THE "SEA" MODEL FOR ASSESSMENT IN MATHEMATICS

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Abstract
Using the framework of the National Council of Teachers of Mathematics (NCTM) reform documents (NCTM, 1989, 1991, 1995), I propose a theoretical multi-dimensional assessment model (the "SEA" model) that should help expand the scope of assessment in mathematics. This model uses a context based on mathematical reasoning and has components that comprise mathematical concepts, mathematical procedures, mathematical communication, mathematical problem solving, and mathematical disposition.

Introduction
Assessment in mathematics has been going through several changes in this decade, especially after the publication of the reform documents of the NCTM (NCTM, 1989, 1991, 1995). Whereas assessment in mathematics used to emphasise status performance, the current trend is to incorporate growth assessment into our assessment practices (Lambdin, Kehle, & Preston, 1996; Romberg, 1992; Stenmark, 1991, 1989). The apparent shift in what must be assessed and how it must be assessed stems from the current belief that the central goal of teaching and learning mathematics should be to foster the development of mathematical power among students so as to make these students mathematically literate (NCTM, 1989; Webb & Romberg, 1992; Zarinnia &
In fact, Zarinnia and Romberg (1992) differentiate between a definition of mathematical power as "the intrinsic power of mathematics" (p. 261) and the perception of mathematical power as "individuals and societies empowered by mathematics" (p. 261). It is the latter, the perception of mathematical power as individuals and societies empowered by mathematics, that is the focus of this model. Accordingly, mathematical power must be seen to involve "the ability to discern mathematical relationships, reason logically, and use mathematical techniques effectively." (Zarinnia & Romberg, 1992, p. 260). That is, students must be seen to be tinkering with mathematics. In other words, mathematical power refers to "all aspects of mathematical knowledge and their integration" (NCTM, 1989, p. 205).

Although several efforts are being made to gather information that should provide evidence of students' mathematical power, there is no model developed within the framework of the NCTM documents that would consistently and systematically guide and promote the expansion of the scope of assessment in mathematics. We need such a model within the mathematics education community to help sustain our efforts towards the realisation of the visions within the reform documents. It is the need to expand the scope of assessment in mathematics that prompted the development of the "SEA" model. It is a theoretical multi-dimensional model that uses a context based on mathematical reasoning and has dimensions (components) that comprise mathematical concepts, mathematical procedures, mathematical communication, mathematical problem solving, and mathematical disposition. Notice that I am not calling for the assessment of either status or growth of mathematical knowledge, but I am arguing for both, using the "SEA" model. It is a truism that what is valued is what gets assessed and that what gets assessed provides an indication of what is valued (see Wilson, 1994). The problem with this position is that students might limit the scope of what they learn to only what they perceive as assessable by teachers. One way to overcome this problem is to expand the scope of assessment and make it integral to instruction. In the pages that follow, I describe a framework for the model, the model and some of its features, its relevance to classroom assessment in mathematics in Singapore and elsewhere, and some implications of the model for classroom assessment in mathematics. Let me state here that my use of classroom assessment in this article is not limited to assessment done only indoors, but also to assessment done outdoors.

Model Framework
There are seven focus areas of student assessment listed in the Curriculum and Evaluation Standards for School Mathematics. These are mathematical reasoning, mathematical problem solving, mathematical communication, mathematical concepts, mathematical procedures, mathematical disposition, and mathematical power. (For details of what constitute these student assessment standards, read NCTM, 1989, p. 205
Although the NCTM places equal emphasis on all seven focus areas of student assessment, it recognizes the need for productive changes in the curriculum and evaluation standards (see NCTM, 1989, p. 189).

The NCTM definition of students' mathematical power encompasses the definitions of what constitutes the other standards of student assessment and their integration (see NCTM, 1989, p. 205).

Furthermore, it is argued in the same document that mathematics is reasoning, and for students to become autonomous in doing mathematics, they need to "gain confidence in their ability to reason and justify their thinking" (p. 29). Kline (cited in Clements & Ellerton, 1991) shares a similar view on the role of reasoning in gaining knowledge when he argues that although authority, revelation, experience, and experimentation are important sources of gaining knowledge, the major method is reasoning. It follows then that if the goal of mathematics education is to develop mathematical power, then mathematical reasoning should be the medium through which students develop that power.

Consequently, mathematical reasoning should also provide the medium through which evidence of students' demonstration of mathematical power can be gathered. So, using mathematical reasoning as a medium, evidence of mathematical power can be provided by the remaining five standards of student assessment, namely mathematical communication, mathematical concepts, mathematical procedures, mathematical problem solving, and mathematical disposition.

The Assessment Standards for School Mathematics document provides "criteria for judging the quality of mathematics assessments" (p. 9). These criteria address the six major issues of mathematics, learning, equity, openness, inferences, and coherence. While the mathematics criterion involves assessment that reflects mathematics that all students should know and be able to do, the learning criterion involves assessment that enhances mathematics learning and becomes part of routine classroom activity. The criterion for equity in assessment focuses on providing each student the opportunity to demonstrate individual mathematical power while the criterion for openness focuses on providing information (from the assessment process) to all those who need it. Finally, while the inferences criterion involves procedures for making valid inferences from evidence of a student's learning, the coherence criterion involves ensuring that the assessment phases and purposes fit together and that the assessment practices are aligned with the curriculum and with instruction.

