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EXPERIMENTAL STUDIES ON CONCRETE ANALOGUES FOR ALGEBRA

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Three experimental studies have been undertaken to contribute to ongoing research on mental processes in early algebra. A major objective was to collect data about teaching methods which differed mainly in terms of the use or non-use of selected concrete analogues. These data were to be collected in the field in a controlled experimental environment and analysed in terms of current theories about the place of concrete analogues in the learning of mathematics. In each study, a comparison is made of two teaching approaches for some aspect of Junior Secondary algebra. The intervention teaching stage includes the use of concrete analogues for one class, while another class follows a parallel concept sequence without the analogues. Both classes are taught by their regular mathematics teacher and both are in the same Year level.
In November 1993, two Year 7 mixed ability coeducational classes in School A were each taught with the aim of developing an understanding of algebraic generalisations which included the distributive law. One class used arithmetic examples leading to generalisations. The other used an objects-and-containers model to assist. Significantly better gains were recorded by the latter class on attitudes and content-specific achievement. No significant differences were detected before the four periods of teaching intervention. A delayed posttest showed that the advantages persisted, although not at a significant level.
The August/September 1994 study in School B examined the use of unit cubes for representing exponential growth, leading to an exploration of properties of indices. Before the Pretest, the class using the concrete analogues was significantly better on attitude and performance measures than the class following a more traditional approach. After the intervention teaching the advantages were generally maintained.
The September 1994 study in School C investigated the effect of using an area approach for finding the factors of first and second degree algebraic expressions. In this case the class following the concrete approach was significantly weaker than the other class at the time of the Pretest. Significant improvements were recorded for both classes, despite some difficulties with the area model.

This paper presents a short report on an ongoing research project. Some of the data was obtained only a few days before this conference. The author invites comment from colleagues. The project is attempting to throw light on the elusive goal of understanding mental processes used by students when they first meet basic principles in the algebra

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of generalized arithmetic. One aspect of this search is to try to tease out the influence(s), whether for benefit or hindrance, of appropriate concrete analogues when they are used efficiently as instructive aids in early algebra. The project is pursuing research directions flagged in Quinlan's doctoral study (1992, pp. 348 - 350). It takes up the recommendation that research is needed on "critical appraisal of the use of manipulatives in current mathematics curriculum documents" (Perry & Howard, 1994, p. 493). The concrete analogues investigated are all recommended in the current N.S.W. Years 7 and 8 Mathematics Syllabus. Data were accumulated to contribute to further study of the qualities of profitable analogues, the influences of concrete approaches on affective factors in learning, and possible interactions between mental capacities and instructional style.

The researcher was well-aware that research focused on comparing teaching approaches or assessing the influence of intervention teaching does not always produce a significant result.

Brophy and Good (1986, p. 329) said of projects prior to the 1970's that ‘there has been remarkably little systematic research linking teacher behavior to student achievement.’ In the 1970's, the well-resourced projects Developing Mathematical Processes and Individually Guided Education (Romberg, 1977) produced the outcome that ‘little evidence is available to substantiate the importance of teacher actions’, according to Romberg and Collis (1987, p. 17). These researchers identified the importance of including observations of teacher actions, pupil actions, and teacher-pupil interactions for productive research. Investigators are challenged not only by these aspects but also by the need to balance characteristics of schools, teachers, classroom groups, and individual students when comparing teaching approaches.
(Quinlan, 1992, p. 16)

However, encouragement came from the fact that, despite the difficulties, successful research in the area has been documented. Brophy and Good (1986) summarized examples of progress made since 1970 in Process-Product Research as well as Correlational and Experimental Studies. Sweller has been a co-author of several papers (cf. Owen & Sweller, 1985; Ward & Sweller, 1990) reporting a variety of recent research projects which identified significant effects of teaching strategies. Presmeg (1986; 1991) documents the interaction between visual learners and teaching styles which varied according to the degree of visualization employed.

Sowell (1989) examined 60 research studies on the effects of manipulative materials in mathematics instruction during the 1960s and 1970s. Her comparison of the concrete versus abstract instructional condition for effects on achievement showed that "when treatments lasted a school year or longer, the result was significant in favor of the concrete instructional condition. Treatments of shorter duration did not produce statistically significant results" (p. 502).

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Furthermore, when instructional conditions were randomly assigned, "attitude measures were significant in favor of the concrete instructional condition" (p. 502). Some research reports in the area clearly admit that little control was exercised by the researchers over the ways concrete analogues were used by the teachers concerned (e.g., Hart & Sinkinson, 1988; Atweh, Boulton-Lewis & Cooper, 1994). The experimental design described in this paper included a close working relationship between the researcher and the three teachers involved, so that the lessons were pre-planned in consultation and the students' worksheets and materials were supplied by the researcher.

