted off-topic almost immediately. Note that there were only two boys involved at this stage (the ones who were throwing the two dice and producing a graph of the results); the third was producing a graph of his own by combining the blocks which recorded the results of the opening dice games. This was one of the groups for which it was difficult to identify individuals for the transcript.

I What would you expect it [the graph showing the performance of the two dice] to look like if both the dice were fair?

S Up and down.

I How far up and down?

S Well I think that really does look fair
[points all over the graph] ... except for 2.

S  Yeah, except for 2.

S  It's rigged.

S  Yeah, it looks pretty fair. [Did he listen to what the other student said?!

S  Umm ... whose go is it? [They sort this out and resume their recording.]

S  1 is probably the fairest number [looking at the record of the throws: both dice have the same number of 1’s recorded, and so it would seem that for this student "fairness" is something which is determined for particular outcomes - here the outcome "1" occurs with the same frequency for each of the dice, and so "1" is fair.]

S  [Forcefully] 2 isn't! [His perception of fairness seems to be the same as previous student; perhaps influenced by him. "Falling" may have occurred. This student did not disagree with the other, so either he had the same interpretation of fairness before the other spoke or else he has quickly interpreted what the other meant and used that interpretation himself. So, he either was down there already or he "fell".]

S  2 and 6 are not the fairest.

S  2's easily going to win this unless there's some [? inaudible].

S  I've hardly got any 2's. I've only got two 2's. He's only got three 6's though [referring to player 1's blue dice which, in addition to being biased towards 2, seems to be biased away from 6. Interestingly the red dice seems to be well ahead of the blue with number 5 as well (even more markedly than the 6
columns), but this does not seem to be noticed at this stage.]

S  There was one like this we did at the Mathematica exhibition [display at the museum] with two dice. [They discuss the various exhibits.]

I  Was that the horse race one?

S  No. [Further discussion of the exhibits.]

I  Which one came up most when you did this with two dice?

[No obvious response. Throws go on; some comments lost by loud music in background.]

S  I reckon it looks really fair, apart from the 5 on the blue dice and 2 on the red [so they did see the difference in the 5 column.]

Interestingly the unfairness is seen here in terms of underperforming when comparing the dice rather than overperforming.]

The red dice is fair. [Well, this is a bit out of left field! Where does this come from? What does he mean by it?]

I  Ok what do you think?

S  3!

S  Oh I don't know ...  

S  I don't reckon ...

S  Looks pretty fair.

S  Perhaps 2's just the lucky number.

S  Either that or there's a fault in the dice.

I  If there was, what would it be?

S  A weight on one side.

I  What numbers would you expect to come up the least if it was a weight on that side?
S The weight would be on this side so ... 5 ...

S No, 5 came up a lot. [Some confusion here: one student seems to be referring to the red dice (for which the result is true), and the other blue (where it definitely isn't).]

I So how does that fit with what you think?

S I think that 5 is good for the red dice and 2 is good for the blue dice.

S And 6 is good for the red dice ...

S And blue dice.

There were a number of other features present in this extract which were commented upon as they arose. They included ignoring/not listening (see the section on "Egocentrism"); a comment that possibly caused "falling"; an indication that the boys were quite capable of carrying on a discussion - it simply appears that the dice protocol itself did not seem to encourage it in this case; an out-of-the-blue comment; and possible misinterpretation when communicating.

Other Social Factors Influencing Collaboration [Extracts 24, 25; 1, 12]
The social forces which influence group dynamics can have a considerable effect on the success of collaboration. The following list is not intended to be exhaustive, but illustrative of some of the social factors which can impinge on the effectiveness or otherwise of the activity.

Strength of pre-existing friendships - For example, the Grade 6 - boys - suburban - data cards group (see also Chick & Watson, 1997) seemed to comprise three boys who were already close friends and who were used to working together as a team. In this case the existence of friendship links worked in the boys' favour. In other cases, where friendships do not appear to be as strong, the extent and success of collaboration may be diminished.

Gender balance - In some circumstances, boys and girls may not be comfortable working together, just because they are of different sex. This difficulty may be particularly obvious in a mixed-gender three-some: one of the group is very obviously the odd one out. See Extract 24 for a possible example of this.

