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# Self-Directed and Peer-Assisted Thinking in a Secondary Mathematics Classroom

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Abstract. This paper examines the metacognitive monitoring and regulatory strategies used both individually and collaboratively by senior secondary school mathematics students. Analysis of interviews and videotaped lesson transcripts revealed that metacognitive activity was organised around routine monitoring of progress and recognition of, and response to, various types of impasse presented by challenging problems. The findings help to extend existing episode-based models of mathematical thinking, and shed light on the social processes of peer collaboration through which students monitor and extend each other's thinking.

The last decade has seen the emergence of an international movement calling for reform in the teaching and learning of mathematics. In both the United States and Australia, for example, new curriculum and policy documents place increased emphasis on problem solving, communication and mathematical reasoning, and endorse greater use of small group work and peer interaction as a means of encouraging students to become self-directed learners (Australian Education Council, 1991; National Council for Teachers of Mathematics, 1989). Such significant curriculum reforms require a sound research base if they are to be effectively implemented. However, our theoretical understanding of problem solving processes, and how students' problem solving abilities are cultivated by these new forms of classroom interaction, is far from complete.

In a recent review of progress in mathematical problem solving research over the past 25 years, Lester (1994) lamented that research interest in this area appears to be on the decline, even though there remain many unresolved issues that deserve continued attention. One such issue highlighted by Lester was the role of metacognition in mathematical problem solving where metacognition refers to what students know about their own thought processes, and how they monitor and regulate their thinking while working on mathematical tasks. Although the importance of metacognition is now widely acknowledged, we still lack an adequate theoretical model for explaining the mechanisms of individual self-monitoring and self-regulation (Schoenfeld, 1992), and, despite increasing research interest in the social and cultural aspects of mathematics learning (e.g. Brown et al., 1993), we have barely begun to examine the possibilities for collaborative metacognitive activity that may occur when students work in small groups.

This paper reports on a study which investigated metacognitive activity in secondary school mathematics classrooms, and connections between self-directed (individual) and peer-assisted (collaborative) monitoring and regulation. Two research questions are addressed:

1. What monitoring and regulatory processes do individual students use while working on classroom mathematics tasks?
2. How are these metacognitive processes elicited and supported during collaborative student-student interaction?

## Mathematical Problem Solving and Metacognition

Frameworks for analysing task-oriented mathematical thinking typically identify phases or episodes representing distinctive kinds of problem solving behaviour. For example, Schoenfeld (1985) developed a procedure for parsing verbal protocols into five types of episodes: Reading, Analysis, Exploration, Planning/Implementation, and Verification. A recent study by Artzt and Armour-Thomas (1992) has modified and expanded Schoenfeld's model in order to delineate the roles of cognitive and metacognitive processes in small group problem solving. The modifications adopted in the present study separate Schoenfeld's Planning/Implementation episode into two distinct categories, and include an additional episode type, Understanding the problem, which overlaps somewhat with Schoenfeld's Reading and Analysis episodes. These modifications may be applied equally well to individual and collaborative problem solving. The ideal characteristics of each episode are described in the far left column of Figure 1.

In addition to identifying and categorising episodes of problem solving behaviour, the analysis frameworks of both Schoenfeld (1985) and Artzt and Armour-Thomas (1992) acknowledge the central role of metacognitive monitoring and regulation in keeping the solution process on track, for example, by noting that the solution status or one's general progress should be monitored and plans modified if necessary. However, a deficiency in both frameworks is the lack of detail in describing the types of monitoring and regulatory activities that would be appropriate and expected in each episode. The suggested scope of these metacognitive activities is detailed in Figure 1, in the columns headed Monitoring and Regulation.

The model offered in Figure 1 also makes a distinction between two kinds of monitoring. First, the routine assessment of activity during each episode (for example, assessing one's understanding of the problem, assessing execution of the strategy) confirms that problem solving is on track. On the other hand, this routine monitoring may alert the student to specific difficulties and signal the need for a pause or some backtracking while remedial action is taken. Possible warning signals, or metacognitive 'red flags', associated with this second kind of monitoring are shown in Figure 1 as shaded boxes. 'Red flags' may be of three types: lack of progress, error detection, and anomalous result. Recognising lack of progress during a fruitless exploration episode should lead students back to analysis of the problem in order to reassess the appropriateness of the chosen strategy and to decide whether to persist, salvage whatever information is useful, or abandon the strategy altogether. In the latter case it is likely that students will need to reassess their understanding of the problem, and search for new information or a new strategy. Error detection during an implementation episode should prompt checking and correction of calculations carried out so far. Finally, if attempts to verify the solution reveal that the answer does not satisfy the problem conditions, or does not make sense, then this anomalous result should trigger a calculation check (assess execution of strategy), followed, if necessary, by a reassessment of the strategy.

