Conceptual Development of Patterns and Relationships

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by

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Introduction

Recognition of patterns in visually presented stimuli poses questions related to conceptual knowledge, cognitive strategies and the nature of intelligent behaviour. In mathematics problem solving, for instance, a recent finding by Shama and Dreyfus (1994) suggests that visual and algebraic strategies are equally shared. Such findings provide the basis for determining relevant teaching and learning approaches in mathematics education (Collis, Watson & Campbell, 1994). In geographic education there is similar interest in cognitive processes. Environmental perceptions (Robertson, 1994a; Walmsley, 1988) are being studied for their influence on the development of mental constructs as diverse as language, number, spatial distribution and density. Euclidean or real world space may rely on similar internal processes. The mutual interests of these disciplines in the study of space cognition provides a rich and complementary source of research findings that can potentially serve the educational needs of both cognate areas (Bishop, 1983).

More recent enquiry in these and other related disciplines has focussed on the role of everyday learning in problem solving. There is growing acceptance that students experiencing difficulty with formal school learning can be assisted by teaching that takes account of their personal meanings and understandings as well as their stage of development (Resnick, 1992). In this project the concepts underlying the conceptual development of patterns and relationships are explored in relation to small scale and large scale space with emphasis on open-ended tasks for construct validation and further theorising.

Informal versus formal learning

The distinction is made between formal and informal learning. The former is viewed in the context of concepts and ideas that rely on strategies for thinking skills that are learned conventions and arguably exclusively within the formal domain (Biggs & Collis, 1991; Collis & Biggs, 1991; Resnick, 1992). By contrast, everyday experiences in informal settings of home, neighbourhood and social contexts with friends and family are rich sources of learning that shape the context of experience in informal and formal environments (Marton, 1988; Robertson, 1994b; Gardner, 1993).

Recognition of the importance of everyday learning is not new for educators but for psychologists the dilemma is how to translate the mental constructs of personal experience into ways that are measurable and transferable (Milgram, 1973). The shift towards research tasks that are open-ended and enable observation of processes is notable in the literature as a means of tackling the problem. Reconstruction of meanings that students attach to a problem in formal learning can provide the key to perceived misunderstandings in conceptual development and inappropriate cognitive strategies. This holistic approach is designed to generate new hypotheses for testing and evaluating. Its strength is that prior assumptions are tested regarding the nature of intelligent behaviour. And, far from viewing one general intelligence or 'g' factor as in the Spearman sense, an acceptance of multiple intelligences (Sternberg, 1983; Gardner, 1983) provides a broader more imaginative basis for the interpretation of findings.

Learning and Multiple Intelligences

Gardner (1993) relates the theory of multiple intelligences to learning through an analysis of what he describes as the general puzzles on learning (p.2). On the one hand there are three kinds of learners. First, the intuitive learner who easily absorbs symbolic systems and is competent at theorising and interpreting the material world. Second, the traditional student (academic learner) who masters school conventions but who may well respond as the intuitive learner when not in the formal environment. Third, the disciplinary expert who rates as the one with genuine understanding. The intuitive learner is described as a naive learner unlike the disciplinary expert who is able to question conventional wisdom. However, both the intuitive and disciplinary learner are able to interpret information and apply theories with apparent ease. It is this recognition that has researchers reconsidering the meaning of intuition as a means of knowing how one becomes a disciplinary learner (Campbell, Collis & Watson, in press; Clement, Brown & Zietsman, 1989).

The search may well explain the apparent inability of many 'experts' to solve problems away from formal contexts. Retreating to 'everyday intelligence' has often been observed (Marton, 1988; Resnick, 1992). Knowing more about the cognitive processing of the disciplinary learner may facilitate the development of traditional learners to their next step in higher order thinking and ultimate competence. As Gardner says: "The understandings of the disciplines represent the most important achievements of human beings" (1993, p.11). School based learning ought to promote learning to achieve this end.

Taken further, Gardner (1993) argues that an assumption of multiple intelligences or diverse ways of acquiring and processing information is a way of strengthening opportunity for understanding of the 'learning puzzles'. 'Gaps' between intuitive, traditional and disciplinary learners are 'bridged' by traditional systems and Gardner concludes these extend "beyond the walls of the school-house, touching on issues of human nature, human institutions, and human values" (1993, p.15).

