The impact of self-selection on patterns of gender difference in mathematics achievement1

Glenn Rowley, Christine Brew (Monash University)

and Gilah Leder (La Trobe University)

Abstract

Gender differences in school achievement have been a fertile field for research, and in mathematics and science, particularly, a matter of public concern. At senior levels of secondary school, where these subjects are no longer compulsory, differences in achievement are confounded with differences in the groups of males and females who choose to study the subjects. In Victoria, three mathematics subjects are offered at year 12, forming a hierarchy of perceived difficulty. This study uses data from the 1994, 1995 and 1996 Victorian Certificate of Education results to examine the extent to which group differences in mathematics achievement can be accounted for by differences in patterns of self-selection between males and females. By modelling different selection patterns, we show that gender differences are particularly sensitive to changes in the pattern of self-selection. In particular, we demonstrate that, as the patterns of self-selection become more similar between males and females, gender differences become smaller, in many cases approach zero, and in some cases reverse. In mathematics at least, it appears that the most significant differences between males and females are between their choices of subject, and not in the achievement of comparable groups of males and females once they make their choices.

Introduction

Gender differences in mathematics achievement have been well documented since the early 1970's, and were recently reviewed by Leder (1996). Comparisons across age groups, school types, cultural background and socioeconomic backgrounds have generally found considerable overlap in achievement levels between girls and boys. In the final analysis, though, boys generally appear to maintain a lead over girls in traditional examinations and girls over boys in more project, school based tasks.

The full implementation of the Victorian Certificate of Education (VCE) in 1992, with its commitment to encouraging a broader curriculum through assessment-driven change, provided an appropriate context in which to re-examine the subsequent assessment results of senior secondary mathematics by gender. Two main studies, (Cox, 1993, & Rowley et al., 1994) documented gender differences in the 1992 Common Assessment Tasks (CATs). The results were consistent with those of previous studies. The girls generally performed better on the new school-based project CATs, while in the traditional examination CATs, boys generally maintained a smaller, but significant lead over girls.

Yet gender comparisons at senior college level, where certain core secondary subjects, such as mathematics, are no longer compulsory, are possibly fraught by a self selection variable. Girls, for example, have traditionally opted out of higher level mathematics and this is a pattern that continues to persist (Teese at al, 1995). Teese et al also point to the apparent contribution of socioeconomic factors to patterns of subject choice for both boys and girls. Recent media coverage of gender differences at the VCE level in the least demanding mathematics, Further Mathematics, highlighted a 19% gap favouring girls over boys in the school-based project CAT (Messina, 1997). The actual gap for the traditional examination CATs were not reported and which were significantly lower, ranging from 4 to 5% in favour of girls. Still, girls in this mathematical study as in others, do outperform boys, and it is this outcome that we explore here statistically in some detail using comprehensive VCE data from 1994 to 1996. Our concern is that VCE results over the past few years may lead commentators to overestimate the success of girls in senior mathematics, and consequently to underestimate the need to continue to encourage higher participation rates.

A simple analogy provides the rationale upon which this research was founded. We start by imagining a society in which males and females are equally successful in mathematics, and equally likely to study it. Mathematics is offered at two levels, A (Advanced) and B (Basic). All students study Mathematics, and they are very wise in their choices; the most able 50 percent of males and females choose the A level mathematics, and the remaining 50 percent choose Level B. Males and females perform equally well, and their mean scores on the two levels are equal. The situation is represented in Figure 1.

Figure 1. Equal distribution of males and females across subject choices

Imagine now that the values of society change, so that females, while equally able at mathematics, become less confident in their ability. The result is that less females decide to study the higher Level A Mathematics, and more study Level B. Specifically, the 10 percent who would have been the least able females in Level A move to Level B, and become the most able 10 percent studying Level B Mathematics. The situation is now as shown in Figure 2.