## Collaborative Problem Solving

The analytical framework used by Artzt and Armour-Thomas (1992) revealed the type and extent of metacognitive behaviour for individual students working together in small groups, but it was not designed to capture the interactive nature of the groups' monitoring and regulation of their collective mathematical activity. Little is known about how collaboration between peers of comparable expertise might mediate metacognitive strategy use, and the few studies which have been reported have produced conflicting results (e.g. Hart, 1993; Stacey, 1992). One source of difficulty may lie in the lack of attention given to the quality of students' interaction, since instructing students to work together does not guarantee that they will collaborate. For example, Damon and Phelps (1989) reserve the term peer collaboration for the interaction that occurs when students with similar levels of competence share their ideas in order to jointly solve a challenging problem. Similarly, Teasley and Roschelle (1993, p. 235) described collaboration as 'a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of the problem'. However, collaborative work can also include periods of individual activity when participants withdraw to grapple with difficult or partially-formed ideas, and turn-taking is not responsive to their partner. Neither is the students' overlap of meaning always complete or certain, producing periods of conflict during which mutual understanding has to be renegotiated.

Thus, although collaboration may involve both cooperation and conflict, its distinguishing feature mutuality the process of exploring each other's reasoning and viewpoints in order to construct a shared understanding of the task. Producing mutually acceptable solution methods and interpretations thus entails reciprocal interaction, which would require students to propose and defend their own ideas, and to ask their peers to clarify and justify any ideas they do not understand.

The study reported in this paper focussed on the metacognitive monitoring and regulatory processes reported by individual students and observed during collaborative work in senior secondary school mathematics classrooms.

Figure 1. An episode-based model of metacognitive activity during problem solving

(Available from the author: see end of paper for details)

## Method

The data for the present study come from a larger project, the purpose of which was to investigate both individual and collaborative metacognitive monitoring and regulation in senior secondary school classrooms, patterns of teacher-student and student-student interaction associated with metacognitive activity, and assumptions about teaching and learning mathematics underlying teachers' and students' actions. The project involved up to eight secondary school teachers and their classes in five schools over a period of three years. Methods used to investigate the metacognitive strategies students employed while working on mathematics tasks included questionnaires (Years 1-3 of the study), individual interviews (Year 3 only), and classroom observation supplemented by audiotaping and videotaping (Years 1-3). This paper draws on interview and audio/videotape data gathered from one independent and one government school (labelled School A and School B respectively) in Brisbane's northern suburbs.

Target students was chosen for interview and observation on the basis of their metacognitive sophistication and preference for working collaboratively with peers, as judged from preliminary observation and responses to questionnaires probing their beliefs about mathematics and metacognitive self-knowledge (see Goos, 1995 for details of questionnaires). Individual interviews were conducted with seven students at School A and eight at School B. The semi-structured interview script is shown in Figure 2. The interview was designed to probe and follow up some of the issues raised in the open ended metacognitive self-knowledge questionnaire. Questioning generally followed the order indicated; however, since the course and substance of interviews were determined by students' responses, some variation in the sequence of questioning occurred. All interviews were audiotaped and transcribed.

One lesson each week was observed in participating classrooms, and groups of target students were audiotaped or videotaped as they worked and discussed their ideas together in class. The tapes were reviewed to identify instances of collaborative metacognitive activity, and these portions were transcribed for later analysis. Only transcripts from School A are discussed in this paper.

Individual Interview

1. What aspects of maths do you find most difficult?

What do you do to overcome these difficulties?

What strategies seem to help?

2. What do you do when you get stuck on a problem?

3. What is the best way you have found to learn and understand maths?

What kinds of things do you do to learn and understand maths while you're in class?

4. (a) When you're working on a problem, how can you tell whether you're doing it the right way? How do you decide whether to change your approach?