Procedure

Central to the structure of the research stage described herewith was making use of an experimental design for three topics in early algebra. In each case, the regular class teacher used different methods to teach two classes in the same school grade. Similar teaching approaches were maintained in each pair of classes. For instance, group work was used in each class in the first two schools, Schools A and B, while individual student work was characteristic of the class work in School C. The researcher exercised considerable control over the teaching intervention stage by choosing the concrete analogues, preparing the worksheets for the teachers, and discussing the teaching procedures, including recommendations for the use of the analogues. The lessons were recorded on videotape and/or on audiotape. For each pair of classes, the same test was used as a Pretest and a Posttest, with four periods of teaching intervention between them. Some months later, a Delayed Posttest was administered, consisting of relevant

extracts from the earlier test. Students in Schools B and C were given a digit span test either by the class teacher or the experimenter. The digits were in three lists, each growing from 2 to 8 digits. These were assembled from information supplied to the author by Boulton-Lewis (personal correspondence, 16th November, 1994), with the addition of 7- and 8- digit lists taken by the author from a set of random numbers. The numbers were read to the students at about one per second, "the rate normally used in memory span tasks" (Dempster, 1981, p. 68).

In November 1993, one class in School A was led towards the acceptance and understanding of algebraic generalizations by the study of sets of arithmetic examples ("Traditional" Approach), whereas the other was led in a similar direction with the aid of concrete manipulatives ("Concrete" Approach). The concrete model chosen was the objects-and-containers model as described in Quinlan, Low, Sawyer, and White (1993), Unit 1 Worksheet 3. Each class used a variety of tasks to lead the students to accept and understand that simple algebraic expressions may be written in several equivalent forms, for example, $2y + 8 = 2(y + 4) = (y + 3) + (y + 5)$.

This experimental stage was reported at the MERGA Conference in Lismore in July (Quinlan, 1994, pp. 515 - 522).

The two classes involved in the study were both of mixed ability and

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both were coeducational classes. Each class had similar background experiences in algebra, having had a few lessons in algebra in April without any substantial follow-up. The worksheets were similar for the two classes in terms of some common exercises and the sequence of development. Differences centred on contrasts in teaching approaches. Each class worked in groups and the teacher kept to a problem-solving approach in each class, giving groups time to discuss ideas and exercises before directing them towards intended goals. In each class, students presented their ideas on the blackboard or at the overhead projector as the discussions progressed. There were no significant differences between these classes at the Pretest time.

The focus of the experimental study in School B was the topic of indices, a topic introduced some months earlier (without any manipulative aids). During the experimental stage in August/ September 1994, one of the teacher's Year 8 classes used "Cubit" cubes to build three-dimensional rectangular prisms to demonstrate the effects of doubling, tripling and multiplying by four, starting with one cube. Their work paralleled that of the other class. Both groups made use of three different calculator methods for obtaining exponential growth figures, they graphed the figures, and used two-sided cards folded as "tents" to record their findings, such as having 23 on one side and 8 on the other. These cards were used to help students discover relationships which led to illustrations of the index laws. For instance, they found that

8 x 4 = 32 could be written as 23 x 22 = 25 .

The class using the Concrete Approach checked such results with the cubes. This class was significantly better than the other class on the Pretest, although the teacher regarded them as of comparable ability. Most of the students were given a digit span test by either the experimenter or the class teacher. During this one-to-one interview situation, were asked about whether or not the cubes helped them learn, if they were in the class that used them.

In School C, the teacher did not use group work with either of his Year 8 classes as they studied factorisation and expansion of first and second degree expressions in September 1994. One class manipulated specially-prepared squares and rectangles to represent factors by means of an area model. They were directed to take a certain collection of shapes, write down their total area and put them together to form one rectangle. The dimensions of the latter gave the factors of the algebraic expression for the area. In this case, the class following the Traditional Approach were significantly better than their counterparts on the Pretest. They were ranked second among the graded Year 8 classes, while the class using the concrete analogues were in the fifth of the graded classes. Most of the students were given a digit span test by either the experimenter or the class teacher. During this one-to-one interview situation, those who used the area model were asked about whether or not the squares and rectangles helped them learn and whether or not they enjoyed using them.

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Copies of the worksheets used by any of the six classes may be obtained from the author.

