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Title: Cognitive mapping of Advanced level Physics students' conceptions of Quantum Physics

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Abstract

Elementary particles seem to be waves on Mondays, Wednesdays and Fridays, and particles on Tuesdays, Thursdays and Saturdays.
Sir William Bragg

Students experience considerable conceptual difficulties in trying to incorporate the ideas of quantum physics into their overall cognitive framework. The preliminary findings of a study investigating students' understanding of quantum phenomena is presented.

The powerful heuristic metaphor of the map is used to construct graphic representations of students' understanding of quantum physics. The nature of students' understanding being represented by their construction of groupings of ideas in a personal psychological space, with underlying dimensions providing a co-ordinate system for their perceptions. The relationships between students' conceptions (at the level of the population group) of quantum phenomena are investigated using a structured questionnaire, and multivariate analytical techniques (multidimensional scaling, cluster analysis, and factor analysis). Groupings of conceptions are identified and related to underlying interpretable dimensions.

Strand: Teaching, learning and cognitive processes
Key words: quantum physics, cognitive mapping, multivariate analysis

Bio-data
Azam MASHHADI's doctoral thesis, at the University of Oxford, addressed the question of What is the nature of the understanding of the concept of 'wave-particle duality' among Advanced level Physics students? Following degrees in Physics and Astrophysics (University of London) and Astronomy (University of Sussex) he taught for several years at a college in London (UK) before completing a MSc in Educational Research Methodology (Oxford). His research interests include student learning,

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Scientists simply were not able to go deep enough into their own

paradigms and free themselves of habits of thinking. They were approaching the atomic world like visitors in a foreign city who cling to their maps and guidebooks and only walk along the most travelled paths. They would have done better to have plunged fully into the heart of the city and attempted, in all their confusion and excitement, to have captured something of its spirit.
F. David Peat (1994: 45), Blackfoot Physics

1 Introduction

Reality is quantum mechanical. The quantum theory is probably the most successful theory in the history of science, yielding descriptions for all the fundamental forces of nature except gravity and accounts for phenomena ranging from starlight to the periodic table. It has also been responsible for technologies spanning nuclear reactors to lasers. However as the physicist Richard Feynman (1965: 129) in The Character of Physical Law famously remarked:

...after people read the paper a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that no one understands quantum mechanics ....

Two immediate implications of the comment is that, firstly, there is something fundamentally different about quantum physics, and secondly it raises the question of what is meant buy 'understanding'. The theory of relativity was in many ways a continuation of 'classical physics' - it is quantum physics that represents a new conceptual revolution (Selleri, 1990). In less than a century physics has abandoned a world view consisting of concepts that were mechanistic, deterministic and largely absolute, and espoused a world view comprising concepts that are relative, frequently non-deterministic and stochastic in nature (Lahti, 1990). Quantum theory has two characteristic features that distinguish it from classical physics. Firstly, quantisation, i.e. physical quantities are not allowed to take a continuous set of values.

Secondly, it is not possible to predict the outcome of an individual measurement.

At present in England and Wales upper secondary school students (ages 16-18) wishing to read for a physical science degree at university will follow the two year Advanced Level Physics course. The quantum physics section of the syllabi for the various examining boards will typically not include the Heisenberg Uncertainty Principle, the Schrödinger wave equation, and there is no explicit mention of introducing students to conceptions of the 'nature of science'. At the heart of quantum physics at both A-level and generally lies the concept of 'wave-particle duality'. The conceptual challenge of coming to terms with quantum physics was commented on by Einstein:

We know that light has certain characteristics which we designate for short, respectively, as undulatory and corpuscular. It has no meaning to say, it is a wave and it is a corpuscle. Up to now we just have no reasonable theory which explains all its characteristics. However there is no contradiction, any more that it signifies a contradiction that a man feels and has weight.
Albert Einstein (In Stachel, 1986:363)

2 What is meant by 'understanding'?

This study is concerned with investigating students' understanding of quantum physics. McCubbin (1984: 67) expresses the twin problems that the acceptance of the importance of understanding gives rise to:

The case for promoting understanding as an explicit educational objective is a difficult one to deny. It is also a peculiarly difficult one to make, in practice, because of the twin problems of defining and assessing understanding.

