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Mathematics and Manipulatives: Views from the Secondary Schools

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Introduction

At the 1994 Conference of the Mathematics Education Research Group of Australasia, Perry and Howard questioned the ways in which manipulatives (or concrete materials) were being used in primary school mathematics learning and teaching. This was done in the light of current research and practice surrounding children's construction of mathematical knowledge. The paper declared that a continuing research program including the role and acceptance of manipulatives in secondary school mathematics classes was needed (Perry & Howard, 1994).

Data concerning the use of manipulatives in primary school mathematics classes and their relationship to teachers' beliefs about mathematics, mathematics learning and mathematics teaching have been analysed and reported in earlier papers (Howard, Perry & Conroy, 1995; Perry, Howard & Conroy, 1996).

This paper seeks to identify current teacher usage of concrete materials (manipulatives) in secondary school mathematics learning and teaching and reasons for this use in a sample of schools in South Western Sydney.

Background to the study

For the most part, the terms 'concrete materials' and 'manipulatives' are taken as synonymous to mean 'concrete models that incorporate mathematical concepts, appeal to several senses and can be touched and moved around by students" (Hynes, 1986, p.11). Other authors use

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

'manipulatives' to incorporate both concrete and pictorial representations, including even images on computer screens (Sowell, 1989; Touger, 1986). In this paper, the former definition and the term 'manipulatives' are used.

Manipulatives play an important role in the learning and teaching of mathematics in primary schools (Atweh & Watson, 1992; Australian Council for Educational Research, 1965; Australian Education Council, 1991, 1994; Grouws, 1992; National Council of Teachers of Mathematics, 1980, 1989, 1991; NSW Department of Education, 1969, 1972, 1989). However, their impact on the quality of mathematics learning and teaching in secondary schools has received little attention. This paper reports baseline data related to the use of manipulatives in secondary mathematics learning and teaching from which this impact might be gauged.

Manipulatives have been useful in the learning and teaching of mathematics (Bohan & Shawaker, 1994; Sowell, 1989; Thompson, 1992). Support for their use has come from curriculum developers, textbook writers and learning theorists. Much of the stimulus for this has arisen from the work of Piaget related to stages of development and their relationship to mathematics learning. It has been reported that the use of manipulatives in mathematics learning and teaching decreases as children move through the primary grades (Gilbert & Bush, 1988; Hatfield, 1994; Suydam, 1984, 1986). This may be directly related to an application of Piaget's stage theory. Hence, it would be of no real surprise if there were very little use of manipulatives in secondary school mathematics classes. Certainly, anecdotal evidence gained through observation of such classrooms would suggest this to be the case. However, further data were needed to substantiate or repudiate such perceptions.

Methodology

Baseline data have been collected on secondary teachers' perceptions of their acceptance and use of manipulatives in their mathematics classrooms. In particular, data have been collected and analysed to help answer the following questions:

1. What manipulatives are employed in the learning and teaching of mathematics in Years 7 - 12?
2. How are these manipulatives used?
3. What factors influence the choice of secondary mathematics teachers to either use or not use manipulatives in their mathematics lessons?
4. Are there differences in the use of manipulatives across different mathematical topics?

The data were collected using a specifically designed questionnaire consisting of both multiple choice and open-ended questions covering the following areas:

1. subject demographics such as gender, age, position in school, nature of teacher training, length of teaching experience, class(es) currently taught, class size, classes taught over the last ten years;

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

2.use of manipulatives in mathematics learning and teaching such as: which are used, why and how they are used, and the areas of mathematics in which they are used;
3.beliefs about mathematics, mathematics learning and mathematics teaching.
The questionnaire relies on the self reporting of the teachers and parallels much of the work reported in Hatfield (1994). It is substantially the same questionnaire as that which was used to gather data from primary school teachers (Perry, Howard & Conroy, 1996).
In September, 1996, the questionnaire was posted, with reply paid envelopes, to 52 secondary schools in the South Western suburbs of Sydney. Fifteen schools were part of the Catholic Secondary Schools System while 37 were Department of School Education (government) schools. Schools were contacted by telephone to gain the Principals' initial approval to undertake the survey in the schools and to ascertain the number of mathematics teachers in each school. Four hundred and sevnty-five questionnaires were posted to cover all mathematics teachers at each school, along with one or two extras. Two hundred and forty-nine responses were received. This equates to a response rate of at least 52%. Data from these responses were analysed using the SPSS-X program to provide descriptive statistics for the demographic data, the use of manipulatives and teacher beliefs about mathematics, mathematics learning and mathematics teaching. As well, inferential statistics dealing with relationships between these variables were generated.

Results
This paper reports on some of the baseline data concerning secondary mathematics teachers' demographics and their use of manipulatives.

Demographic data
Of the 249 respondents, 112 (45%) were female.
The respondents appear to be a relatively well experienced group with 64 (26%) of the respondents having in excess of 20 years teaching experience, while 83 (33%), 51 (21%), 42 (17%) had from 11 to 20 years, 6 to 10 years, 1 to 5 years teaching experience respectively. Only 8 respondents (3%) had less than one year of teaching experience.