Types of Knowledge

Biggs and Collis (1991) suggest that competent behaviour may be a measure of available knowledge. The differences between novices and experts are viewed as a function of familiarity with the appropriate knowledge and competent strategies for application in relevant settings (Chase & Chi, 1981). Knowledge can be declarative or procedural (Anderson, 1993, p.18). Procedural knowledge, like Sternberg's (1991) definition of 'tacit' knowledge, is concerned with performance whereas "declarative knowledge is factual knowledge that people can report or describe" (Anderson, 1993, p.18). The former is that which enables people to adapt to their surroundings and is not necessarily measured by traditional intelligence tests. Other ways of knowing as identified by Biggs and Collis (1991) are through intuitive knowledge and

theoretical knowledge. The latter one might suppose is akin to that which Gardner (1993) attributes to 'disciplinary' learners, thus representing thinking at the highest levels of abstraction.

In brief, assuming there are diverse ways of knowing and sources of knowing and competence, then experiences that shape everyday as well as academic intelligence need to be considered when interpreting responses to problems in learning.

Development Issues: The SOLO Taxonomy

Part of this conception of intelligence involves an understanding of human development. Piagetian theory would have provided a view that equated competence with ages and stages of development. Neo-Piagetians (Fischer, 1985, Biggs & Collis, 1982) retain the central notion of stages but provide more flexible interpretations. Illustrative is the multi-modal theory put forward by Biggs and Collis (1991). In their view there are five developmental stages expressed as modes or levels of abstraction: sensori motor, ikonic, concrete symbolic, formal and post formal. Most school learning occurs in the concrete symbolic stage with the acquisition of formal symbolic systems in language, writing and number. Formal thinking or that related to abstract theorising develops towards the end of formal school years. Post formal learning is considered a further step and that which characterises the thinking of researchers at the highest levels of abstract hypothesising. Learning progresses in cycles of competence from prestructural to unistructural, multistructural, relational and finally extended abstract levels. These hierarchical outcomes that distinguish learning within each mode, known as the SOLO Taxonomy (an acronym for structure of observed learning outcomes) identify increasing competence towards expertise within a mode. Modes accumulate or coexist thus making available a wider range of skills to the developing person.

The unlimited application of the SOLO Taxonomy makes it a valuable tool for classifying responses in children's problem solving. Sources of knowledge can be identified and judgements are possible about observed competence. As a consequence, the SOLO Taxonomy was adopted in this project as the framework for analysis of children's responses and patterns described in relation to visually presented stimuli.

Visual Stimuli: Meanings and Understandings

Recognition of patterns in visually presented materials suggests a process of meaning construction that relies, in part, on spatial cognition. Piagetian theory would advance the idea that the child's understanding reflects a developing ability to interpret space. The first stage is one of "vague awareness" (Golledge and Stimson, 1987, p.53). Next there is the stage where the "object is given an existence or location in space and time". Third there is perceptual recognition of parts of the object, and finally, " the attachment of meaning" (p.53) to the object. Longitudinal studies typically use this developmental perspective for analysis (Bishop, 1983).

Another perspective may be traced to Tolman's (1948) seminal paper on cognitive mapping titled the Cognitive maps in rats and men. In this he explains how schema or mental maps are drawn from information selectively extracted from the environment. Mental maps are seen as a way of representing the complex relationships of information in memory and, as such, have been receiving considerable attention in recent publications on teaching and learning strategies. Concentration on spatial mapping tasks is now viewed as a means for assisting learning in logical rational fields such as arithmetic and reading and one that is recognised as being largely unexplored (Holley and Dansereau, 1984).

This perspective has provided the groundwork for much of the research on real-world imagery. Such cognitive processing is the concern of geography educators who consider the ways in which children represent large scale environments (Downs & Stea, 1973; Golledge & Stimson , 1987; Walmsley, 1988). A development in this field is socio-spatial cognition (Lynch, 1973; Ley, 1983; Russell & Ward, 1982; Stokols & Shumaker, 1981; Wohlwill & Koln, 1976, Zube & Pitt, 1983) or that which takes into account the interaction of people and places as a means of understanding spatial behaviour. Socio-spatial research recognises cultural differences in experiences of the environment that help shape language concepts, attitudes and beliefs (Goodnow, 1970; Tuan 1974)) as well as mental constructs such as distance cognition (Walmsley, 1988).