Studies of student understanding in science tend to tacitly assume some meaning for the term. Understanding is generally accepted to be an active process in which meaning is constructed, with new information being interpreted with regard to currently activated knowledge (Bransford, 1979). As Carey (1986: 1123) expresses it:

To understand some new piece of information is to relate it to a mentally represented schema, to integrate it with already existing knowledge.

A concept is understood, ultimately, through its relations with other concepts (Sowa, 1983). A new concept therefore cannot be explicitly understood until it is linked in a meaningful way to pre-existing concepts (Ausubel, 1963; Gagné, 1985; Novak and Gowin, 1984). All associations, which would include images, expectations, emotions and sensory experience, add to concept meaning and understanding. A concept is the collection of memory elements that are associated with the label (e.g. the photon) and the pattern of their links. Two students' understanding of a particular concept is given by the similarity of their sets of elements, i.e. their concepts will be the same if they have identical sets of images, propositions, episodes and so forth about the label. A possession of a concept (e.g. the electron) is, therefore, not a dichotomy in the sense that the student either has it or has not. It is the elements that are possessed or not possessed, the concept can be held to a greater or lesser degree.

Concepts may be viewed as cognitive devices for classifying objects in an economical way. Meaning is attached to concepts and to the relationships between concepts, and the aim is for students to learn selected networks of meaning. As a consequence Lewis (1973) argues that knowledge in the human and social sciences needs to be seen as a network or `string bag' rather than as a hierarchy.

White and Gunstone (1992) point out that a viewpoint which defines understanding as the ability to use knowledge and to cope with situations forms the basis of the use of problems in tests, and of transfer tasks in research, as measures of understanding. However, this definition and the tests are to do with overt performance, not with an internal state of mind.

Ausubel and Robinson (1969: 50) refer to two essential factors influencing meaningful learning or understanding:

...the most important factor influencing learning is the quantity, clarity and organization of the learner's present knowledge. This present knowledge, which consists of the facts, concepts, propositions, theories, and raw perceptual data that the learner has available to him at any point in time, is referred to as his cognitive structure...The second important focus is the nature of the material to be learned.

The definition of 'cognitive structure' referred to by Ausubel and Robinson is perhaps a description of the contents of cognitive structure. This definition of cognitive structure needs also to be augmented by White's (1988) suggestion that it should also make reference to the arrangement of knowledge.

This study (along with much of science education research) is making a number of assumptions that need to be made explicit:

(1) that concepts are in some way 'stored' or represented in a learner's brain,
(2) and that there is some form of organisation of these representations (i.e. we accept the existence of cognitive structure);
(3) that therefore the notion of two concepts being more or less

closely linked, connected or integrated in cognitive structure is a meaningful and sensible one;
(4) that we do not have access to a learner's cognitive structure;
(5) that a learner's behaviour (statements, responses to questions etc.) may be considered to reflect aspects of her cognitive structure;
(6) that we may construct models to represent cognitive structure in terms such as the various conceptions that a learner holds, and how they appear to be inter-related.
(Taber, 1995: 5)

The aim of this study is to try and go behind students' overt performance and describe the organisation of knowledge that underpins overt performance, and define understanding in terms of elements of memory and the pattern of association of these elements (White, 1985 and 1988).

The previous discussion has highlighted the difficulties of describing what could be meant by understanding. The word 'understanding' can have a continuum of meanings depending upon the context. This study has adopted an operational definition or limited `measure' of understanding at the level of the population group in which understanding is represented by the relationships or groupings of students' ideas (or conceptions). It should be emphasised that the unit of analysis is taken as the population group, and not the individual. The research findings, therefore, reflect the tendencies of the group, and not necessarily the perceptions of individuals.

3 Representing understanding: The conceptual map

To represent the 'understanding' of the population sample required the construction of a 'conceptual map'. The 'metaphor of the map' is a powerful heuristic device to represent the psychological structure of knowledge in the area of quantum physics as perceived by the sample population of A-level physics students.

