Benchmarking Student Performance: exploring the potential with an example based on a Year 12 cohort

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Abstract

Industry uses ‘benchmarking’ as a method of organisational comparison. Performance levels, benchmarks, for an industry group or successful firm are identified and these are used to identify best practice. At a simplistic level the comparison can be in terms of outputs. Clearly a more comprehensive approach requires significant evaluation of the production procedures which means that the focus is on the process by which inputs are converted to outputs.

Despite the many problems associated with mapping industry-based jargon and procedures on to education, there is a trend, promoted by the federal government, to consider benchmarking in schools in Australia. Despite the potential for abuse of such there does seem some value in a considered benchmarking exercise, i.e., having general data on how large numbers of students perform in certain tasks, the types of strategies they employ, the types of errors they make and, where possible, linking such findings to a theoretical position concerning cognitive development.

This paper reflects on the issue of benchmarking, and in addition, provides information on a large benchmarking exercise, undertaken with support from the NSW Board of Studies, on Year 12 student performance in mathematics as indicated by an analysis of students' examination scripts.

Introduction

In July 1996 a meeting of the Ministerial Council on Employment, Education, Training and Youth Affairs (MCEETYA) the members resolved, in part, on national reporting of achievement in literacy and numeracy. This reporting would be benchmarked against stages in a child’s schooling. The project was overseen by a Benchmarking Taskforce comprising nominees from all States and Territories. Benchmarks were defined to encompass: standards describing student achievement at a year level, and targets setting the percentage of the cohort which it is intended will achieve each standard. It was anticipated that standards would consist of a set of indicators or descriptors. Each standard was expected to describe a cut-off between ranges of achievement, such as a high-achieving cut-off point, a proficiency cut-off point, and a minimum cut-off point. [The above information was taken from a memorandum of unnamed authorship titled: Literacy and
Background

Benchmarking is a term drawn from the business world. It is a technique that can be used to provide information on how a particular company’s or organisation’s products compare in, say, price, quality, reliability, to their competitors. The purpose behind the technique is to be a catalyst for organisational improvement. At its best this improvement should imply ongoing and systematic evaluation of all aspects relevant to production. Two basic types of benchmarking have been isolated [see e.g., Zairi (1992) and Bartos (1994)]. The first is based upon developing “a score card” of comparison. This can have the advantage of leading to cost reductions but usually implies only short-term improvements, often of a superficial nature. The second approach takes a more considered look at inputs as well as outputs. It involves a deeper analysis of the underlying processes as well as a more considered exploration of outputs.

The underlying issue addressed in this paper is whether benchmarking in education should be of the first or second type, i.e., should the focus be on outputs or is it more valuable in terms of long-term advantages to look at the processes leading to outputs? This paper looks at one ongoing attempt to consider some issues leading to a deeper analysis of outputs in education as a contrast to a limited simplistic output-driven agenda.

The Study

In order to explore aspects of the benchmarking process, and ways that such information might be most appropriately used to enhance teaching, permission was granted from the NSW Board of Studies to retain students’ HSC scripts for further analysis. With the support of the Board and its officers a sample of scripts (approximately 600) were selected for each of the ten questions in the 1995 NSW Higher School Certificate (HSC) 2 unit Mathematics paper. The selection process was random and all student and examination centre identification was removed from each script. The scripts were then sent to the Centre for Cognition Research in Learning and Teaching at the University of New England for analysis.

The 2 Unit Mathematics paper is undertaken by two distinct candidature’s in the HSC, namely, those undertaking the 2 unit mathematics course and those undertaking the 3 unit course. The 3 unit and 2 unit groups consist of 15.95% (8,774) and 32.45% (17,845) students, respectively, of the students doing mathematics. Together the two groups represent approximately the top 50-95% of academic achievers in mathematics. These two courses are distinguished by a strong focus on algebra and calculus topics. These courses are designed, in the case of 2 unit, for students wishing to undertake mathematics related subjects at University such as economics or biological sciences, and, in the case of 3 unit, the course is designed for students who wish to study mathematics in its own right at University.

While some statistical evidence is available to the education community on students’ performances, such as question means, only impressions about strategies and common errors are obtained from the markers. While such evidence is often insightful, it does not allow for a precise feeling for how students are actually performing on different topics or aspects of topics.

Design

A sample of 75 scripts was analysed for each question to identify correct and incorrect methods of approach. This was facilitated by the marks allocated for questions by the examiners. This initial coding set up a series of categories (although these were not
necessarily exhaustive). The 600 remaining scripts were then allocated to approaches and new categories were developed as required. As a result of this procedure (which is very time consuming) a series of benchmarks on performance was established. This paper reports on aspects of one of the questions analysed, namely, Question 2 in the 2 unit examination paper from the 1995 HSC. The question, testing aspects of the topic in the syllabus referred to as Linear Functions and Lines, is provided below in Figure 1.

Question 2

The line l cuts the x axis at L(-4,0) and the y axis at M (0,3) as shown. N is a point on the line l, and P is the point (0,8).