Psychological space whether conceived in relation to imaging Euclidean space, rotated or real space is where meanings, manipulations and interpretations exist as mental constructs. An interest in meanings related to patterns, therefore, is best considered in terms of a range of tasks that incorporate both real world space and modelled, drawn, photographed, and filmed or computerised small scale space. Correlation analysis will enable some assessment of the medium of instruction as well as the processes involved.

In his analysis of projective space, Bishop (1983) notes that space and geometry research in mathematics education as opposed to research on number is rather slim. He suggests that mathematics educators could learn from geographic educators. In his review of the broad fields of research on mental imagery of space he notes: "Many ideas in mathematics and specifically in geometry have relationships with words used in everyday contexts, and quite clearly the colloquial meaning which a child associates with a word will affect the meaning associated with that world when it is in the geometric sense" (p.179-180). Resnick (1992) provides similar evidence with respect to tasks in arithmetic. Implicit in these observations is recognition of the importance of viewing the child in a social context whereby problem solving is partly reliant on existing constructs and learning.

As part of his analysis Bishop (1993) provides a workable understanding that could apply to visualisation of small-scale and large-scale real world space. He suggests two kinds of 'ability constructs'. One is the ability for interpreting figural information which "involves understanding the visual representations and spatial vocabulary used in geometric work, graphs, charts and diagrams of all types" (p.184). The other is the ability for visual processing or "visualisation and the translation of abstract relationships and nonfigural information into visual terms" (p.184).

Several case studies support this conclusion
In a study of first year university students in Papua New Guinea, Bishop (1983) found that tasks that related to interpreting figural information were generally poorly done. However, tasks that related to visual processing were completed with much more competence because the students "used a very visual, Gestalt-like approach" (p.194). They tried to 'see' letters and other symbols. In Australia, studies of Aboriginal people have shown a strong link between spatial relationships and the environment, and a slower development of "logico-mathematical operations" when compared with nonaboriginal Australians (Ross, 1984). Similarly culturally different approaches to

problem solving are illustrated in Posner's (1982) study with two contrasting West African tribes, the Dioule and the Baoule. While the Dioule people are mostly subsistence farmers, many of them are involved in trading and therefore have knowledge of monetary transactions. By

contrast, the Baoule have an almost entirely subsistence livelihood with all members of the family involved in what are primitive agricultural practices. Students aged nine and ten from each tribe were first asked to make judgements about quantities. There were no significant differences in performance on these tasks. However, when the task required decisions about equivalent (in number) but differently illustrated quantities, the Dioule successfully employed counting strategies while the Baoule appeared to use length. The latter appeared to reflect the relationship of the people to the land.

Such contrasts at the macro level could be expected but research also exists to show variations in visualising competence that seem related to general competence or academically more and less able children. Bishop (1993) reports on research that suggest the ability of the more able to integrate parts of a problem and 'see' the whole whereas the less able child tends to be much more concrete in thinking and less able to differentiate. The field for the latter tends to be static or two dimensional rather than fluid and more like the visualising ability observed in cross-cultural studies.

An analogy exists with the cognitive style of the global versus sequential or analytical learner (Kirby, 1988; Miller, 1987). Such preferences in learning may considerably complicate the learning process for the child if the teaching strategies do not reflect their skilled preferences. A preference for visual processing may require trained thinking skills that enable a shift from holistic to differentiated thinking.

In brief, visualising in the spatial domain suggests a competence that can enhance learning and problem solving across diverse knowledge bases. Cross cultural studies and studies that focus on individual differences in learning provide insights for mathematics and geography educators. Here again is the chance for complementary research in these fields. As Bishop (1983) observes geography educators have been interested for some time in the issue of moving from a holistic view of space to a fully articulated and differentiated interpretation (Hart and Moore, 1973). Mathematics and geographic educators appear to have similar concerns when considering visualisation processes. As a consequence, their experiences ought to be viewed as a strength to pursue for developing better teaching and learning approaches. A more varied approach to the selection of research materials that takes into account small scale and real-world space may provide correlates that assist eductors in mathematics education, geography education and potentially other disciplines. The assumption is that people process visual arrays in ways that reflect prior knowledge and learning.