The general aim of this study is to arrive at a representation of the multidimensional virtual world of students' understanding of quantum physics through the construction of a 'common mental geography'. The generation of a map involves the construction of a bounded graphic representation that corresponds to a perceived reality. Robinson (1982: 1) points out that the act of mapping involves the 'combination of the reduction of reality and the construction of an analogical space', and enables structures to be constructed or discovered that would remain unknown if not mapped.

All maps are approximations and involve distortions of perceived reality, as they inherently involve the use of a projection. The map's intended intellectual function and the desired visual structure are used to determine which projection is most appropriate for a given application. A number of points about maps, however, need to be borne in mind:

(a) mapping and knowing are closely intertwined; (b) maps are excellent heuristic devices; (c) both the map maker and the map reader have

important responsibilities to fulfil if communication is to occur; (d) every map reflects both its data and its designer; (e) changes in maps reflect changes in understanding; (f) the prior knowledge of the map maker can have a great influence on the maps he or she produces; (g) all maps distort reality, both because of the very nature of mapping and because map makers have learned how to exploit distortion to achieve their communicative goals; and (h) maps have great cognitive, integrative, summative, and generative power.
Wandersee (1990: 930)

This study is applying the heuristic metaphor of the map to construct graphic representations of a population group of A-level students' understanding of quantum physics, with understanding being represented by the relationships or groupings of students' conceptions. The reference frame being provided by the co-ordinate axes of the map. For instance, as in the diagram below:

The coordinate axes can be interpreted as perceptual dimensions. The dimensions are orthogonal, and their interpretation can be considered independently of each other. The labels given to the dimensions or axes of the map result from interpretations depending on the nature and location of specific conceptions. The post-modern self-consciousness of educational research emphasises that the process of interpretation is the result of an unavoidable interaction between the researcher and the researched.

The aim of this project was, therefore two-fold: to elicit students' conceptions, and investigate the relationships between conceptions.

4 Methodology

The definition of understanding adopted in terms of the structural relations between conceptions immediately raises the methodological question of how to access and represent such an implicit conceptual structure if it is present. A possible way to access such relationships would be to try to find some regularities in their responses to a series of statements. The data being analysed to see if there were any underlying 'dimensions' or 'factors' (see Child, 1970; Everitt and Dunn, 1983; O'Muircheartaigh and Payne, 1977).

With regard to the implementation of this project there were two phases (implemented from May 1993 to May 1995). Phase 1 was concerned with identifying students' conceptions, and Phase 2 with identifying groupings of conceptions and any latent dimensions of thinking.

For Phase 1 the strategy adopted was that of using a series of three studies to elicit students' conceptions. Questionnaires utilising directed or free questions were used, and students encouraged to write freely in their own words. This approach enabled a considerable amount of significant data to be acquired in a relatively short time. The use of a questionnaire maximised the sample size. A large sample size enabled a wide range of students' writing, and consequently a wide spread of students' conceptions to be obtained. Since the study is concerned with understanding at the group population level it was important to obtain as much data as possible from as wide a range of students as practically possible. The usual technique of identifying students' conceptions via interviews with a small number of students was therefore not appropriate with regard to the research questions. The use of a reasonably large sample, and the emphasis on the confidentiality of respondents helps to validate the notion that they are replying honestly. The empirical work, therefore, involved using these studies to test the feasibility of the research, the likelihood of getting useful results, to develop methods for the analysis of data, and to elicit students' conceptions in the required domain area. Each

study informed the subsequent study and gave a further insight into the nature of the research question, reflecting the fact that research is not a linear process. It should be borne in mind that the aim of Phase 1 of this study is confirmatory, in the sense of seeing if the conceptions held by the population sample are similar to conceptions identified in previous studies (see Fischler and Lichtfeldt, 1991, 1992; Niedderer, 1987; Niedderer, Bethge and Cassens, 1990; Mashhadi, 1993).