Physical position in groupings - Physical constraints may force one or more of the group members to be in an awkward position for collaboration, and this may have a detrimental effect on the extent of that person's contribution. For example, in the Grade 9 - boys - suburban - data cards group (see, in particular, Extract 25 and, to a lesser extent, Extracts 1 and 8), student S3 is at the short end of the table while S1 and S2 sit next to each other on the long side. Location of interviewer/teacher - The proximity of someone outside the group may also influence the interactions and activity of the group. A particularly strong influence is the presence of the interviewer or the teacher. Interestingly, this can work two ways: the close monitoring role of a teacher/interviewer may have a positive effect on "time on task," but as the role is a naturally dominant/authoritarian one it may inhibit collaborative interactions. In observing the Grade 6 - boys - suburban - data cards group the transcriber
independently annotated the transcripts with a comment that it seemed that the
group worked together well because the Interviewer had moved to sit away from
them.

**Extract 24 [Grade 3 - Mixed - Rural - Data Cards] [Falling - Pulling Down,
Charismatic Intellectual, Other Social Factors].**

This was the only mixed-gender group in the study. At this young age they were
happy to be "tally-ers" of straightforward information (single variable analysis). For
example, they produced frequency histograms for each of how many people have
the different quantities of fast foods, eye colour, and favourite activities. This came
from grouping the information into categories and then producing a bar graph (in the
first two cases) or a table (in the last case). There was a hint on a couple of
occasions that they did appreciate the existence of relationships but it was not
pursued. The main interest in this example, however, is a consideration of the roles
played by the girls and the boys: she seemed isolated from them, even though one of
them was sitting next to her, and later in the activity the boys became distracted
building things from the Unifix cubes. As mentioned earlier in the description of the
groups, she was happy to work on her own and was able to focus for extended
lengths of time, but although she had some good ideas she was not able to involve
the boys with them, either because of a lack of leadership skills or because the boys
withdrew. Indeed, on occasions she tended to take over the activity, perhaps
because she felt she knew what needed to be done and did not trust the others to do
it. It might be postulated that perhaps she was a non-charismatic intellectual and
that, furthermore, this may have been influenced by gender issues.

Two extra points should be borne in mind which may mitigate the above
interpretation. Firstly, the group members did appear capable of working together
collaboratively and there did not appear to be any "bad vibes." Secondly, the
Interviewer also had an influence over the interactions, as she provided the students
with some guidance on task allocation. The students' initial ideas were nebulous and
undirected, and this direction from the Interviewer enabled them to focus on aspects
that would lead to identifiable outcomes in the time-frame of the interview.
Unfortunately it also split the group since the allocations assigned led to each
individual having a turn at graphing one of the categories of information. The extract
below follows their progress from an initial observation about the relationship
between weight and fast food consumption.

[They place the cards on the table in age order and then go through them reading the
weights aloud. There is a sense that they can see the weights increasing but no
comment is made. Shortly thereafter the following remark occurs.]

B2

[Who actually doesn't contribute greatly throughout the whole
proceedings] No wonder he weighs 74 because he eats 12 fast
food meals. [Laughs from others, although G is concentrating on
something else].

This understanding exhibited by B2 reappeared later in the interview, although
possibly prompted by the Interviewer:

I

You were looking at weight and you said that somebody who
ate a lot ...

B2

Weighed a lot.
Five minutes later B1 and B2 had diverted themselves by playing with blocks while G sorted the cards into order of weight. At one point B1 offered assistance to G, asking, "Do you want a hand there? We're just sitting around, watching her," but G either did not hear or decided not to respond. The following exchange involved the Interviewer and G, with B1 and B2 distracted.

I {B2} was interested in how many fast food meals they ate; whether or not lighter people ...

G It seems to go ... except there ... it's zero [looking at the cards in front of her, in particular the number of fast food meals] ... yeah, that's because they're probably older as well ... [continues checking] the weight of it's sort of like how much they eat and how old they are.

I Right ... that's got something to do with how much they weigh ...

G Yeah.

Shortly thereafter G clarified this, but the boys, who had lost interest earlier, were unable to appreciate what she was saying and responded at a very simplistic cognitive level by giving personal stories, a phenomenon referred to by cognitive psychologists (e.g., Biggs & Collis, 1991) as "ikonic support." B2 may have lifted briefly at the end (although this may be because of the Interviewer's "leading question"), but he quickly fell again, and perhaps pulled B1 down with him.