Ways to proceed

This introductory sketch has drawn on current research in educational psychology, developmental psychology, mathematics and geographic education. The conceptual framework is summarised in Figure 1. The central concern that synthesises this review is an interest in children's understandings and meanings of patterns in space - both Euclidean and real. Three avenues are suggested for initial study. First, an exploratory study of children's problem solving with simple open ended, visually presented tasks, part of this to include a comparative analysis of data collected in schools located in contrasting neighbourhoods. Second, and based on findings from the first study, a longitudinal study that considers questions of developmental understanding and novice to expert growth as incorporated in the SOLO Taxonomy. Third, a study of children's understanding of patterns in real world space.

Figure 1: Conceptual framework

Some of the research questions of specific interest are as follows:

- Is there one mode which is most commonly preferred by students?
- Does the mode in which the task is presented influence the mode in which the student responds?
- Does the student work in just one mode or in a combination (e.g. ikonic mode, multi-modal, intra-modal, as in Watson and Collis (1994); - can we identify any other modes to add to this model)?
- How does conceptual development influence the problem solving strategies used by students?
- Does the mode of presentation affect the (SOLO) level of generalisation?
- How can we classify the modes of students' internal representations of the tasks ?
- Are there differences in the ways in which students process small-scale space as opposed to large-scale, real world space?
- How do time, motions and change influence visual processing of patterns and relationships?

These questions and the three steps suggested have provided the framework for ways to proceed. They form part of a project that is designed in three phases. Phase 1 is completed and the preliminary findings are reported in subsequent sections of this paper.

Phase 1: Exploratory research - Pilot study

An attempt was made in the pilot study to gather data that reflected the major issues raised in the literature review. In so far as visual processing and conceptual development of patterns and relationships were concerned research questions needed to be exploratory with the potential for data rich responses. To this end the following tasks were formulated:

Tasks

Four tasks (see Figure 2) were selected in which students were asked to express generalisations from patterns. These tasks were selected for the following reasons: they are spatial in nature, they are suitable for representation in different modes, namely 3-dimensional modelling, 2-dimensional diagram and verbal modes, and, they are typical of patterning tasks being used in schools.

Figure 2: Diagrammatic representation of Task 1-4

Task 1: Match problem

Task 2: The Step Pattern

Task 3: The Path Pattern

Task 4: The U-Shape Pattern

Three modes of presentation of these tasks were selected to be consistent with those proposed be earlier researchers and described earlier in this paper. They were:

(i) concrete modelling, in which a representation of the pattern was made from blocks or other materials, to cater for the kinaesthetic learners,

(ii) diagram, to cater for the visual learners,

(iii) word description, presented both verbally and in writing, to cater for the linguistic learners.

In order to ensure that the nature of the task was not likely to influence the student's mode of response, Tasks 1-3 were presented in a cyclic rotation of modes, as shown in Table 1. Task 4 was presented verbally throughout. Students were also asked in Task 5 to make their own pattern using any of the available materials, including blocks, squared paper, plain paper and matches. Finally, students were asked in which mode they found the actual presentation of the problem easiest and which would be their preferred mode for similar problem solving.

Table 1: Modes in which tasks were presented

Tasks: Match	Step	Path	Student order
concrete	diagram	verbal	Students 1, 4, 7, 10 etc.
diagram	verbal	concrete	Students 2, 5, 8, 11 etc.
verbal	concrete	diagram	Students 3, 6, 9, 12 etc.

Sample

The sample consisted of forty, Year 7 students, with equal numbers of males and females aged 12 and 13 years. The sample was split into two sub-samples of twenty students (10 female and 10 male) each: one was from a school in a lower middle class area, and the other from a school in a predominantly working class area. These schools will be referred to as School A and School B respectively. They were selected randomly from several classes in School A and formed one class group in School B.

Method

Data were collected in individual clinical interviews, each of approximately twenty to thirty minutes duration, using a replicated samples technique (Sawin, 1992). The items were administered separately to the two sub-samples by different people, and each person independently scored the responses received. The experimenters later conferred to ensure consistency in scoring. Observation and "Teachback" (Pask, 1976) strategies were used to monitor the students' responses and interviews were tape-recorded for later analysis. The students were advised that the purpose of the project was to learn more about how students approach patterning tasks and that they could withdraw from participation at any time.

The interviews were divided into three stages: (i) responses to tasks in given modes, (ii) selection of preferred mode to devise task, and (iii) mode of internal representation of task.