Phase 2 involved representing the conceptions elicited as specific statements in order to develop a structured questionnaire. The students responded to each statement on a 5-point ordinal response scale. The questionnaire, and the data analytical techniques were piloted, and then fully implemented in the final study with a sample population of 319 students (in eight schools and colleges). The final research instrument consisted of 54 statements representing students' conceptions of quantum phenomena, models, and the ontological and epistemological status of theoretical entities. This paper will report on the analysis of students' responses to statements on quantum phenomena (see the Appendix). The process of elicitation of students' conceptions, and the construction of statements is reported elsewhere (Mashhadi, 1995, 1996).

Multidimensional Scaling can be used to determine if there are any underlying structure or 'dimensions' to students' responses to the statements. The principal factors generated by Principal Components Analysis should correspond to the dimensions generated by Multidimensional Scaling. Cluster Analysis can be used to further define and help interpret any groupings. All three methods are used, as confidence in the results is enhanced if different techniques give similar results.

5 Interpretation

5.1 Underlying dimensions of thinking

The responses by the students were entered into an EXCEL spreadsheet, and the data converted into a proximity matrix. Since the grouping of statements is being investigated, not the grouping of students, the statements are treated as variables, and not the respondents. The Multidimensional Scaling program, ALSCAL, represents the structure in a proximity matrix by a geometrical model. A 3-dimensional solution or model is chosen through considerations of `goodness-of-fit', parsimony and interpretability of the dimensions generated. The dimensions are orthogonal, and their interpretation can be considered independently of each other.

Figure 1 describes the location of statements on quantum phenomena located in the multi-dimensional space generated by MDS, and provides a plot of Dimension 2 versus Dimension 1.

Figure 1: Location of statements on quantum phenomena in 3-dimensional space (Dimension 2 versus Dimension 1)

The greater distribution of statements along the horizontal Dimension 1 clearly indicates that its influence is greater than the vertical Dimension 2. Successive dimensions account for a smaller proportion of the variance. Overall Dimension 1 is the most influential, then Dimension 2, and Dimension 3 is the weakest.

For the horizontal Dimension 1 the statements at one end of the

dimension refer to the definite nature or behaviour of entities (e.g. light is always a wave [B12], electrons are fixed in their shells [B45], and the electron is always a particle [B08])2. At the opposite end of Dimension 1 the statements emphasise the indefinite nature of entities (e.g. labelling an electron as a particle or a wave depends on the nature of the experiment [B35], and electron clouds provide a probabilistic picture [B15])3. Dimension 1 is, therefore, interpreted as referring to the Definite to the Indefinite nature of entities.

For the 'weaker' vertical Dimension 2 the statements at one end of the dimension indicate a certainty in knowledge about the nature of an entity or certainty about a property or behaviour (e.g. electrons have a definite trajectory [B40] or the photon is a spherical entity [B51])4. The statements at the other end refer to uncertainty in knowledge - for instance, the position of an electron is not known accurately because of its high speed (B27) and the nature of light depends on the experiment (B20)[5]. Dimension 2 is, therefore, interpreted as ranging from Certainty to Uncertainty in knowledge about the nature of entities or their property and behaviour.

The Multidimensional Scaling program also generated a plot of Dimension 3 versus Dimension 1 (Figure 2).

Figure 2 Location of statements on quantum phenomena in 3-dimensional space (Dimension 3 v Dimension 1)

From Figure 2 the statements at one end of Dimension 3 are concerned with the visualisability of entities or their behaviour (e.g. an image of the electron [B02] and the planetary model of the atom [B01 and B31])[6]. The statements at the other end propose that an atom cannot be visualised (B10) or that electrons are waves (B18) (i.e. refer to non-visualisability)[7]. Dimension 3 is interpreted as ranging from Visualisability to Non-visualisability of behaviour and of entities.

The implication of the tentative interpretation of the model generated by ALSCAL is that the location of the statements is 'determined' by three latent dimensions: Definite to Indefinite nature of entity, Certainty to Uncertainty in knowledge of the nature of an entity or of its behaviour, and Visualisability to Non-visualisability of behaviour or of entities.