Fifty-four (22%) of the respondents had been at their current school for more than 10 years, while 89 (36%) and 74 (30%) had taught in their current school for 6 to 10 years or 1 to 5 years respectively. Only 29 (12%) had been in their current school for less than 1 year. The school positions held by the respondents are reported in Table 1.

Table 1
School positions heldn=249

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

Current position	Number	Percentage
Principal	3	1
Leading Teacher	7	3
Head Teacher(Mathematics)	40	16
Classroom Teacher (Mathematics)	193	78
Other	5	2

Two (1%) of the respondents described themselves as two year trained teachers, 13 (5%) as three year trained, 57 (23%) as four year trained with a Bachelor of Education, 154 (62%) as four year trained with a Bachelors degree followed by a Diploma of Education and 21 (8%) as teachers with more than four years of training.

An attempt was made to ascertain where the respondents saw their major teaching focus. They were asked in which of the Year ranges 7 and 8, 9 and 10, 11 and 12, they had mostly taught in the past 10 years. Sixty-two respondents (25%) saw their major emphasis in Years 7 to 10 while 40 (16%) saw the major emphasis of their teaching being in the senior secondary school (Years 11 and 12). Of the remainder, 127 (51%) reported that they had taught across all Years in the last ten years and data were missing for the remaining 20 respondents.

Use of manipulatives data

A significant number (86%) of the respondents generally felt confident in using the manipulatives available to them. As well, teachers were asked if they would like more training in the use of manipulatives. One hundred and fifty-two (61%) indicated that they would. Interestingly, respondents were as likely to want training if they were confident in the use of the materials as they were if they answered "No" to the confidence question.

Twenty of the respondents (8%) indicated that they used manipulatives in their mathematics classes 'often' and 210 (85%) 'sometimes'. Eighteen teachers (7%) responded that they did not use manipulatives at all in their mathematics teaching. Of those who responded that they used manipulatives 'often', 2 (10%) said that this meant 'every mathematics lesson', 14 (70%) said that it meant 'approximately once a week' and another 4 (20%) 'once a fortnight'. On the other hand, of the 210 who responded that they used manipulatives 'sometimes', 47 (23%) said that this meant 'approximately once a week' while another 64 (31%) said that it meant 'approximately once a fortnight' and 96 (46%) 'approximately once a month'.

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

Table 2 shows the mathematical topics in which manipulatives were used.

Table 2
Mathematical topics using manipulativesn=249 (Multiple responses possible)

Mathematical Topic	Number	Percentage
Measurement	213	86
Geometry	183	74
Algebra and Number	111	45
Chance and Data	142	57
Calculus	20	8
Other	18	7

The respondents were asked to identify what manipulatives they used in their mathematics classes. Table 3 shows the distribution of responses for the different types of manipulatives.

Table 3
Manipulatives used in mathematics classesn=249 (Multiple responses possible)

Manipulatives	Number	Percentage
Base 10 material	56	23
Unifix	32	13
Algebra materials	98	40
Polydrons	92	37
Pattern blocks	99	40
Environmental materials	82	33
Others	84	34

A variety of manipulatives was being used by the respondents. These results do not indicate specifically how often each manipulative was being used, only that they were used. Further, the results do not tell us for what areas of mathematics specific manipulatives were being used. What is apparent is that no manipulative was being used by more than 40% of teachers - 40% of the respondents used algebra materials and 40% used pattern blocks. Base 10 material and Unifix, which could be identified as manipulatives for Number, were used to some degree by

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

23% and 13% of respondents respectively.
There is a variety of reasons why teachers used manipulatives in their mathematics lessons. These are reported in Table 4.

Table 4
Reasons for the use of manipulativesn=249 (Multiple responses possible)

Reason for use	Number	Percentage
Syllabus says you have to	8	3
I believe they benefit students' learning	201	81
Students enjoy using them	129	52
It is in school mathematics policy	19	8

The overwhelming response is that teachers use manipulatives because they believe that the materials benefit students' mathematics learning. The next strongest response indicated that teachers use manipulatives because they believe students enjoy using them. Whether or not students in secondary mathematics classes actually do enjoy using the manipulatives or whether students believe that they benefit their learning was not investigated in this study. School mathematics policies and the prescribed syllabus appear to have little impact on most teachers' use of manipulatives in their mathematics lessons.

Table 5Ways in which manipulatives are used in mathematics lessons
n=249 (Multiple responses possible)

Classroom use of manipulatives	Number	Percentage
By the teacher as demonstration	164	66
By the students as they wish	53	21
By studts as agreed betwn teacher& them	136	55
By students to check their work	45	18
By students as a means of remedial help	66	27

Manipulatives are being used for a variety of purposes in secondary mathematics classrooms. The wide use of manipulatives by teachers for demonstrations in mathematics lessons is of particular interest. The responses in Table 5 do need to be substantiated through teacher interviews and classroom observation of the teacher and student use of manipulatives in the learning and teaching of mathematics across the secondary years.