I [Trying to involve the distracted B1 and B2 in the proceedings again.] What have you noticed about how old people are when they have lots of fast food meals?

G Well ...

B1 Basically young and old. So ...

B2 The little kids are the ones who get spoilt and the big kids ... because they get more money.

G [Plaintive, but not assertive!] No ...

B1 Cos they get more money I suppose. [B1 echoes B2 and ignores G.]

I [To G] What do you think?

G It depends if they're older than the others and they eat more fast foods they'll be near the end.

I What do you boys think ... {G} said their weight has got something to do with their age and something to do with how much fast food they eat.
I agree pretty well but umm ...

B1 I have no idea.

B2 I eat fast foods and I'm not fat.

B1 I eat lots of food and I'm not fat. [B1 echoes B2 - is this "pulling down"? B1 and B2 then continue to discuss their eating habits.]

Here B1 repeated ideas expressed by B2, and neither boy gave much consideration to what G had said. B2's "fade-out" here is interesting in light of the fact that he was the first to propose a connection between food consumption and weight.

**Extract 25 [Grade 9 - Boys - Suburban - Data Cards] [Lifting, Charismatic Intellectual, Other Social Factors].**

The three boys in this group gave the impression of being friends, and they worked well together. They were seated with S1 and S2 along one side of a desk while S3 was on the end. S3 seemed to contribute less than the others; this may have been because of the seating arrangement. This, of course, invites the question: "Why did he end up in the odd seat?"

Was it just an accident, and then his position there affected the extent of his participation, or was he normally the quiet member of this group (if, indeed, they were a group of friends outside this context) and then subconscious social pressures caused him to end up in the odd seat, compounding the problem? Unfortunately there is not enough information available to answer these questions.

Part of the way through the interview the Interviewer introduced them to the idea of a scatter graph and a line of best fit. The boys were able to interpret the Interviewer's demonstration graphs readily enough and later, as the following excerpt shows, they produced their own from the information on the data cards, with, perhaps, the hint of a prod from the Interviewer. As it happens, it was not a standard scatter graph, because they still provided separate positions on the horizontal axis for all of the examples that have the same age value (age being represented on that axis). Note that S2 provided some leadership through this excerpt, again highlighting his possible dominant role (see also Extracts 8 and 22). S3's contributions are highlighted in bold type.

I Do you want to choose one more aspect of the information on there and perhaps do another representation of it?

Ss Yup.

S3 Still do the age probably ... and something else ... and compare it with ...

I Is there some sort of a theory ... perhaps you can have a hypothesis that something may be related to something else and you draw your graph and represent it and test it out and see what happens.

S1 Maybe the younger the less fast food you eat because your
parents make you stay at home and you eat what they make for you. The older you get the more you can do what you like.

S2 When you are younger you play more ball games.

S3 And read a lot more ...

I OK you've got three things there to look at.

S2 [to S3] You can draw this one.

S3 OK.

S1 So it's the age. Going to do what they do or what they eat?

S2 Fast food I think.

S1 OK ... Going to put how much they eat ... like 8 times? ... Or are we going to put them in age again?

S2 What?

S1 Going to put them in age? Like we've got two 18 year olds ... are we going to put two ate 8 fast foods a week or ... ? [They seem to be having difficulty deciding on an appropriate representation.]

I Is there another form of graph you could use?

S2 Could do one like that one [refers to the pocket money scatter graph] if we wanted to.

S1 OK, righty-o. Going to put them in age groups again ... or ... ?

S2 Up the side we can put fast foods eaten per week.

S1 OK there's one.

S2 ... then on the bottom their age.

S3 Not enough room on the page.

I You could turn the page around! Or you could start again. [S1 and S2 are sorting cards while S3 is drawing up a graph.]

S2 What are we doing age for? [having glanced at S3's work]

S3 We don't have to. [Going to throw aside his effort with which he is dissatisfied.] So how are we going to lay it out?
S1  With difficulty! Put age ...

S2  Just keep that one. [Asides inaudible. Eventually they keep the graph S3 began.]

S2  This bloke's 8. Like ... put spaces along there for ... going from 8 upwards. [Points along horizontal axis.]

S3  How many 8 year olds?