The data was also subjected to a Principal Components Analysis. The three dominant factors that account for the largest percentage of variance in students' responses to the statements were consistent with the interpretations of the three principal dimensions identified in the MDS model. Principal Components Analysis therefore supports the interpretation of the underlying dimensions identified using MDS.

The results indicate that there is an underlying structure to the responses given by the students, determined by three underlying dimensions: Definite-Indefinite, Certainty-Uncertainty, and Visualisable-Non-visualisable;

The dimensions constitute perceptual axes which are implicitly referred to by students in thinking about the behaviour or properties of quantum entities. For instance, in considering the question of how to come to terms with the concept of the electron or photon a number of questions are possibly implicitly posed by students. Does it have a definite or fixed nature? How certain is knowledge about its behaviour? Is it visualisable?

5.2 Clusters of ideas

Cluster Analysis indicated the groupings of statements or conceptions. The Cluster Analysis using the Complete Linkage method produced a dendogram showing how the statements cluster or group together (see Figure 3). Inspection of the dendogram suggested three broad groupings of statements.

Figure 3: Clusters of statements on quantum phenomena

Inspection of the clusters suggests that they are interpretable and internally consistent and coherent. Cluster 1 consists of the following statements:

B15 Electron clouds provide a probabilistic picture of the likelihood of finding an electron at a particular point.
B17 The photon is a sort of 'energy particle'.
B43 It is not possible to continuously observe the motion of an electron.
B20 How one thinks of the nature of light depends on the experiment being carried out.
B35 Whether one labels an electron a 'particle' or a 'wave' depends on the particular experiment being carried out.
B27 Nobody knows the position accurately of an electron in orbit around the nucleus because it is very small, and moves very fast.
The statements comprising Cluster 1, for instance, describe the 'quantum' behaviour of phenomena - the cluster is therefore labelled as Quantum.

Cluster 3 consists of two sub-clusters which comprise the following statements:

B18 Electrons are waves.
B37 If a container has a few gas molecules in it, and we know their instantaneous positions and velocities then we can use Newtonian mechanics to predict exactly how they will behave as time goes by.
B02 It is possible to have a visual 'image' of an electron.
B13 In passing through a gap electrons continue to move along straight line paths.
B51 The photon is a small, spherical entity.
B03 The energy of an atom can have any value.
B10 An atom cannot be visualised.

B08 The electron is always a particle.
B45 Electrons are fixed in their shells.
B12 Light energy always behaves as a wave.

Cluster 3 consists of two sub-clusters in both of which the statements describe quantum phenomena in 'mechanistic' terms , and is therefore labelled a Mechanistic cluster.

Cluster 2 consists of two sub-clusters which comprise the following statements:

B04 The atom is stable due to a 'balance' between an attractive electric force and the movement of the electron.
B46 Light energy travels from a lamp to a zinc plate as a wave but is absorbed as a packet of energy or photon.
B32 When a beam of electrons produces a diffraction pattern, it is because the electrons themselves are undergoing constructive and destructive interference.
B21 Electrons move along wave orbits around the nucleus.
B23 The photon is a 'lump' of energy that is transferred to or from the electromagnetic field.
B24 Electrons consist of smeared charge clouds which surround the nucleus.
B07 Coulomb's law, electromagnetism, and Newtonian mechanics cannot explain why atoms are stable.
B19 When an electron 'jumps' from a high orbital to a lower orbital, emitting a photon, the electron is not anywhere in between the two orbitals.
B29 Since electrons are identical it is not possible to distinguish between them.
B33 Electrons move randomly around the nucleus within a certain region or at a certain distance.
B47 Orbits of electrons are not exactly determined.
B39 A photon has no mass or charge.

B01 The structure of the atom is similar to the way planets orbit the Sun.
B31 Electrons move around the nucleus in (definite) orbits with a high velocity.
B28 It is possible for a single photon to constructively and destructively interfere with itself.
B42 Individual electrons are fired towards a very narrow slit. On the other side is a photographic plate. What happens is that the electrons strike the plate one by one, and gradually build up a diffraction pattern.
B40 During the emission of light from atoms, the electrons follow a definite path as they move from one energy level to another.