Results

School A: Equivalent Expressions. The outcomes for this controlled experiment were classically in favour of the concrete approach used rather than the arithmetic approach to equivalent algebraic expressions. Reasons for the outcomes will be discussed later in the paper. Before the teaching intervention of four periods, the two classes showed no significant differences either in attitude to algebra or in performance on algebraic tasks. After the intervention, the class helped by the objects-and-containers model were significantly ahead both on attitude and on performance on the content-specific algebra questions dealing with equivalent expressions.
Figures 1 and 2 present graphically the relevant information.

Figure 1. Graphs of means and medians for items on the distributive law across three tests, showing the significantly faster growth rate for the Concrete group during the intervention teaching.

Figure 2. Graphs showing the relative changes in attitude to algebra during the period of the research project.
The Concrete group showed positive attitudinal growth to the teaching sessions, whereas the Traditional group became more negative.

The statistical details of t-tests and analyses of variance are given in Tables 2 and 3.

Table 2
Significant Differences Between Concrete & Arithmetic Groups on Posttest

Note. Matched, mixed ability groups. Max. = maximum possible score.
Conc. = Method B, Concrete Approach (n = 25 for Distributive Law and 23 for Attitude);
Arith. = Method A, Arithmetic Approach (n = 25 for Distributive Law and 21 for Attitude).
Numbers vary because of blanks on response sheets. Pretest results are shown in brackets.

Table 3 gives the statistics which show that analyse of variance identified the differences in teaching approaches as having a significant effect on the stronger gains made by the group using the objects-and-containers model. The grand mean is the average of the posttest results for all students in the analysis. The statistics show that the methods of teaching had a significant effect on the outcomes: The F values for method effect are significant at p = .010 and .002 respectively, and the multiple R2 values indicate that methods

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accounted for, respectively, 55.8% and 62.1% of the total variance (cf. Nie, Bent, Jenkins, Steinbrenner, & Hull, 1975, 404).

Table 3
Significant Analysis of Variance Outcomes for Posttest Scores Using

Pretest Scores as Covariates

Note. Matched, mixed ability groups. Max. = maximum possible score.
Conc. = Method B, Concrete Approach (n = 25 for Distributive Law and 20 for Attitude);
Arith. = Method A, Arithmetic Approach (n = 25 for Distributive Law and 19 for Attitude).
Numbers vary because of blanks on response sheets.

School B: Indices. At the time of the Pretest, the class following the concrete approach performed significantly better than the other class on most algebraic measures and on attitude towards algebra. Some advantage was generally maintained over the intervention teaching period and at the time of the Delayed Posttest (21st November, 1994), as the graphs of means and medians in Figures 3 and 4 record. Two students in the Traditional Approach class took up the offer of withdrawing from the video-taped intervention lessons. Outcomes recorded for these two are presented as coming from a control group.

Figure 3. Graphs showing better performance by Concrete group on Question 10
which required students to apply the concept of a variable to compare 2x with 8y.

Figure 4. Graphs showing that the Concrete group was slightly better on Question 7
in the early stages but lost their ascendency at the Delayed Posttest. The question tested the understanding of indices and three index laws.

Both Figures 3 and 4 lend some support to the view that, although the use of the unit cubes took up students' time, this concrete analogue did not overload their brain capacity and so hinder learning: For instance, both classes improved significantly on Question 7 over the teaching intervention period.
Figure 5 records results in terms of a common error, namely, multiplying the base number by the index. An instance of this misunderstanding was shown by those who wrote that 50150 was the same as 15050, considering each as the product of 50 and 150. Those who modelled the index concept using cubes were less inclined to make this error by the time of the delayed posttest than were their counterparts. This is one indication that the concrete analogue they used was beneficial in the long term.

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Figure 5. Graphs recording the sustained improvement of the Concrete group on avoiding the error of multiplying a base number by its corresponding index,
whereas the Traditional group regressed in this matter by the Delayed Posttest.

Attitude towards algebra was measured by 7 sematic differential items, each item being scored out of 4. The Concrete group retained their better attitude throughout the research period, as depicted in Figure 6. They recorded consistently better attitudes at the .05 level of significance when a one-tail t-test was applied.

Figure 6. Graphs showing that the Concrete group retained better attitudes to algebra than the other students.

The effect of the concrete analogue used was not statistically obvious, except that a significant effect level was shown for the mode of

teaching in an analysis of variance for the following question:
10.This question is about 2x and 8y.
(a)Which is larger, 2x or 8y ? WHY? ...
(b)When are they equal (if at all)? ...
(c)When is 2x larger (if at all)? ...
(d)When is 8y larger (if at all)? ...

Table 4
Significant Analysis of Variance Outcomes for Posttest Scores Using Pretest Scores as Covariates

Note. Unmatched, mixed ability groups. Max. = maximum possible score.
Conc. = Concrete Approach (n = 15);
Arith. = Arithmetic Approach (n = 18).