Figure 2. Unequal distribution of males and females across subject choices

How will the mean scores of girls and boys be affected? We note first that in Level A Mathematics, the mean score of girls will be elevated, since the least able 10 percent are no longer studying the subject. But also, in Level B Mathematics, the mean score of girls will also increase, as a result of the addition of 10 percent more students at the upper ability levels in the group. Consequently, the movement of girls from Level A to Level B Mathematics will have elevated the mean scores of girls in both subject, and created a gender difference that was not there before. Ironically, the apparently superior performance of girls arises from decisions made by individuals in response to their perceptions of themselves as being less able at Mathematics.

We see, then, that even in a situation where ability and performance are equal in males and females, differences in take-up rates can lead to gender differences, as reflected in mean scores at the two levels. In this case, the effect of females making less ambitious choices than males is to elevate the mean scores of females at both levels, and, in relative terms, to depress the mean scores of males at both levels.

The example is not chosen light-heartedly. We know that the participation rates in senior mathematics are higher for boys than for girls, and that this is most acute for the highest level mathematics - in Victoria this is Specialist Mathematics. We also know that girls generally obtain higher mean scores than boys in most subjects, including mathematics. To what extent the higher mean scores can be explained by differences in participation levels is the focus of the investigations described in this paper.

In Specialist Mathematics in particular, girls are severely under-represented compared to boys. In the circumstances, it seems more than likely that the group of girls studying Specialist Mathematics are more able, on average, than the boys who study Specialist Mathematics, because they are more highly selected.

Participation rates

In Victoria, Mathematics at the highest level (usually taken at year 12) is offered at three levels. Specialist Mathematics, Mathematical Methods and Further Mathematics. Specialist Mathematics is perceived to be the most demanding, and is normally taken in conjunction with Mathematical Methods by students who wish to continue to tertiary studies in Mathematics or Engineering. Mathematical Methods alone serves the needs of student who need mathematics to prepare them for other tertiary studies, while Further Mathematics is a practically-oriented Mathematics subject which is not designed to prepare students for further studies in mathematics. Almost all Specialist Mathematics students study Mathematical Methods concurrently; the overlap between Mathematical Methods and Further Mathematics is small and probably atypical.

Each subject is examined by three Common Assessment Tasks (CATs); in each case, CAT 1 is a project or investigative task, centrally set, completed in the students' own time, and assessed by teachers with some central monitoring. In Specialist Mathematics a test component was introduced for CAT 1 to alleviate concerns about authenticity.

There are two ways in which participation levels can be presented within each mathematical study. In Table 1, the number of students enrolled in each study was taken as the population and here we can see that in 1994 and 1995, the proportion of girls and boys taking Further Mathematics was close to 50% for both with girls participation slightly increasing in 1996. For Mathematical Methods, there has been a consistent pattern of participation over the period 1994-1996, with boys participation remaining about 8% higher than girls. In Specialist Mathematics is the greatest gender difference in participation with a general 2:1 ratio of boys to girls. But we note that that girls' participation does appear to be increasing in this study, if only very slowly.

Table 1: Participation in each Mathematical Study by gender

CAT 3 figures have been used in each case as these better represent the number of students who completed the requirements of the study.

Apart from Specialist Mathematics, the difference in the participation rates for the two, more popular mathematical studies, are small. Yet there is another way to examine participation rates, and that is to consider participation in mathematics with respect to retention rates. As shown in Table 2, more girls have been consistently taking the VCE than boys for the period 1994-1996, despite an overall fall in their participation generally. Boys participation also declined over the period 1994-1995 but picked up in 1996.

Table 2: Participation in Mathematics with respect to participation in VCE generally.

CAT 3 figures for the mathematics studies have been used in each case as these better represent the number of students who completed the requirements of the study. For the English studies, in 1994 there were four CATs and CAT 4 figures have been used here, otherwise CAT 3 English figures are used. na means not applicable, as these studies were introduced in 1994 replacing the six mathematical studies introduced in 1992.

When girls' level of participation in the VCE is compared with their participation in mathematics, only six in every ten girls take at least one mathematics, while nearly eight out of every ten boys do so. This level of participation has not changed notably since 1988, when participation in at least one mathematics was 60 percent for girls and 75 percent for boys (Lamb, 1977). When the participation of girls in the individual mathematics studies is examined, their participation is lower than boys across all studies, with less difference in the least demanding mathematics, Further Mathematics. The different participation results by gender might well mean that those girls choosing to take at least one mathematics may, on average, have a higher mathematical ability than the boys who do and that differences in performance favouring girls may be explained by a self-selection variable. A graphical representation of the subject choices of boys and girls (Figures 3 to 5) illustrates that the pattern of choices is more complex, but not fundamentally different to the hypothetical case represented by Figures 1 and 2.