Cluster 2 consists of two sub-clusters which combine both 'quantum' and 'mechanistic' descriptions of phenomena (i.e. an Intermediate cluster).

The statements are located within these three perceptual dimensions or group psychological space, and grouped in three broad clusters: Mechanistic, Intermediate, and Quantum.

Figure 4: Dimensions and clusters for statements on quantum phenomena

6 Discussion of findings

A number of points arise from the project.

6.1 Summary of findings

The following conceptual map or three dimensional 'epistemological' space summarises the underlying dimensions and clusters for statements on quantum phenomena:

In responding to propositions concerning a range of phenomena students' responses were modelled by a reference frame of three dimensions: Definite versus Indefinite, Certainty versus Uncertainty, and Visualisable versus Non-visualisable. The dimensions may constitute reference axes which are implicitly referred to by students in thinking about the behaviour or properties of quantum entities. For instance, in considering the question of how to come to terms with the concept of the electron a number of questions are possibly implicitly posed by students. Does it have a definite or fixed nature? Is it visualisable? How certain is knowledge about its behaviour?

Within these three perceptual dimensions or group psychological space there are located three broad clusters, reflecting Mechanistic, Intermediate and Quantum viewpoints. The Mechanistic cluster refers to properties as being of a definite nature (e.g. electrons as being fixed in their shells, light always behaving as a wave, or photons as a spherical entity). The Quantum cluster consists of statements describing quantum properties or behaviour of entities (e.g. electron clouds provide a probabilistic picture, and that it is possible to have more than one description of the same thing). The Intermediate cluster consists of statements that represent a range or mix of 'mechanistic' and 'quantum' ideas (e.g. light being described as a wave involves the use of a model or a photon has no mass or charge).

6.2 Scale
Most of the previous studies of students' conceptions have used the clinical interview with individual students or questionnaires with a small number of students. If the analogy of a fisherman's net is used, the mesh size determines the size of fish caught.That type of methodology leads to insights into the microscopic, and as a consequence macroscopic patterns are not discerned. Microscopic descriptions, and macroscopic patterns are not, of course, contradictory but complementary. This study is designed to provide an additional level of insight to that obtained in previous studies in this area.

6.3 Levels of description
This study has provided a description of the common features of the underlying thinking of a group of A-level students concerning quantum behaviour. Eventually this description needs to be linked to that of the description of the underlying thinking of just one student. There is, of course, no reason to suppose that the descriptions will be identical. In fact, if anything it might be expected that descriptions of patterns in the underlying reasoning of a group would be different from that of the individual student, since the former is concerned with the macroscopic and the latter is concerned with the microscopic.

6.4 Conceptual maps
An operational definition or limited `measure' of understanding, at the level of the population group, was adopted in which understanding was represented by the relationships or groupings of ideas (conceptions). This gave rise to the use of the powerful heuristic device of adopting the 'metaphor of the map' to construct 'conceptual maps' to represent the holistic understanding of A-level Physics students, at the level of the population group, of concepts associated with quantum physics. Conceptual maps have been constructed of territory that has had few previous explorers.

The maps produced are approximations and involve distortions of perceived reality, as they inherently involve the use of a projection which constitutes a systematic reference frame (i.e. orthogonal dimensions). The use of a projection is necessary to communicate effectively. Distortion is in fact not only unavoidable, but necessary to allow the map reader to comprehend the meaning of the map. The map's intended intellectual function and the desired visual structure were used to determine which projection was most appropriate. The maps are also a reflection of both the data and the researcher's interpretation of the data. The conceptual maps generated have great cognitive, integrative, summative, and generative power.

6.5 Visualisation and change
At the heart of the notion of successful teaching and learning is the

idea of progression or at least change. The methodology developed enables a visual 'snap-shot' to be obtained for a group of students about their ideas concerning quantum phenomena. A visual picture provides a powerful method for the holistic representation of knowledge. In this case the visual snap-shot or conceptual map consists of latent dimensions and clusters of conceptions. A succession of snap-shots enables changes in the conceptual maps of either the same group or different groups to be discerned.