(b) How can you tell if you've made a mistake?

(c) How can you tell whether you've solved the problem correctly?

5. If you and a friend got different answers to the same problem, what would you do?

6. Is it possible to get the right answer to a maths problem and still not understand the problem? Explain.

Figure 2. Individual interview script

## Results

### Research Question 1: What monitoring and regulatory processes do individual students use while working on classroom mathematics tasks?

Analysis of students' interview responses was guided by the episode-based model of problem solving shown in Figure 1. From the interview script (Figure 2), Questions 2, 4 and 5 were identified as those probing metacognitive self-monitoring and self-regulation strategies that might be used while working on a mathematical task: specifically, during Implementation (Questions 4a, 4b), Exploration (Question 2) and Verification (Questions 4c, 5) episodes. Responses to only these questions are included in the following analysis. Note that the interview did not include questions specifically probing metacognitive activity during the initial Reading, Understanding, Analysis and Planning episodes of problem solving. However, students' responses to the question 'What do you do when you get stuck on a problem?' (Interview Question 2) indicate that this lack of progress can trigger a reassessment of their task-specific knowledge and understanding of the problem monitoring activities which are associated with these initial episodes (see Figure 1). Thus the interview deals either directly or indirectly with all of the episodes typically found during problem solving.

The interview implicitly investigated students' ability to recognise and act on metacognitive 'red flags' (see Figure 1). The analysis began by identifying the 'red flags' targeted by each interview question:

| Interview Question | 'Red Flag' |
| --- | --- |
| 2. What do you do when you get stuck on a problem? | Lack of progress |
| 4. (a) When you're working on a problem, how can you tell whether you're doing it the right way? How do you decide whether to change your approach? | Lack of progress<br>Error detection<br>Anomalous result |
| 4. (b) How can you tell if you've made a mistake (while working on a problem)? | Error detection |
| 4. (c) How can you tell whether you've solved the problem correctly? | Anomalous result |
| 5. If you and a friend got different answers to the same problem, what would you do? | Anomalous result |

Students' responses to each interview question were then collected together and grouped into categories corresponding to the monitoring and regulatory strategies identified in the episode-based framework of Figure 1.

Since each 'red flag' was targeted by more than one interview question, the next step in the analysis procedure drew together all strategies triggered by each of the three 'red flags'. The results are presented below, together with a sample of actual responses to exemplify the range of monitoring and regulation strategies mentioned by students. The proportion of students (n=15) who reported each type of strategy is also noted.

'Red Flag': Lack of progress

¥ Assess progress towards goal (Monitoring), 60%

Leading nowhere, going round in circles.

No pattern forming.

Realise you're not heading in the right direction, run out of patience with it.

¥ Assess strategy appropriateness (Monitoring), 13%

See if I'm doing it the right way first.

Go back to the beginning and check my reasoning.

¥ Change strategy (Regulation), 67%

Try different approaches, start afresh.

If I'm attacking it one way, change directions.

If I know any other ways, try them.

¥ Assess knowledge and understanding of problem (Monitoring), 20%

Reread the question, make sure I've understood it properly.

Make sure I haven't missed anything the question gives.

¥ Identify new information, reinterpret problem (Regulation), 53%

Jot down all information given, try to piece together to find pattern.

Look for underlying similarities to something you've done before.

Think about what we've learned, see what could apply to that.

Additional regulatory strategies triggered by lack of progress included individual persistence (47%), seeking help from the teacher (33%), or collaborating with peers (40%).

'Red Flag': Error detection

¥ Assess strategy execution (Monitoring), Correct calculation errors (Regulation), 33%

Redo every calculation a few times.

Pause at critical stages in the problem to check what you've done before going on.

'Red Flag': Anomalous result

¥ Assess result for accuracy and sense (Monitoring), 93%

Check it another way, or work backwards.

Does it make sense.

Ask someone else what they got.

¥ Assess strategy execution (Monitoring), Correct calculation errors (Regulation), 60%

Check my own working for errors, then check each other's working.

Explain our working to each other.

Decide whose working matches what we've been taught.

¥ Assess strategy appropriateness (Monitoring), Change strategy (Regulation), 53%

Compare approaches and decide which makes most sense.

Criticise my own work, and my friends'.

Try it together and see where the two ways branch off.