In Figures 3 to 5, the hierarchy is not one of subjects, but of subject-choice combinations:

SM+MM: Specialist Maths & Maths Methods

MM: Maths Methods only

MM+FM: Maths Methods and Further Maths

FM only: Further Maths only

NM: No mathematics

Figure 3. Participation rates by gender, 1994

Figure 4. Participation rates by gender, 1995

Figure 5. Participation rates by gender, 1996

Over the three-year period, boys clearly demonstrated a greater willingness to undertake the choices perceived to be more difficult: the Specialist Maths/Maths Method combination and Maths Methods only. Girls have consistently been more inclined to take the mathematically "easier" options - Further Mathematics and no mathematics. Although girls have slightly increased their participation in the higher-level offerings, so have the boys. These differences must be taken into account as we interpret mean differences in achievement between genders.

VCE mathematics results

Table 3 provides the mean raw scores for the individual CATs across all three mathematical studies, for the period 1994 to 1996 (as for other data presented, those awarded zero CAT scores have been excluded from the analyses). In the most demanding mathematics study, Specialist Mathematics, females did achieve higher scores than males for all three CATs in both 1994 and 1995, and in CATs 1 and 2 for 1996. Statistically, only in the 1994 CAT results were all gender differences significant; in 1995, in CAT 2 and CAT 3, and in 1996, only in CAT 1. While there is some variation present across the years, the general pattern is that females are doing better than males in Specialist Mathematics though this advantage has declined since the introduction of the study in 1994. This has been in association with an increasing participation rate by females.

Table 3. Summary performance of males and females in mathematics, 1994-1996 (raw scores)

All gender differences are statistically significant ($p < .001$) except where otherwise indicated.

In the most popular study, Mathematical Methods, females achieved significantly higher mean scores than males in CAT 1, and males did significantly better than females for both CATs 2 and 3. This pattern is consistent across all three years and is associated with a constant participation rate between 1994 and 1995, and a slight rise in 1996 for both genders.

For the second most popular, and least demanding study, Further Mathematics, females achieved higher mean scores than males for both CAT 1 and CAT 2 across all three years and all were significantly different except for CAT 2 in 1994. For CAT 3 there has not been a consistent gender pattern across the three years. In 1994, there was no difference; in 1995, males out performed females; and in 1996 females out performed males. In general, females over the past three years have been maintaining an advantage over males in Further Mathematics and this is in the context of declining enrolments. What is interesting is that males have recently been opting out of Further Mathematics while the proportion of females has remained steady (Table 2).

In the 27 CATs across three years, females achieved significantly higher scores than males in 15 of the CATs and the males achieved significantly higher scores in seven. Females outscored males in eight of nine CAT 1s, four of nine CAT 2s and in three of the nine CAT 3s. Males outscored females in three CAT 2s and four CAT 3s.. The pattern of results is consistent with that apparent in the 1992 VCE mathematics, in that the school-based project CAT continues to be completed more successfully by females and the examination CATs generally by males.

What if? Simulating the effect of different retention rates

Each year a large cohort of students takes the Specialist Mathematics/Mathematical Methods combination2 This provided us with the opportunity to ask the question: How well could the students in Mathematical Methods who did not take Specialist Mathematics have been expected to perform had they taken it? Regression analyses were run to assign predicted SM scores for students who had taken MM but not SM. For each year, the three MM CATs were used as independent variables and the dependent variables were SM1, SM2 and SM3 in succession. Thus predicted SM scores are based on the total MM information available; no assumptions were made about which of the MM CATs should contribute to the prediction. The results are shown in Table 4.