Copy the diagram into your writing booklet. Marks

(a) Find the equation of the line l 2

(b) Show that the point (16,15) lies on the line l 1

© By considering the lengths of ML and MP, show that _LMP is isosceles 3

(d) Calculate the gradient of the line PL. 1

(e) M is the midpoint of the interval LN. Find the coordinates of the point N. 2

(f) Show that _NPL is a right angle. 2

(g) Find the equation of the circle that passes through the points N, P, and L. 2

Figure 1: Question 2 in the 1995 2 Unit Mathematics HSC examination.

The overall performance on this question was good. For the 3 unit students the mean was 11.08 (out of a maximum of 12) with a standard deviation of 1.61. The mode and median score were both 12 with 5257 students scoring full marks while 18 scored zero. For the 2 unit students, the mean was 8.28 with a standard deviation of 3.14. The mode and median were 10 and 9, respectively, with 2792 students scoring full marks while 526 scored zero.

The following report of the analysis considers two parts, namely, 2a and 2f of Question 2, separately.

Part 2a

This question was worth 2 marks. It required the student to find the equation of a line given the intercepts on the x and y axes.

Correct Strategies

Four successful strategies were identified and these are summarised in Table 1 together with the percentage of the responses. In overview the strategies are:

1. Correctly using either (a) the intercept form of a straight line, where a and b are the x and y intercepts, respectively, or (b) the two point form of a straight line, .

2. Determining the gradient first by applying the formula and then using either (a) the point-slope form of a straight line, where m is the slope and is one point, (b) the slope y-intercept form of a straight line where m is the slope and b is the y-intercept, or © substitution of a point on the line as well as the gradient into to find m and b.

3. Determining the gradient first by applying the rule f(rise, run), and then using either (a) the point-slope form, or (b) the slope y-intercept form.
4. Using a general equation for a straight line, such as \( y = mx + b \), and then using substitution to determine the constants \( a \) and \( b \).

Table 1: Student breakdown of correct strategy for Q2a

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student (%) usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Formula of straight line</td>
<td></td>
</tr>
<tr>
<td>(a) intercept form</td>
<td>0</td>
</tr>
<tr>
<td>(b) two point form</td>
<td>23</td>
</tr>
<tr>
<td>2. Gradient by formula</td>
<td></td>
</tr>
<tr>
<td>(a) point-slope</td>
<td>33</td>
</tr>
<tr>
<td>(b) slope, y-intercept</td>
<td>3</td>
</tr>
<tr>
<td>© - substitute</td>
<td>2</td>
</tr>
<tr>
<td>3. Gradient by rule</td>
<td></td>
</tr>
<tr>
<td>(a) point-slope</td>
<td>8</td>
</tr>
<tr>
<td>(b) slope y-intercept</td>
<td>1</td>
</tr>
<tr>
<td>4. A general equation and substitute</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
</tr>
<tr>
<td>Incorrect Strategies</td>
<td>71</td>
</tr>
</tbody>
</table>
Five unsuccessful approaches were identified and these are summarised in Table 2, together with the percentage of responses. In overview the strategies are:

1. Determining the gradient to have the wrong sign but otherwise proceeding correctly.

2. Confusion associated with coordinates. This involves using either (a) as opposed to being substituted into \( y = mx + c \), or (b) the and values being swapped, i.e., \((0,-4)\) being written as \((-4, 0)\).  

3. Correct formula produced but the application used incorrectly or incompletely. Examples include (a) \((-4,0)\) and \((0,3)\) substituted into to find \(m\), or (b) and \((0,3)\) substituted into without further working. 

4. Formula written incorrectly and either (a) the working is correct, or (b) some reasonable attempt.

5. No correct or appropriate formula used.

Table 2: Student breakdown of incorrect strategy for Q2a

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student % usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Gradient given, sign incorrect</td>
<td>5</td>
</tr>
<tr>
<td>2. Confusion with coordinates</td>
<td>3</td>
</tr>
<tr>
<td>3. Correct formula but incorrect or incomplete approach</td>
<td>8</td>
</tr>
<tr>
<td>4. Formula incorrect</td>
<td></td>
</tr>
<tr>
<td>(a) Working is correct</td>
<td>1</td>
</tr>
<tr>
<td>(b) Working is incorrect</td>
<td>1</td>
</tr>
<tr>
<td>5. No correct formula</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3</td>
</tr>
</tbody>
</table>

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Discussion

This part of the question was relatively straightforward for this cohort of students. Overall 71% of students sampled were correct, 21% were incorrect and 8% failed to provide an attempt. Of surprise was the small number who applied the intercept form of a straight line (none detected in this sample) or even the two-point form. The students appeared to prefer to use the two-step process of finding the gradient first and then using a less ‘complex’ form of a straight line.

An important source of error was the use of -4, the value of the x-intercept, to determine the slope. This usually resulted in an incorrect sign for the gradient.