6.6 Dimensions and complexity

In order to gain an insight into phenomena the physicist often copes with the complexity of the situation by using the technique of orthogonality or mutual independence. For instance, in considering the forces experienced by charges moving in magnetic fields or projectile motion vector analysis is used to investigate separately horizontal and vertical components of force or motion. In an analogous manner in order to gain an insight into students' thinking Multidimensional Scaling generated orthogonal dimensions. Each dimension could be considered independently of the other dimensions, and thereby reduce the complexity of the situation.

The number of dimensions chosen was guided by statistical measures which indicated the 'goodness-of-fit' in order to obtain the best compromise between fit, parsimony and interpretability.

In responding to the large number of propositions, present in the final research instrument, describing a range of possible behaviours or phenomena a quite startling finding is the small number of underlying dimensions needed to model students' perceptions of quantum phenomena (e.g. Definite versus Indefinite, Certainty versus Uncertainty, Visualisable versus Non-visualisable etc.). This tends to indicate an unexpected level of simplicity in trying to come to terms with what is normally regarded as an incredibly complex phenomena. The implication is not necessarily that student thinking is imprisoned by these dimensions. However the dimensions may constitute reference axes which are implicitly referred to by students. For instance, in considering the question of how to come to terms with the concept of the electron a number of questions are possibly implicitly posed by students. Does it have a definite or fixed nature? is it visualisable? How certain is knowledge about its behaviour?

6.7 Dimensions and their interpretation

The label assigned to a dimension (e.g. Definite versus Indefinite) is the result of the process of interpretation by the researcher. With any set of data, in this case a series of statements, there are theoretically a number of possible interpretations. All studies, whether employing qualitative or quantitative methodologies, are inherently interpretive. The epistemological question arises, therefore, on what basis was one particular interpretation put forward? The principle of 'Occam's Razor' was used (i.e. the simplest of a number of possible explanations for the same facts is taken as 'true'). However Occam's Razor is still trapped with the hermeneutic circle -

the interpretation chosen is dependent on the researcher's conceptual framework. To address this issue the process of interpretation was made

as transparent as possible by making the data available for peer review in order to establish the validity of the interpretation proposed.

6.8 Range of statements

It could be argued that the range of statements presented to students to respond to determines what dimensions might emerge. Furthermore that additional important dimensions may have been overlooked or distorted. It was precisely for this reason that the statements were informed by the research literature but were primarily developed from an analysis of students' responses to open questions in a series of studies. The studies used open questions with reasonably large samples of students in order to obtain, at the end of a process of interpretative analysis, as wide a range of conceptions as possible.

6.9 Robustness of dimensions

In addition even though, as a result of the pilot study, the number of statements were reduced for the Main Study the same latent dimensions were obtained. For two reasonably large samples of students the same dimensions were identified, and are therefore reasonably robust.

6.10 Dualistic thinking

The most obvious aspect of the dimensions is their dualistic nature (e.g. Definite versus Indefinite). The model generated by Multidimensional Scaling reflects the pattern of students' responses by the spatial distribution of statements. The fact that the location of the statements leads to a dualistic interpretation of the dimensions emphasises again the primacy of the metaphor of dualism in both science and society. Phenomena are being made sense of in terms of either/or, in terms of polarities. Either-or categories are presumably constructed not simply for convenience, but because by defining one pole as the negation of the other, it is asserted not only what is, but what it is not (i.e. A is defined by being not-B). Conceptions arise from the interpretation of perceptions. In other words the interpretation results from the unconsciously utilised conceptual grid that is being applied. The conceptual maps generated are inherently dualistic. The problem for trying to move from a classical to a quantum framework is that quantum physics is not necessarily inherently dualistic.

6.11 Description at the level of the group

The description of the common features of the implicit thinking of A-level students has been carried out at the level of the group. It does not follow that each individual student clusters conceptions or has exactly the same dimensions as the group. If the analogy of a fluid is used the macroscopic description of the movement of the fluid will not be reflected by the microscopic motion of a particular molecule. The group conceptual maps constructed, however, can provide an insight into the possible thinking of an individual student.