It is clear from Table 4 that Mathematical Methods provide a reasonable basis for prediction of performance in Specialist Mathematics. In general, prediction is best for the examination CATs, 2 and 3, and equally it is these two CATs that contribute most to the prediction. This is consistent with previous work by Rowley, Leder and Brew (1994), which showed that in its early years, the examination CATs and the school-assessed CATs assessed different and statistically distinguishable skills. The proportion of variance accounted for was around 0.6 for CAT 1 and generally above 0.7 for CATs 2 and 3. The best prediction in all three subjects came in 1996 (.605 for CAT 1, .773 for CATs 2 and 3). This is a sufficiently high multiple correlation to conclude that Mathematics Methods provides a good basis for prediction of performance in Specialist Mathematics, although when we deal with students lower in the ability range, our confidence in the predictions must drop.

Table 4. Prediction of Specialist Maths performance from Mathematical Methods: Regression summary

The procedure followed was as follows. Each student was assigned an estimated score for CATs 1,2 and 3 of Specialist Maths, which was either

the score they obtained if they actually did Specialist Maths, or

their predicted scores from Maths Methods, if they did not do Specialist Maths.

These "estimated scores" were then used to assess what might have happened had the take-up rates been different to what they were. In the process, it was observed that a small number of students at the bottom of the Maths Method score distribution had predicted scores in Specialist Mathematics less than zero. Any such scores were adjusted to zero.

Scenario 1: All Mathematical Methods students take Specialist Mathematics

Under this scenario, we ask "What would have been the pattern of gender differences if all students of Mathematical Methods had been persuaded (or compelled), however unwisely, to take Specialist Mathematics?" Means were computed for males and females separately for each Specialist Maths CAT in each of the three years 1994-1996. Table 5 shows the actual means (SM1, SM2 and SM3) for the students who did Specialist Maths, and the estimated means for the whole sample. The second set of means demonstrate the pattern of gender differences that may have been expected had all students taken both subjects, that is, completely eliminating the factor of self-selection from the Maths Methods cohort. Figures 4, 5 and 6 display the effects graphically. For ease of visual comparison, scores in the figures have been converted to percentages. (Scores in the tables are raw scores; different CATs had different maximum scores.)

Table 5. Pattern of estimated gender differences under the assumption of no self-selection.

We note from these results that females generally have higher actual mean scores in the range of CATs across all three years. Among estimated scores, the female advantage is either nullified, or reversed. The conclusion we draw from that is that the apparent superior performance of females across the tree-year period is explainable in terms of the self-selection factor. When the effects of self-selection from Maths Methods are removed, no female advantage remains. In some cases, particularly the examination CATs, the superior performance is by males.

Figure 4. Actual and estimated gender differences under the assumption of no self-selection, 1994

Figure 5. Actual and estimated gender differences under the assumption of no self-selection, 1995

Figure 6. Actual and estimated gender differences under the assumption of no self-selection, 1996

Scenario 2: Males and female elect to study Specialist Mathematics in equal proportions

The previous scenario looks at what might be expected to happen if all students of Mathematics Methods opted (or were required) to study the higher-level Specialist Mathematics. While this may be equitable, it is not realistic. Specialist Mathematics was not designed to be suitable for all students, and mathematics teachers would be aghast at the prospect of teaching it to all students. The analyses involve a significant amount of extrapolation, in which we have estimated scores on Specialist Mathematics CATs for students who would not and should not be studying Specialist Mathematics. The lower estimates are shaky, since they require us to assume that a linear relationship in the populated part of the score range will continue into parts of the score range that are presently unpopulated. The fact that regression yielded some negative score estimates (which were subsequently adjusted to zero) indicates that the extrapolation is not reliable.

Consequently, we look at a second scenario, and ask what would happen if males and females opted for Specialist Mathematics in equal proportions. To do this, we looked first at what proportions of each Mathematics Methods cohort also studied Specialist Mathematics. From Table 2, we find that female enrolments in Specialist mathematics, as a percentage of female enrolments in Mathematics Methods, have increased from 20.4% in 1994 to 23.7% in 1995 and 26.2% in 1996. On the other hand, male enrolments in Specialist Mathematics have increased from 20.4% to 23.7% to 26.2% of their Mathematics Methods enrolments in the same period. For the purposes of analysis, we shall take 25% as typical of female enrolments trends, and 40% as typical of male enrolment trends. We then ask how would males and females mean achievement scores have compared if

females had been persuaded to enrol for Specialist Mathematics at a rate equal to that of males, and typical of males today (i.e., 40 percent), or

males had been persuaded to enrol for Specialist Mathematics at a rate equal to that of females, and typical of females today (i.e., 25 percent)?