Analysis and discussion
The frequency of the use of manipulatives in secondary mathematics

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

classrooms is low, particularly when compared to that found in primary schools. Howard, Perry & Conroy (1995) reported that 62% of the primary teachers in their sample responded that they used manipulatives 'often' and another 36% used them 'sometimes'. The corresponding figures for the secondary sample are 8% and 85%. Further analysis of the secondary data shows that less than 1% of all of the secondary respondents used manipulatives in every mathematics lesson, 24% used them approximately once a week, 27% once a fortnight and 39% once a month. It would seem that many secondary mathematics teachers do not consider manipulatives central to their teaching. Those who do use manipulatives indicated that they use them primarily because they feel that the manipulatives benefit the students' learning and that the students enjoy using them. These reasons were also the most dominant among primary teachers.

There are several reasons which could be put forward for the relatively low rate of use of manipulatives by secondary mathematics teachers. The very structure of many secondary schools, with their rigid timetables, movement of students and teachers around the school and firm, school-wide programs may make it difficult for individual teachers to organise the supply of manipulatives to their classes. Even though the Year 7/8 Mathematics Syllabus (Board of Studies, NSW, 1988) advocated the use of manipulatives in mathematics learning and teaching, they are not always easily available for secondary classes. The dominance of text book lessons in secondary mathematics classrooms and the ease with which the use of such texts can be arranged could also effect the regular use of manipulatives.

Though a variety of manipulatives is used by secondary respondents, no particular manipulative is used by more than 40% of them. That is, there is not a dominant manipulative in secondary mathematics. This is in stark contrast to the primary schools where Howard, Perry & Conroy (1995) found that 84% of the respondents used Base 10 blocks. No doubt, the importance placed on Base 10 blocks in the K-6 Mathematics Syllabus (NSW Department of Education, 1989) and text books and in professional development over the last ten years as effected this level of adoption.

There are some particular results which are worthy of note. Firstly, the fact that 40% of respondents reported that they used algebra materials in their mathematics lessons should be pleasing to those Australian mathematics educators who have advocated such action for some time (Atweh, Cooper & Boulton-Lewis, 1996; Quinlan, Low, Sawyer & White, 1993). Less pleasing, however, might be the fact that only 74% of the respondents claimed that they used manipulatives in the teaching of geometry. Perhaps the explanation for this surprisingly low figure could lie in an assumption by the respondents that some of the more commonly used geometric materials, such as compasses, rules and set squares, constitute measuring instruments rather than manipulatives. The only way of checking on this supposition would be to ask the teachers directly. This has not been done in the study reported here.

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

Data concerning the nature, regularity and manner of use of manipulatives in secondary mathematics classes were analysed against variables such as gender, years of service, position in school, school system and years most commonly taught over the last 10 years. The only finding of significance is recorded in Table 6.

Table 6Frequency of use of manipulatives and gendern=249
Number (Percentage of gender group)

Gender of Teacher	Frequency of use of manipulatives		
	Often	Sometimes	Don't use
Female	14 (13%)	96 (86%)	2 (2%)
Male	6 (4%)	114 (84%)	16 (12%)

P2 = 13.43449 with 2 degrees of freedom, p=0.00121

No other significant differences were found. This may simply be an indication of the homogeneity of the sample of secondary mathematics teachers who responded to the questionnaire. Eighty-five per cent of the respondents were four year trained with either a degree / diploma in education or a Bachelor of Education. Much of their training would have been content based through university mathematics subjects. There is a high likelihood that this mathematics training may not have involved manipulatives. There is only a small number of different textbooks used in secondary mathematics classrooms in New South Wales and most of these are very similar in their approach. Most secondary teachers would use such a text as the basis of their mathematics lessons. There is very little difference in these approaches either across years in a school or across schools, even if they are in different systems of organisation. Consequently, perhaps it is not too surprising that there were no significant differences identified across years, systems and the teacher training of the respondents.

Conclusion

This paper has presented some initial baseline data on the use of manipulatives in secondary school mathematics classrooms. It would appear that the use of manipulatives in these classrooms is low, particularly compared to such use in primary school mathematics lessons. Nonetheless, just as their primary colleagues, the secondary mathematics teachers who do use manipulatives in their mathematics lessons often do so because they believe that the manipulatives benefit the students' learning and that students enjoy using them. Teachers reported a high degree of confidence in their use of manipulatives, although 61% of all respondents indicated that they would like more training in this use.

The data and their analysis generate more questions than they answer. There needs to be further investigation into the actual use of manipulatives in the secondary mathematics classroom, through direct

Paper Presented at the Joint ERA/
AARE Conference, Singapore, 1996
Australian Association
for Research in Education

observation and interview. This would allow some of the finer distinctions between year level, school system and teacher training to be illuminated, if there are, in fact, any distinctions to be found. As well, it would allow further analysis of ways in which manipulatives are used in secondary mathematics classrooms to be compared with this use in primary mathematics lessons. Finally, the relationship between secondary mathematics teachers' use of manipulatives and their beliefs about mathematics, mathematics learning and mathematics teaching still need to be analysed from the data collected. There is much still to be done.

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Paper Presented at the Joint ERA/
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