S1  You need two gaps for the 8 year olds ... Like two squares ... Two for the 9 year olds. [S3 puts age labels along the horizontal axis, but leaves gaps of 1, 2 or 3 depending on how many data cards there are with that age.] [...]

S2  How are you going to do this? Up 15 and across.

S1  OK, how much fast food?

S2  This 15 year old has none [no fast foods]. So just put a dot where the 15 is. [S2 directs S3 to show zero fast foods against the 15 year old.]

S1  Each square represents one fast food.

S1  [who has been looking at all the cards] 12!

S2  That's more than 1 a day! [Inaudible comments which seem to be related to what it would mean to consume 12 fast food meals in a week.]

S2  [to S3] ... so put a dot where 15 is. Put a dot where the 8 is. Put a dot where the 10 year old is. This is for zero. And a zero where the 12 is.

S1  Right ... now there is one for a nine year old; one for an 11 year old; one for a 17 year old. That's it.

S2  2 for a 15 year old and 2 for a 16 year old. [S1 and S2 continue to read out entries (fast food consumption and ages) to S3 who is producing the scatter graph.]

As can be seen from the transcript S3, although he completed the actual physical work, was very much in the background while S1 and S2 did most of the decision-making.

**Task Factors [Extracts -; 5, 16, 17, 20, 26]**

The nature of the task itself can influence the nature and success of collaborative behaviour. It is not possible to present a single extract to definitively illustrate this, but by taking the whole collection of extracts provided here and comparing the collaborative performance of those students engaged in the data cards task in
comparison to those engaged in the dice task, it would appear that in the latter case the students are much more likely to behave as a group of individuals, uninfluenced by the others in the group, than they are to behave as a team. It is hypothesised that this is because the dice task is less open-ended, and that students may believe that there are no right or wrong answers to the questions posed. This last point is exemplified by the fact that students offer individual - and often contradictory and undeathed - opinions, as seen in Extracts 16, 17 and 26.

Another illustration of the influence of the task itself occurs in the data cards activity because there is only one set of cards available for the whole group to study. This can force the group to work together, but on the other hand, as can be seen in Extracts 5 and 20, if the group divides the cards between the group members then they may lose sight of any overall trends, and so the level of performance of the group may drop.

**EGOCENTRISM [EXTRACTS 26, 27; 12, 14, 16, 23]**
This is a potentially negative behaviour which becomes obvious in a collaborative context. Individuals may become locked into their own world-view, to the extent that they do not listen to or refuse to be influenced by others' opinions because they are so fixated on their own. Although most readily apparent in group environments, it may also arise in an individual context.

In the group situation, there may be other related factors causing the phenomenon of egocentrism to arise. Firstly, it is often exemplified by tenacity of ideas: students exhibiting egocentric behaviour in the sense described cling to their own ideas so tenaciously that there is no hope of persuading them of any alternatives. Secondly, egocentrism may be particularly prevalent when there is no cognitive conflict in the group, because no one is forced to reevaluate his or her understanding, and hence ideas remain unchanged within individuals.

For example, individuals may put forward disparate and contradictory ideas, but if they remain unchallenged then each individual is unlikely to have his or her perceptions altered by the contributions of others and so there may be no transfer of ideas (see, for instance, Extract 14). Tenacity and egocentrism will be distinguished as follows: tenacity is a more active phenomena, exhibited by repeated expression of an idea until the group recognises it, while egocentrism may be more passive, perhaps exemplified merely by a student being "wrapped up in his or her own little world."

The extract below gives a minor example of egocentrism; see the other extracts noted in the heading for more substantial evidence of this phenomenon.

**EXTRACT 26 [GRADE 6 - BOYS - SEMIRURAL - DICE] [TASK FACTORS, EGOCENTRISM]**
In the next excerpt (which continues directly from Extract 23) there are some contradictory statements which do not elicit further comment or debate. It may be that the students are so wrapped up in their own opinions and see themselves as individuals offering those opinions in an interview scenario, rather than as a group trying to obtain an understanding about what is going on.

I If you were given those dice and you'd done all that and someone came along looking for fair dice so that they could open up a casino, which one would you give them?

S Red.
S Red.

S Umm ... I'd give them blue then I'd tell Mum to go along and use the [?] dice!

S I'd give them both.

S I'd give them the red.