In stage (i), the students were given Tasks 1-3 consecutively. Each task was presented in a different mode, as described in Table 1. The interview technique was based on that used by Orton and Orton (1994). Students were shown the patterns and asked to represent the fifth and tenth terms in the sequence in their preferred ways. They were then asked to describe their predictions of the 100th term and a generalisation for "any term". Based on a strategy described by Shield and Swinson (1994) after completing all three items, the students were then asked which mode of representation was more interesting and easier to work in, and their reasons for this decision.

The purpose of stage (ii) was to investigate the mode of presentation which students automatically chose to use. They were asked to invent a pattern of their own, represent it for the experimenter in their chosen way, and describe it (Task 5).

The procedure for phase (iii) was adapted from that used by Thomas and Mulligan (1994) and was included to add another dimension to the data by exploring the nature of the students' internal representations of a pattern. They were given Task 4 and asked to close their eyes and visualise solutions for the 5th, 10th, 100th and "any" terms. They were then asked to describe their internal representations to the experimenter.

Results

It must be noted that the analyses presented in this section are still in progress. Statistical analyses have not yet been completed, so the trends presented here are based on conjecture and must be interpreted with caution.

Table 2 shows the overall numbers of times students used each mode of representation, irrespective of the mode in which the task was presented for Tasks 1 - 3. This refers to each student's first representation of the problem. Most switched to verbalisation as soon as they had established, or thought they had established, a pattern. At this stage of the analysis it is apparent that, on Tasks 1-3, the most frequently chosen mode of representation was concrete modelling. The use of the diagram was second, being used slightly more frequently in School A than in School B. Verbal description was given thirteen times by one group and fifteen times by the other, but analysis of the interviews suggests that this occurred when the students had already established a pattern from which they were able to work. Three students wrote down a numerical pattern.

Table 2: Modes of representation of tasks

SCHOOL A

REPRESENTATION	TASKS 1-3			OWN PATTERN		
	No. Times Mode Used			No. Times Mode Used		
MODE	BOYS	GIRLS	TOTAL	BOYS	GIRLS	TOTAL
concrete	13*	14	27	7	9	16
diagram	9	8	17	2	2	4
verbal	7	6	13	0	0	0
written/numerical	1	2	3	0	0	0

*Two of these students modelled 5th step but drew 100th

SCHOOL B

REPRESENTATION	TASKS 1-3			OWN PATTERN		
	No. Times Mode Used			No. Times Mode Used		
MODE	BOYS	GIRLS	TOTAL	BOYS	GIRLS	TOTAL
concrete	21	14	35	10	10	20
diagram	5	5	10	0	0	0
verbal	4*	11	15*	0	0	0
written/numerical	0	0	0	0	0	0

*5 girls and 1 boy switched from verbal to concrete mode after first attempt at task

It is interesting to note that, in School A, the response modes of boys and girls were fairly evenly distributed. However, in School B, there is a tendency for more boys than girls to choose concrete modelling and for more girls than boys to give verbal responses. Perceived differences in motivation of the boys and girls was offered as a possible explanation by the teacher of the School B group of students. His observations of the students' learning behaviour suggest that the boys see no real purpose for school or formal learning. In his view the boys are pessimistic of their futures as they "can't see the point of trying hard at school". By contrast the girls are still reasonably well

motivated. This is possibly reflected in their greater confidence with verbal reasoning.

When asked to develop their own patterns in Task 4 the majority of students in both schools chose to make concrete representations. A small number drew their patterns. None chose to represent them either verbally or numerically. Hence, in relation to the first research question there appeared to be considerable evidence to support a conclusion that students prefer to work in the concrete mode when processing spatial information.

The second research question was to explore whether the mode in which the task was presented affected the mode which the student selected to represent the task. Table 3 shows the response modes of students by gender who responded to tasks presented in each of the three modes, concrete, verbal and diagram.

Table 3: Modes of representation of tasks (by presentation mode)

SCHOOL A

PRESENTATION MODE	RESPONSE MODE	BOYS	GIRLS	TOTAL
concrete	concrete	4	6	10
	verbal	2	0	2
	diagram	3	2	5
	written/numerical	1	2	3
verbal	concrete	6*	5	11
	verbal	1	3	4
	diagram	3	2	5
	written/numerical	0	0	0
diagram	concrete	4	3	7
	verbal	4	3	7
	diagram	2	4	6
	written/numerical	0	0	0

*One of these students modelled the 5th but drew the 10th.