The students who were incorrect had not sought to compare their answer to the diagram. While teachers would probably expect this error, the confusion concerning coordinates and the inability to complete correctly the substitution would probably come as a surprise for students of this age.

Part 2(f)

This question involved showing that a certain identified angle was 90°. There were several different approaches identified but most resulted in error.

Correct Strategies

Three successful strategies were identified and these are summarised in Table 5 together with the percentage of responses. In overview the strategies are:

1. Applying the rule, that two lines are perpendicular if the product of the two gradients is -1.
2. Applying the distance formula to all sides and confirming Pythagoras’ Theorem.
3. Applying the cosine rule.

Table 3: Student breakdown of correct strategy for Q2f

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student % usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Product of gradients equals -1</td>
<td>28.7</td>
</tr>
<tr>
<td>2. Distance plus Pythagoras’ Theorem</td>
<td>8.2</td>
</tr>
<tr>
<td>3. Cosine rule</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.48</td>
</tr>
</tbody>
</table>
Incorrect strategies

Nine unsuccessful approaches were identified and these are summarised in Table 6 together with the percentage of responses. In overview the strategies are:

1. Finding correct gradients but unable to proceed further. 2. Applying incorrect gradients to the rule that the product of gradients is -1.

3. Recording the correct rule but gradients not found. 4. Stating the answer by using the result that an angle in a semi-circle is a right angle but failing to prove that M is the centre of a circle, radius 5.

5. Finding the distance of several intervals but failing to link it with Pythagoras' Theorem.

6. Assuming that and or that are isosceles.

7. Attempting to use either incorrectly or inappropriately one of a number of methods such as Pythagoras' Theorem, Euclidean geometry facts, perpendicular distance formula, or gradients. 8. Relying on the look of the diagram.

9. Stating as fact what is to be proved.

Table 4: Student breakdown of incorrect responses for Q2f

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Student % usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correct gradients</td>
<td></td>
</tr>
<tr>
<td>(a) Incorrect rule</td>
<td></td>
</tr>
<tr>
<td>(b) No evidence of rule applied</td>
<td></td>
</tr>
<tr>
<td>© Language difficulty</td>
<td></td>
</tr>
</tbody>
</table>

(language excludes minus sign)

0.16
1.3
0.48

2. Incorrect gradients

(a)Correct rule
(b)Incorrect rule
(c)No rule

2.2
0.95
3. Correct rule, no gradient

4. Incomplete application of angle in a semi-circle

5. Distances, no Pythagoras

6. Considers triangles

7. Incorrect or inappropriate method/formula
   
   (a) Pythagoras  
   (b) Euclidian  
   (c) Perpendicular distance formula  
   (d) Gradients  
   (e) Trigonometry  
   (f) Distance formula  
   (g) Attempt unclear

8. Visual verification

9. Lines are perpendicular, therefore angle is 90°
Total
0.48
28.69
Discussion
Approximately 37% of the candidates sampled obtained the correct answer, 29% were incorrect and 34% failed to provide a response. While the number of successful strategies identified was limited, there were numerous ways in which candidates who were incorrect chose to attempt the question.

Implications and Considerations
Question 2 was not seen, to be a difficult question for both groups of students. This is consistent with the brief of the examination committee that the ten questions should be ordered in terms of difficulty with the initial questions being the easiest. In terms of the mean mark for a question it was ranked the second easiest out of the ten questions posed. Nevertheless, the analysis undertaken highlights a number of important features.

Most significantly, the analysis of student responses is an important way of highlighting the inadequacies in students’ understanding, even in a question that is considered easy for the candidature in general. The analysis showed that many incorrect strategies are employed, resulting from misconceptions about areas of mathematics that students have been studying for 3-4 years.

When considered from a cognitive development framework such as the SOLO model, one can interpret the questions in terms of cognitive load. For example, Question 2a required a sequential linking of a number of ideas or steps. Hence the level of response required was a high multistructural response. Those who were unsuccessful failed to include all the necessary elements. Question 2f required higher cognitive demands as the student had to collect all the elements together before commencing the question. This could be coded as a relational question. The added difficulty appears to be in the ability to recall and use the formula m1. m2 = -1.

Conclusion
An in-depth analysis of student responses can give significant insight into the methods they are using and their misconceptions. It can form a basis for benchmarking to give indications of the proportion of students who are successful in answering questions and solving problems for each small section of the mathematics courses that are tested in the HSC and thus can be invaluable for curriculum developers for several reasons: ú such information provides a picture of what the candidature can and cannot do as a result of the teaching of the current syllabus. If student understanding is to be improved, then the curriculum needs to facilitate improved methods of teaching. ú in the situation that large proportions of students have significant misconceptions in an area, curriculum developers need to take this on board. It may be the case that certain concepts are too difficult for the candidature, or the foundation of understanding has not been sufficient to support such concepts. ú analysis of student errors gives valuable insight into how students are thinking in mathematics. It would be beneficial for curriculum developers and teachers to take note of such errors so that efforts can be made to reduce them through the facilitation of enhanced student understanding.

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References