The pattern of estimated means is displayed in Table 5. In each case, the assumption is that the 40% or the 25% opting to study Specialist Mathematics are the most able 40% or 25% respectively.

Table 5. Estimated means on three CATs under the assumption that equal proportions of males and females select Specialist Maths

Under these conditions of selection, the data indicate that males would perform consistently better than females on all CATs, and that the margin is smallest on CAT 1 and greatest on CAT 3. These results are displayed graphically in Figures 7A and 7B. The pattern is consistent: if males and females opted to study Specialist Mathematics in equal proportions, the male "advantage" would reassert itself. Not surprisingly, the advantage would be greater in CATs 2 and 3. In relative terms, females still fare best in the school-administered and school-assessed CATs.

Scenario 3. Students opt to study Specialist Mathematics (or not to) purely on the basis of their performance in Mathematical Methods.

Under this scenario, which many would hope would be the case one day (it is not yet so!), students would opt to study higher-level mathematics purely on the basis of their performance on lower-level mathematics. The fact that the Specialist Mathematics cohort is a subset of the Mathematics Method cohort gives us a way of addressing the question. We have chosen to ask what gender differences there would have been had 50 percent, 60 percent, 70 percent or 80 percent of the cohort chosen to study Specialist Mathematics, assuming that the decision was made entirely on mathematical ability as demonstrated in Mathematics Methods and regardless of gender. The estimated mean scores under each of these selection scenarios are shown in Table 6, and in Figures 7A, 7B and 7C in Appendix A.

Table 6. Estimated mean scores on Specialist Maths if selection is based purely on Maths Method scores (independently of gender)

Under these circumstances, we find that the estimated gender differences were almost non-existent in 1994, but have increased slightly to 1996. Equally, we note that the gender differences are non-existent in CAT 1, but begin to appear in CAT 3. By 1996, male students would have had a mean score about2% greater than female students on CAT 2, and 3% greater on CAT 3. The pattern is quite consistent, and appears to be independent of the level of performance that triggers the decision.

Discussion

The study of gender difference will continue to fascinate all observers of human behaviour, and long may it be so. But those who focus their attention on gender differences in achievement need to be aware of the complexities of what they study. Changes in patterns of achievement can have many causes, and these causes may interact in complex and contradictory ways. In this study, we have shown that some, but not all, of the differences in mean achievement levels between males and females are explainable by differences in choice of subjects. In mathematics, females have been relatively reluctant to undertake the more demanding levels of study, and we have shown that this has contributed to an apparently higher level of mathematical achievement among females compared to males. But this is an illusion. Nobody gained a higher score in anything because of this pattern of choice. It looks as if females are doing better than males in mathematics, but what is happening is that those females who judged themselves to be likely to perform poorly are not there. They are either doing a lower level of mathematics, or choosing to study other subjects, in which they judge themselves to have a higher probability of success. Unless we judge (as some of us may be inclined to do) that the study of mathematics is inherently more valuable than the study of other subjects, we may view this with equanimity. What is happening is that girls are adopting a strategy of subject choice that is closer to maximising their aggregate scores than the strategies adopted by many boys. On the other hand, we may choose to believe that the restriction in subject choice inherent in such a strategy buys a short-term advantage at the cost of a long-term restriction in career choice.

Our study was based in mathematics because it is only in this subject area that there is a clearcut hierarchy of subjects, and it is only in Specialist Mathematics and Mathematics Methods that there is sufficient overlap of students for us to have conducted the analyses that we did. But the kinds of decisions that students make when they opt to study Specialist

Maths, Maths Methods, Further Maths or no maths at all, are not restricted to mathematics. Every time a student chooses to study one subject and not another, these kinds of considerations apply. Some students will bravely tackle subjects that they fear because they are advised that they should, while others will avoid subjects that they may need, out of fear of performing poorly. Gender differences in any subject can arise from these patterns of choice, and do not necessarily imply that one gender is more able, or more interested in the subject than the other. This study has shown how fear of performing poorly in mathematics is a possible cause of apparently superior performance by a group.