I refer to these criteria as external to students but in-built into the model. They are external to students because they are expectations of someone else (not students); they are things to be done by someone else (the teacher, for example) to ensure that the assessment of students is appropriate. They are in-built into the model because they should necessarily form part of the model if the assessment should be appropriate. Notice that it is the responsibility of the user of the model to ensure that these criteria are satisfied. Meanwhile, the
model components are considered internal to students because students are to provide evidence of these components and their integration to demonstrate mathematical power. So, with the model, student assessment in mathematics should be such that, through mathematical reasoning, students provide a comprehensive evidence reflecting and integrating mathematical communication, mathematical concepts, mathematical procedures, mathematical problem solving, and mathematical disposition.

The Model

Figure 1. The "SEA" Model

In the diagram, C stands for mathematical communication, MC stands for mathematical concepts, MP stands for mathematical procedures, PS stands for mathematical problem solving, and MD stands for mathematical disposition. Each of the five lines provides a link between mathematical reasoning and each of the five components of the model. The overlap of the five dimensions should be viewed as their integration. The five dimensions and their integration represent mathematical power.

Model Features
In this section of the paper, I describe some of the features of the model. Also, I argue for the model's flexibility in encompassing some of the most pressing utility issues of assessment in mathematics. Notice that issues of reliability and validity are not addressed by the model. These are issues for the various techniques and instruments used to gather information.

Multi-dimensionality
The "SEA" model's five components, although having some overlaps, provide dimensions of mathematical learning that can be focused on and assessed. The many assessable dimensions therefore make it a multi-dimensional model.

Use of Multiple Sources
The different dimensions of mathematical learning permit the use of multiple prevailing data-gathering multiple techniques and instruments to provide evidence of students' demonstration of mathematical power. For example, students can be assessed while performing a mathematical task or investigating a mathematical problem. They could be observed while on task, they could be listened to, interviewed, asked to write a report on their investigation, or self-assess themselves using journal entries. In addition, students could be assessed using standardised testing techniques.
Integrating with Instruction
The model is to be perceived as dynamic and providing information from multiple sources. Since the model is to help expand the scope of assessment, information from such multiple sources (and on different learning dimensions) should be useful for planning subsequent instruction.

Contextual Significance
If we want evidence of students' ability to, for example, solve mathematical problems, then they must be assessed within the context of mathematical problem solving. The model dimensions provide such contexts. For example, students having the ability to use mathematics to solve problems should provide evidence that they can:
i) formulate problems;
ii) apply a variety of strategies to solve problems;
iii) solve problems;
v) verify and interpret results;
Furthermore, evidence of integration can be gathered within the same context. For example, while students are on task solving a problem, evidence of how they are communicating mathematically, their use of mathematical concepts and procedures, and their disposition towards mathematics, can all be documented. Evidence of such integration is possible whether students are working individually or cooperatively in small groups (see Anku, 1996).

Grading and the Model
Although the model does not suggest how grading should be done, it is pertinent to state that information gathered from the model dimensions can be graded by either using already developed rubrics (see Lambdin, Kehle, & Preston, 1996; Stenmark, 1991, 1989) or by developing one's own. What is needed here is an expertise in the use of existing rubrics or in the development of new ones. As such, information gathered using the "SEA" model, even in the form of grades, can still be provided to those who need it.

Flexibility of the Model
An important feature the model should have, if it should remain useful in the future, is that it must be flexibility but versatile. Flexibility is seen in terms of the model accommodating educationally defensible changes or modifications to the constituents of its dimensions. Also, the model should be able to accommodate any educationally defensible evolving dimensions and still promote the primary role of helping to expand the scope of assessment. The model's versatility should then be seen in terms of how it can respond to such flexibility. This flexibility is possible, since as noted earlier, the NCTM recognises the need for any productive changes to be made to the student assessment standards. In fact, changes to the constituents of the model dimensions are possible if the changes are educationally
Relevance of the Model to Singapore
The current framework for mathematics teaching and learning in Singapore (Curriculum Planning Division, 1992) focuses on mathematical problem solving. Other components of the framework are concepts, skills, attitudes, metacognition, and processes. Broadly, mathematical problem-solving component of the framework is covered by the mathematical problem-solving dimension of the model while the concepts component is covered by the mathematical concepts dimension of the model. Attitudes and metacognition components of the framework are covered by the mathematical disposition dimension of the model. Finally, the skills and processes components of the framework are adequately covered by mathematical communication and mathematical procedures dimensions of the model. As such, the model is appropriate for use within Singapore to expand the scope of assessment in mathematics. I hope that similar arguments can be made for frameworks of the mathematics curriculum of other countries, thus making the use of the model universal.

Implications for Assessment in Mathematics
There are several implications for assessment in mathematics if the "SEA" model is used to guide assessment in mathematics. Three of such implications are provided below. First, instead of gathering limited information on students' mathematical competency, the "SEA" model guides teachers to collect information not only from multiple sources but also from multiple dimensions so as to make informed decisions on students' mathematical power. Second, if such comprehensive information will have to be gathered, then the information must be valued and used for decision making, especially when the decisions affect students. Third, teachers will have to be trained to use assessment techniques that will enable them gather appropriate information from the different dimensions of the model.

Concluding remarks
The proposed "SEA" model is a theoretical multi-dimensional assessment model to help expand the scope of assessment in mathematics. In view of the current goal of developing mathematical power in our students and with the realisation that it is the convergence and integration of information from multiple sources that are appropriate for making educationally defensible decisions regarding the attainment of such goal, the "SEA" model should be seen as timely. It is open to modifications if and when necessary, but for now it provides the mathematics education community with a way to ensure a consistent and systematic expansion of the scope of assessment in mathematics.

References