S Red's just about even.

I So if you had some money and you were going to bet on it ...

S I'd still give them the red.

S Blue, but ...

S I'd give them both.

I So is it just chance or ...

S Yep.

S Yep.

S I think it's just luck.

S With red ... [then seems to indicate the blue, although this may be a different student entirely, because we can't tell who's talking] I reckon it's rigged still.

Towards the end of the extract the boys were, in essence, offering their own opinions about the fairness of the dice. Apparently there was no problem for them in the fact that their opinions were contradictory; so either they were not paying great attention to what was being said by each of the others (egocentrism) or else - and here the nature of the task may be influencing the group's behaviour - they believed that there was nothing to discuss. It is possible that they believed that there was no right or wrong answer about the fairness, and regarded it as perfectly acceptable for each to have his own opinion. It would be interesting to observe this group's interactions if, instead of giving differing choices for which die is adjudged fair, they had given different answers to a numerical problem. One would think that considerable discussion would have ensued in the latter case about the correctness or otherwise of each of the alternatives. Thus it could be the nature of the task which led to what looks like egocentrism: they saw "fairness" as a subjective thing rather than objective and so, as far as they were concerned, there was no need to debate the contradictory answers.

**Extract 27 [Grade 6 - boys - semirural - dice] [Conflict, Egocentrism].**

There were some contradictory opinions offered in the following exchange, and in this case they did not lead to further development or discussion of ideas.
I What do you think, is it a fair game?
S Yes, it's fair.
S Oh come on ... you need new dice!
S You've rigged the 2's.
S 2's won every one.
I Which dice is rigged, then?
S [Points to the blue] This one.
S The blue one.
I It's unfair?
S * Uh, no, I think it's fair ... it's just who rolls it.
S That's because you had that dice twice in a row. [S* had been the player with the blue die in the first two games.]
S * It's just a coincidence.
I You think it's just a coincidence.
S * Mmmm.
I But [one of the others] thought it was unfair?
S I think it's just a roll fluke.
S Probably the weight of the dice.

Shortly after this the following exchanges occurred, in which not only did no discussion ensue but also some comments were not even acknowledged: there was just a collection of different opinions.

I Do all numbers have the same chance or do some numbers come up more often than others ... in a FAIR dice?
S Well yeah.
S 4 or 5 come up often.
S Sixes come up more. [They also make some difficult to decipher comments about the colours/numbers that have prevailed on
Discussion, Implications and Conclusions

It is clear from the examples that both positive and negative cognitive changes are possible as a result of collaboration. As a learning technique collaboration can thus be beneficial, a conclusion which supports many of the earlier studies of cooperative learning. In contrast, collaboration can also have undesirable outcomes. This suggests that - like all learning techniques - it must be used with caution because it is not catch-all approach to education.

Nevertheless, some quite significant gains in cognitive functioning can be achieved through group work.

It is not irrefutably evident that the accomplishments of any of these groups demonstrated higher functioning than could have been achieved by picking a suitably qualified individual from within the group to work on the problem. This is not to say that in an ideal collaborative environment the group will not do better than any one of its individual members, rather that the group as a whole may generally accomplish only the best of what its best individuals can offer. Successful collaborative events are very dynamic: in tackling a task one group member may lift the others and then from that cognitive platform a different member of the group may lift them still higher, but it seems to be a very rare occurrence for a group to rise above the point that the most cognitively able of its members can take it. In all the examples of lifting illustrated here and in Chick and Watson (1997) the group lifted only when someone knew a skill or an approach that could be taught or introduced to the group. Nevertheless, the synergy arising from the contributions of the individuals does seem to be able to lift each of the participants beyond the levels they could have attained on their own.

A number of characteristics have been identified which contribute to the success - or otherwise - of collaboration, and if teachers are aware of these then they may be able to influence positive outcomes. For example, the presence or absence of a "charismatic intellectual" has considerable impact on a group's performance, as does the previous experience of the group's members. In the classroom environment the composition of a group is rarely ideal - determined as it is by decisions and constraints concerning gender balance, cognitive backgrounds, homogeneity, compatibility, group size and the actual task - and although it might be regarded as desirable for each group to have a capable leader or members with suitable background knowledge such may not always be possible. Consequently, it is clear that teachers may be justified in having different expectations for different groups, and that some groups will require more intervention and guidance than others.