6.12 Methodology

This project has utilised a quantitative methodology to provide a qualitative insight into students' understanding of complex phenomena. The study has abstracted from the data a hidden structure that results from some basic typology (using Cluster Analysis), and latent dimensions (using Multidimensional Scaling complemented by Principal Components Analysis). It should be pointed out that although quantitative methods were employed the aim was not to arrive at or build quantitative laws.

A `conventional' viewpoint might regard the use of multivariate statistical techniques as inadequate for eliciting any deep or significant aspects of thought, especially as the statements utilised a fixed (five-point) response scale. Answers to statements might seem to need more subtlety of response. However the way students reason is not easily available to them by reflection. The use of a five point response scale provided students with a means of expressing more subtlety (and uncertainty) in their responses than simple 'yes' or 'no' answers. The study indicates that regularities and similarities in forms of reasoning shared by groups of individuals are not too much affected by individual differences. Building on previous work a methodology has been further developed that is able to detect structures or relationships between ideas and enable comparisons to be carried out of such structures between groups, without knowing in advance whether the structures are comparable at all.

7 Conclusion

A number of insights have been gained into a complex and entangled situation without sacrificing completely the complexity and richness of students' thinking. The construction of conceptual maps enabled what is ultimately a reductionist approach to present an holistic picture.

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9 Appendix: Statements on quantum phenomena

Statements on quantum phenomena

AtomB01 The structure of the atom is similar to the way planets orbit the Sun.

B04 The atom is stable due to a 'balance' between an attractive electric force and the movement of the electron.

B07 Coulomb's law, electromagnetism, and Newtonian mechanics cannot explain why atoms are stable.

B10 An atom cannot be visualised.

Photon

B12 Light energy always behaves as a wave.

B03 The energy of an atom can have any value.

B51 The photon is a small, spherical entity.

B46 Light energy travels from a lamp to a zinc plate as a wave but is absorbed as a packet of energy or photon.

B17 The photon is a sort of 'energy particle'.

B39 A photon has no mass or charge.

B20 How one thinks of the nature of light depends on the experiment being carried out.

B23 The photon is a 'lump' of energy that is transferred to or from the electromagnetic field.

B28 It is possible for a single photon to constructively and destructively interfere with itself.

ElectronB08 The electron is always a particle.

B13 In passing through a gap electrons continue to move along straight line paths.

B02 It is possible to have a visual 'image' of an electron.

B18 Electrons are waves.

B24 Electrons consist of smeared charge clouds which surround the nucleus.

B32 When a beam of electrons produces a diffraction pattern, it is because the electrons themselves are undergoing constructive and destructive interference.

B35 Whether one labels an electron a 'particle' or a 'wave' depends on the particular experiment being carried out.

B42 Individual electrons are fired towards a very narrow slit. On the other side is a photographic plate. What happens is that the electrons strike the plate one by one, and gradually build up a diffraction pattern.

B29 Since electrons are identical it is not possible to distinguish between them.

Trajectory/ probabilityB37 If a container has a few gas molecules in it, and we know their instantaneous positions and velocities then we

can use Newtonian mechanics to predict exactly how they will behave as time goes by.
B31 Electrons move around the nucleus in (definite) orbits with a high velocity.
B27 Nobody knows the position accurately of an electron in orbit around the nucleus because it is very small, and moves very fast.
B40 During the emission of light from atoms, the electrons follow a definite path as they move from one energy level to another.

B45 Electrons are fixed in their shells.
B47 Orbits of electrons are not exactly determined.
B33 Electrons move randomly around the nucleus within a certain region or at a certain distance.
B21 Electrons move along wave orbits around the nucleus.
B43 It is not possible to continuously observe the motion of an electron.
B15 Electron clouds provide a probabilistic picture of the likelihood of finding an electron at a particular point.
B19 When an electron 'jumps' from a high orbital to a lower orbital, emitting a photon, the electron is not anywhere in between the two orbitals.

1 All correspondence should be addressed to this author.
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