The analysis of students' interview responses summarised above has identified all the appropriate metacognitive monitoring and regulation strategies proposed in the episode-based model of mathematical thinking of Figure 1 (although no single student mentioned every type of strategy). It is possible to map students' reported strategies onto the model, as shown in Figure 3. Monitoring and regulation pathways triggered by each of the 'red flags' are plotted on the model. Path 1 describes the sequence triggered by lack of progress: the appropriateness of the strategy is reconsidered and a decision may be made to modify it; and the problem may need to be reinterpreted and new information identified. Path 2 begins with error detection, followed by assessment of strategy execution and correction of calculation errors. Path 3, followed when an anomalous result is recognised, involves eliminating any errors in the execution of the strategy, and trying a new approach if the original strategy was responsible for the incorrect answer.

Here it is worth recalling that 'red flags' will not be recognised without routine monitoring to check whether problem solving is on track. If students are to avoid persisting with an inappropriate strategy, detect and correct errors, and reject nonsensical answers, then continuous assessment of progress, strategy execution, and results is essential.

Research Question 2: How are metacognitive processes elicited and supported during collaborative student-student interaction?

Three transcripts from a senior secondary mathematics classroom in School A were selected for analysis to illustrate common features of collaborative metacognitive activity. Each transcript was drawn from a different year of the study, and each was obtained from a different group of students as they worked on problems set by the teacher in the normal course of a lesson. Students were not specifically told to work together, but were free to choose whether, and how, to interact with their peers. Since a full description of the analysis of each transcript is beyond the scope of this paper, only a summary of the findings is presented here. The three problems involved:

¥ Spreadsheet calculations of compound interest (one male and two female Year 11 students, first year of the project),

¥ Hooke's Law (three male Year 12 students, second year of the project), and

¥ Projectile motion (two male Year 11 students, third year of the project).

The transcripts were first parsed into macroscopic episodes which represented the distinctive kinds of problem solving behaviour shown in Figure 1.

Figure 3. Monitoring and regulation pathways identified from student interviews

(Available from the author: see end of paper for details)

A second, microscopic, level of analysis focussed on the metacognitive function and collaborative structure of the conversational turns of all speakers. A coding scheme developed in an earlier study (Goos & Galbraith, 1996) was used to identify metacognitive acts where new information was recognised or an assessment of particular aspects of the solution was made. The first type of metacognitive act, New Idea, occurred when potentially useful information came to light or an alternative approach was proposed. The second type of metacognitive act involved making an Assessment of a strategy (execution or appropriateness), a result (accuracy or sense), or of one's knowledge or understanding.

Conversational turns were then coded a second time to identify transactive dialogue, defined as discussion in which an individual's reasoning operates on a partner's reasoning, or significantly clarifies his or her own reasoning (Kruger, 1993). Following Kruger, three types of transacts were coded: transactive statements and questions, and responses to transactive questions. The orientation of each transact was also noted, with operations on one's partner's ideas being labelled other-oriented, and reasoning directed at one's own ideas coded as self-oriented. This procedure produced six transact codes: (three types) x (two orientations). Kruger's coding scheme was then extended to highlight the reciprocal nature of collaborative interactions. This was done by grouping the codes to produce an operational definition of collaboration possessing the following three elements:

Self-disclosure Self-oriented statements and responses that clarify, elaborate, evaluate, or justify one's own thinking. (Here is what I think.)
Feedback request Self-oriented questions that invite a partner to critique one's own thinking. (What do you think about my idea?)
Other-monitoring Other-oriented statements, questions and responses that represent an attempt to understand a partner's thinking. (Is this what you mean? Here is what I think of your idea.)
A balance of Self-disclosure, Feedback Requests and Other-monitoring, indicating mutual engagement with each other's reasoning, would confirm that students' interaction was collaborative.

Metacognitive Function of Students' Dialogue

In the Hooke's Law problem, routine monitoring of progress prompted a disagreement over the meaning of the problem's conditions, and the students initiated Understanding and Analysis episodes in which they clarified their interpretation of the problem and chose an appropriate solution strategy. The other two transcripts involve monitoring and regulation triggered by the recognition of metacognitive 'red flags'.
For the projectile motion problem, careful monitoring of strategy execution during Implementation and Verification episodes resulted in error detection and correction. In the compound interest problem, an

anomalous result recognised during a Verification episode triggered a series of Analysis and Verification episodes, involving assessments of strategy appropriateness and of the accuracy and sense of the results these strategies produced.