In addition, teachers need to bear in mind that the extent to which different tasks lend themselves to collaborative work varies; Davidson (1990) provides some discussion of this issue. It appears from this study that a more open-ended task has greater potential to be conducive to productive collaborative exchanges than closed, narrow tasks. Educators also need to be aware that students may choose less satisfactory approaches because these are seen as easier or quicker, and thus it may be important to monitor a group's progress and provide redirection if required.

It may be possible to heighten students' awareness of what can take place during the collaborative process and to model strategies which are likely to lead to successful outcomes. In particular, the following could be drawn to the attention of groups of learners.
1. The role of conflict can be described so that students can see that, if used appropriately, it is an important aspect of the process of gaining understanding. In mathematics, perhaps more so than other disciplines, there is usually a need to resolve conflict and hence groups may need to be encouraged to determine why different individuals believe different results and to clarify and reconcile any contradictions arising from them in an attempt to gain a more correct understanding of the situation.

2. Students can be reminded to be more confident about expressing their doubts, again highlighting that the resulting discussion will lead, in general, to improved cognition. They can also be urged to be mindful of misunderstanding, so that they can stop and resolve it before continuing.

3. The extracts highlight the importance of tenacity and point out possible associated risks. As with the other phenomena, students can be encouraged to persist in bringing up an idea if they feel it is valuable, and the rest of the group needs to be aware of the importance of giving reasons for that idea's rejection if the idea turns out to be less helpful than the proposer thought.

4. Group members should listen to others' opinions, and consciously ascertain where those opinions fit in their own personal cognitive frameworks, in order to avoid the pitfalls of egocentrism.

5. A group can be reminded to review its progress regularly in light of the task that it is trying to accomplish. This can help the group avoid losing sight of the big picture.

In many cases students find group work more enjoyable than individual work, perhaps, in part, because it provides opportunity for social collaboration as well as cognitive collaboration. As the following examples show, students are also aware of the potential educational benefits. Only one of the groups in this study - namely the Grade 6 - boys - suburban - data cards group - was specifically asked for its opinion about group work, and two of the responses are particularly illuminating.

S3 Cos you get it done quicker ... like I'm doing the colouring in and he's doing the calculator and he's drawing up another graph [pointing to the others in turn] [...] 

S1 You get shared ideas and you get to listen to other people's ideas and you probably come up with a solution which would be better ... or better than what you were thinking ... not all the time, but some of the time you would ...

The following quote is the Interviewer's account of an unsolicited comment from one of the Grade 9 boys who worked on the dice activity in a group and, later, the data cards as an individual. He made the remarks at the end of his individual data cards interview.

I [At end of tape, after boy had left the room] He just commented that it would be far better to work in a group because some of the ideas of the other people would help. He found it a lot harder this week working on his own and said that it was better last week when he was working in a group and he could bounce ideas off other people.

Of interest is the fact that of all the groups observed in this study his dice group seemed to be among the least collaborative and interactive. It may be that students'
perceptions of the success, or otherwise, of working in a group differs from those of people observing the group. This is also asserted by Good et al. (1992, p.190).

Alternatively (or, perhaps, in addition) students feel less threatened in a group situation and are more comfortable about participating. A one-on-one exercise with an adult interviewer is probably fairly intimidating, and a student in such a situation may be more reticent about expressing ideas, especially those about which the student feels some doubt. This self-doubt may also be present even when the interviewer is absent and the student is simply working alone. On the other hand, in a group environment, even when there are still only three students with the teacher/interviewer, the student may be able to block out the perhaps threatening figure of the adult because of the proximity of supportive friends. It need not be the case, however, that any of the students in the group pay particular attention to the contributions of the others or even expect the others to pay attention to them, and this often seemed to be the case for the group from which this boy came.

To conclude, those educators who use collaborative experiences must be mindful of why they are doing it and what they expect from it. The purpose of collaboration can vary from learning a mathematical topic, exploring an open-ended problem, to simply having a collaborative experience per se. This raison d’être, together with a consideration of other influencing factors as discussed above, should govern the expectations for and assessment of the outcomes which eventuate as a result of that collaboration. What is more, those outcomes are not always guaranteed to be positive, implying that group work is not a panacea promising successful cognitive development every time it is used.
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