SCHOOL B

PRESENTATIONRESPONSE
MODEMODE
BOYSGIRLSTOTAL

concreteconcrete7815
verbal12 3
diagram20 2
written/numerical00 0
verbalconcrete7310
verbal26 8
diagram01 1
written/numerical00 0
diagramconcrete63 9
verbal13 4
diagram34 7
written/numerical00 0

Two patterns seem to be emerging at this stage. One is a tendency for the majority of students to use concrete modelling regardless of the mode in which the task is presented. An exception to this is the comparatively large number who preferred to respond verbally to the tasks presented in diagram form at School A, and verbally to the verbally presented tasks at School B. There may also be a slight tendency for girls to represent the pattern in the same mode in which it is presented, whereas boys are more independent of the presentation mode.

Table 4 shows the students’ expressed preferences for modes of presentation and representation of Tasks 1-3. Once again the predominant number of responses for both formats was concrete modelling. In School A, students' preferences for concrete modelled tasks was particularly evident for the girls, with the boys being more evenly divided between this and diagrammatic representation. Concrete modelling was also the most common way in which the students preferred to represent their solutions. It is notable that, in School B, it was the boys who showed a greater tendency to use this mode.

The students were asked to explain their preferences. Reasons given for preferring the concrete mode included:

The model, because you can see it. Drawing is OK it is on squared paper. The description was harder to picture in my head before it went away. (girl)

Making it is easier to see (girl)

It’s more fun working with shapes (girl).

You can see if you forget the description or can’t do it (boy).

Table 4: Students' expressed preferences

SCHOOL A
PREFERRED MODEPRESENTATIONREPRESENTATION
BOYSGIRLSTOTALBOYSGIRLSTOTAL

concrete4812459
diagram40 4325
verbal12 3314
no preference10 11**1*1
*One of these students found it easiest to represent in the same way as the task was presented.
**This student’s preference depends on what the pattern is and how complicated it is.

SCHOOL B
PREFERRED MODEPRESENTATIONREPRESENTATIONBOYSGIRLSTOTALBOYSGIRLSTOTAL

concrete55108210
diagram52 722 4
verbal01 101 1
no preference01* 104* 4
*This depends on the task.

One reason for preferring drawing was that it “doesn’t go away if you think about something else”. Those who preferred to draw indicated that this was because drawing was quicker for them. A small number of students indicated that they preferred to describe the patterns because they had already established “rules” in their minds and further representation was unnecessary. One girl indicated that her preference changed according to the difficulty of the task: if she was sure of the pattern she preferred to describe it, if she was unsure she preferred to model it, and if she was reasonably confident she liked to draw it for confirmation.

Discussion, Implications for the Classroom and Further Research Questions

The analysis presented here is in response to just two of the research questions for the total project. Nevertheless, if the statistical analyses confirm the patterns suggested here, there will be a number of questions which will warrant further exploration.

The first of these questions will be to explore whether the predominant preference for concrete modelling is a phase in the student's development or whether it is a preferred learning style which does not change over time. It will be necessary to collect longitudinal and cross-grade data to explore this issue.

A second question, which will have direct classroom implications, is whether there is any mis-match between the way children choose to represent their thinking and the way in which material is delivered to them in class. At this stage, our conjecture is that much material is delivered in diagram form, which may not be consistent with the mode most suitable to enhance the students' learning.

Since there is some suggestion not only of gender differences, but also of between-school differences, it is important to attempt to account for these differences. Further studies will be needed to explore the differential impact of cultural backgrounds and neighbourhood effects on these outcomes. In this respect the data are encouraging for the subsequent phases proposed for this research project.

Summary

This paper has presented a conceptual overview of a project that is designed to be completed in several phases. The aim is to explore the ways in which students process visual arrays through presentations in varied modes including small scale or Euclidean space and large scale or real world space. Such knowledge should contribute to current understandings of students' learning in mathematics and geographic education. Influences such as formal and informal learning, or classroom and out of class learning, are being explored in relation to the preferred modes of problem solving along with the quality of the observed responses. Phase 1 the exploratory study is complete. Preliminary analysis of the data suggests the possibility of gender differences and neighbourhood differences in actual and preferred modes of processing information. Competence with patterning in visual-spatial problem solving is complex. By exploring students' meanings with open-ended tasks there is some chance of reconstructing their understandings. More appropriate teaching materials may be a positive outcome.

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