Although each transcript illustrates different functions of metacognitive monitoring, taken together they also reveal that consistent monitoring patterns accompany the distinctive problem solving behaviours represented as episodes. Table 1 shows the numbers and functions of metacognitive acts recorded in all three transcripts, grouped according to episode type. (No Reading episodes were observed, possibly because the students had read the problems before the camera and microphone were positioned.)

The main patterns which emerge can be summarised as follows:

1. Understanding episodes were monitored by students assessing their understanding of the problem (four Assessments), and it was often necessary to identify new information or reinterpret the problem in order to make further progress (eight New Ideas).

2. Analysis episodes featured assessments of strategy appropriateness (twelve Assessments) as well as understanding (six Assessments), and New Ideas (seventeen) were proposed to help reformulate the problem.

Table 1. Number and Function of Metacognitive Acts in Each Type of Problem Solving Episode

Metacognitive Act Understanding Analysis Implementation Verification Total

New Idea 8 17 1 6 32

Assessment

¥ knowledge - - - 1 1

¥ understanding 4 6 2 1 13

¥ strategyÐappropriateness - 12 6 7 25

¥ strategyÐexecution - - 10 3 13

¥ resultÐaccuracy - - - 2 2

¥ resultÐsense - - - 4 4

Total Metacognitive Acts 12 35 19 24 90

3. Students monitored Implementation episodes by assessing strategy execution (ten Assessments) and appropriateness (six Assessments). (New Ideas and Assessments of understanding were also recorded in small numbers, but these monitoring activities were observed across all episode types, and did not contribute significantly to metacognitive activity during Implementation.)

4. Because Verification episodes are intended to review the entire solution process, almost all metacognitive functions were represented in the transcript analyses. Results were assessed for accuracy and sense (six Assessments in total), the execution of strategies was checked (three Assessments), and the appropriateness of the solution method evaluated (seven Assessments). If the solution could not be successfully verified, further task-related knowledge (six New Ideas)

had to be identified and work on the task continued.

The monitoring strategies students used during collaborative problem solving are consistent with those reported by individual students, illustrated in the episode-based model of Figure 3. However, the analysis has demonstrated that multiple strategies were employed to monitor each episode, rather than the single strategies suggested by the model. As students worked through the stages of solving a problem, from Understanding the problem to Verification of the solution, they needed to call on an ever increasing range of metacognitive strategies to keep track of progress and deal with obstacles (see Table 1). It may be useful to think of this process as backtracking, so that episode types and their corresponding monitoring and regulatory strategies from the earlier stages of problem solving are accessible at later stages when difficulties arise.

Collaborative Structure of Students' Dialogue

Although the teacher did not impose a formal group work structure, the transcripts show that students did choose to work together in class, particularly when they faced difficulties. In the Hooke's Law problem, it was a disagreement which prompted collaborative debate, while students' initial uncertainty over constructing a solution to the unfamiliar projectile motion problem led them to work together. For the compound interest problem, one student consulted with others in order to deal with an obstacle in the form of an answer which did not make sense.

The collaborative quality of students' interaction was measured by coding conversational turns to identify transacts produced by each speaker. Because speakers had unequal opportunities to contribute to the discussion, the six transact codes (self- and other-oriented statements, questions and responses) were counted as both frequencies and proportions. The proportion of transacts in each whole dialogue was calculated as the total number of transacts divided by the total number of conversational turns. As described earlier, the transact codes were then grouped to reflect three elements of collaboration: Self-disclosure (self-oriented statements and responses), Feedback Request (self-oriented questions) and Other-monitoring (other-oriented statements, questions and responses). The proportions of grouped transacts for all three transcripts are displayed in Figure 4.

Figure 4. Distribution of transact groupings by transcript

(Available from the author: see end of paper for details)

The balance of Self-disclosure, Feedback Requests and Other-Monitoring transacts found in each dialogue suggests that the interaction of each group of students was consistently collaborative. However, when individual students' contributions were examined they did not always display the same balance. There were times when individual students were mainly occupied with explaining their ideas to a peer (predominantly Self-Disclosure), or with trying to understand a partner's ideas (predominantly Other-Monitoring). These different findings are not contradictory instead, they highlight the various roles individual students may take in order to initiate and maintain collaborative interaction. It seems possible that students' roles in contributing to collaborative dialogue might change according to the circumstances in which they find themselves.