Practical implications of our results include the following:

On the surface, it appears that girls are very successful in the study of mathematics at year 12. This should not lead teachers and researchers to conclude that the achievement of girls in mathematics is no longer a matter of concern.

While it is legitimate to be concerned at the low level of participation of girls in mathematics once the subject ceases to be compulsory, we need to be aware that successful attempts to increase girls' participation are likely to be accompanied by decreases in the average performance of girls in mathematics.

Equally, we may be concerned that boys, persuaded that mathematics is a "manly" thing to do, may have made subject choices that are not in their interests. A lesser degree of success in persuading boys of the value of mathematics could even lead to an apparent increase in boys;' performance (as indicated by mean achievement levels).

If we were to aim to increase girls' participation in mathematics to the same level as boys', it would probably lead to a situation in which girls are seen to achieve substantially less well in mathematics than boys (as Scenario 2 demonstrates).

A wiser strategy may be to aim at a situation in which students make subject choices on the basis of their perceived abilities, remaining as far as possible removed from societal expectations about what is appropriate for a girl or a boy to do (as Scenario 3 demonstrates).

References

Cox, P. (1993). Sex and CATs: gender related CAT bias in VCE mathematics. In J. Mousley, M. Rice (eds). Mathematics: Of Primary Importance. Brunswick: Mathematical Association of Victoria.

Messina, A. The Age Newspaper, 18 August, Victoria, Australia.

Rowley, G., Leder, G. & Brew, C. (1994). Learning from Assessment: Mathematics in the VCE. Paper presented at the Australian Association for Research in Education. Nov. 1994, New Castle.

Teese, R., Davies, M., Charlton, M., & Polesel, J. (1995). Who wins at school? Melbourne: Department of Education Policy and Management, The University of Melbourne.

APPENDIX A.

Additional Figures

Figure 7A. Estimated gender differences under the assumption that equal proportions of males and females elect to study Specialist Mathematics (CAT 1)

Figure 7A. Estimated gender differences under the assumption that equal proportions of males and females elect to study Specialist Mathematics (CAT 3)

Figure 7C. Estimated gender differences under the assumption that equal proportions of males and females elect to study Specialist Mathematics (CAT 3)

Figure 8A. Estimated mean scores on Specialist Maths CAT 1, 1994, if selection is based purely on Maths Method scores (independently of gender)

Figure 8B. Estimated mean scores on Specialist Maths CAT 2, 1994, if selection is based purely on Maths Method scores (independently of gender)

Figure 8C. Estimated mean scores on Specialist Maths CAT 3, 1994, if selection is based purely on Maths Method scores (independently of gender)

Figure 8D. Estimated mean scores on Specialist Maths CAT 1, 1995, if selection is based purely on Maths Method scores (independently of gender)

Figure 8E. Estimated mean scores on Specialist Maths CAT 2, 1995, if selection is based purely on Maths Method scores (independently of gender)

Figure 8F. Estimated mean scores on Specialist Maths CAT 3, 1995, if selection is based purely on Maths Method scores (independently of gender)

Figure 8G. Estimated mean scores on Specialist Maths CAT 1, 1996, if selection is based purely on Maths Method scores (independently of gender)

Figure 8H. Estimated mean scores on Specialist Maths CAT 2, 1996, if selection is based purely on Maths Method scores (independently of gender)

Figure 8I. Estimated mean scores on Specialist Maths CAT 3, 1996, if selection is based purely on Maths Method scores (independently of gender)

1 The authors express their gratitude to the Victorian Board of Studies for its generosity in providing access to the data that were used in this research.

2All Specialist Mathematics students take Mathematical Methods concurrently, except for students who have taken it in a previous year. The numbers involved were 4752 in 1994, 4791 in 1995 and 5342 in 1996.