Collaborative Metacognitive Activity

Metacognitive activity which was collaborative in nature was described by examining conversational turns double coded as having both metacognitive function and transactive structure. The numbers and types of metacognitive transacts for all three transcripts are shown in Table 2.

These results indicate that joint metacognitive activity was characterised by:

¥ students clarifying, elaborating and justifying their New Ideas for the benefit of a partner (Self-disclosure);

¥ students asking their peers for help in finding errors by inviting critique of strategies and results; and students seeking feedback on the New Ideas they proposed (Feedback Request);

¥ students making an effort to understand their partners' thinking by offering critiques of their strategies, and also by elaborating on and monitoring their understanding of partners' ideas (Other-monitoring).

For the three transcripts in total, 26 of the metacognitive-transacts were self-oriented (Self-disclosure or Feedback Request) and 26 were other-oriented (Other-monitoring). Thus, when the students interacted with each other, their monitoring activity was directed at both their own thinking and the ideas of their peers.

Table 2. Conversational Turns Coded as Metacognitive Acts and Transacts

Transactive Structure (Frequencies)

Metacognitive Function Self-Disclosure Feedback Request Other-Monitoring

New Idea 14 2 4

AssessmentĐstrategy 2 5 16

AssessmentĐresult Đ 3 Đ

AssessmentĐunderstanding Đ Đ 6

Total 16 10 26

Not all of the students' metacognitive activity was jointly transacted. Monitoring sometimes took the form of self-directed 'thinking aloud' which did not seek acknowledgment or response from a partner. On other occasions students did seek a response to their New Ideas and Assessments, but were ignored by their partners. There were also times of conflict and disagreement, when Assessments simply rebutted a partner's strategy and New Ideas merely reasserted the speaker's position. This finding is not surprising, since collaboration does not preclude periods of individual reflection, nor is it necessarily based on immediate or continuing consensus (Kruger, 1993; Teasley & Roschelle, 1993).

## Discussion

Empirical evidence from students' interview responses and videotaped classroom interactions has been presented to support a theoretical framework which extends existing models of the ideal cognitive and metacognitive characteristics of mathematical problem solving behaviour. Most interview questions asked students to report on the strategies they used while working individually on problems; however, some evidence of collaborative strategies was also obtained, for example, in seeking the help of peers to locate errors, evaluate strategies, and deal with obstacles to progress. Analyses of student interviews and of transcripts of collaborative problem solving were guided by the episode-based framework developed here. As well as demonstrating that the same monitoring and regulatory processes reported by individuals are used when students work with peers, the analysis also suggested mechanisms through which peer interaction mediates metacognitive activity.

The three problem solving transcripts record spontaneous student-student interaction during regular classroom lessons. Collaboration was initiated by the students, without being imposed by the teacher or researcher for organisational or experimental purposes. It is therefore worth asking why the students chose to collaborate in these specific instances. The answer is found in the various types of impasse presented by these problems a disagreement, uncertainty, or an obstacle where progress was either halted or slowed and the students turned to their peers to work their way through the difficulty by monitoring their own and each other's thinking. It is important to recognise that collaborative interaction stimulated by challenging problems may feature periods of individual activity or conflict as well as agreement and cooperation. Consensus may only be reached after students have debated each other's perspectives and retired from the discussion to reconsider their own.

The mathematics education reform movement has identified new goals for school mathematics and recommended changes in teaching and learning practices which call for increased attention to be given to small group work and student communication of mathematical ideas. This paper has suggested that reforms which favour such peer interaction have the potential to develop the metacognitive aspects of students' mathematical thinking. Individual metacognition is organised around routine self-monitoring and the recognition of, and response to, 'red flags' which warn that problem solving has gone astray. Collaborative metacognitive activity involves the same processes, but proceeds through offering one's thoughts to others for inspection, and acting as a critic of one's partner's thinking. If students are encouraged to engage with each other's reasoning in this way, the resulting discussion makes visible the processes that individuals could appropriate to monitor and regulate their own thinking, and become self-directed learners.

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