The analysis summarised in Table 4 shows that the difference in teaching methods did have an effect which was significant at the .05 significance level ($p = .004$). However, the variance explained was only 19.1% ($R^2 = .191$). The class using the concrete approach performed better than the other class on this question, one which tested the students' grasp of the concept of a variable in the context of indices. The overall level of performance was understandably not high: only 3 students (2 from the concrete approach group) scored more than 1 out of a possible 4; another 9 students scored 1 (7 of these from the concrete approach group). The analysis shows that the Concrete group performed better on this question at the Posttest, having taken the Pretest scores into account as covariates.

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School C: Factorisation and Expansion. The class following the concrete approach started well behind the other class both on attitude to algebra and success in factorising and expanding first and second degree algebraic expressions. However, after a painstaking introduction to the area model, they improved significantly on at least parts of several questions, as recorded, for instance, in Figures 7 and 8. They even outscored their "bright" counterparts in the Delayed Posttest (18th November, 1994) on a substitution question in part (iii) of Question 1:
1.If y = 3, what is the value of (iii)2(y + 5) ? ....

Figure 7. Graphs showing that both groups made significant gains on Question 7 during the teaching intervention stage.

Figure 8. Graphs of means and medians recording the fact that the Concrete group overtook the Traditional group on Question 1 (iii).

Generally the Concrete Approach group were outclassed by the brighter group on most of the algebra questions. However, they registered a more positive growth in attitude to algebra over the period of the project, as shown in Figure 9..

Figure 9. Graphs of attitude scores showing the better gains made by the Concrete group.

Discussion

School A: Equivalent Expressions. The group following the Traditional Approach were confronted with the difficult task of identifying a pattern from a series of arithmetic examples. They had to create their own algebraic variables. The challenge is discussed in Quinlan (1994).

The clear-cut advantages of the objects-and-containers model give cause for a close consideration of the model and how it was used. It stands up to analysis on the characteristics of scientific analogies as outlined by Gentner (1982). Quinlan and Collis (1990, pp. 445 - 448) discuss the suitability of the objects-and-containers model and show that, in the context of this project, it has the strengths of commutativity, transferability and isomorphism. It appears to have the qualities to match the principle enunciated by Boulton-Lewis and Halford (1991, p. 37):

The value of a concrete representation is that it mirrors the structure of the concept and the child should be able to use the structure of the representation to construct a mental model of the concept.

The students are on familiar ground once they understand the use of the model to represent algebraic expressions, giving the analogue a sound

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base specificity. It is able to model a variety of algebraic expressions with clarity and the students soon can explore higher-order relationships, thanks to the richness of the model. It wa this richness which led the students to nominate great varieties of equivalent algebraic expressions, as was required in the topic taught. The model was used to explore mappings both from the algebraic symbolic form to the concrete form and also in reverse. This was because there was clear isomorphism between the structure of the algebra and the structure presented by the model.
The evidence points to the likelihood that this concrete analogue assisted cognitive development and was neither redundant (Sweller, 1993) nor a distraction (Halford, 1993). It is worth considering that, as depicted in Figure 10, the model contributes directly to the development of the desired conceptual understandings.

Figure 10. Model contributes directly to understanding of concept

The supportive outcome was from just two classes. Further data collection is planned in the near future. Those following a Traditional Approach will be given workheets that more closely parallel those used by the Concrete group, and they will be supplied with algebraic symbols rather than expect them to create their own.

School B: Indices. The concrete analogue consisted of unit cubes which were fitted together to build rectangular prisms. Of the 15 students interviewed, 46.7% said that the cubes helped them learn, and another 13,3% said that they helped sometimes. The 40.0% who said they did not help were mainly put out by the time required to build the shapes. An initial analysis of this analogue is now given in terms of the characteristics described by Gentner (1982). The base specificity of the analogue was quite adequate as the concept of doubling was familiar and the number of cubes could easily be counted. The cubes were useful in communicating the concept of exponential growth rates: doubling the shapes required considerably more cubes once you passed, say 24. By constructing differently-shaped rectangular prisms for the same number of cubes, higher - order relationships were readily observable. For example, 32, or 25, cubes could be represented by a prism with dimensions 2 x 4 x 4, or 21 x 22 x 22, giving evidence for accepting that 21 x 22 x 22 = 25. With an 8 x 2 x 2 prism another relationship leading to 2 5 was demonstrated. This activity built upon the clarity of the internal structure of the model and the systematicity of the mappings required. The model showed a weakness as regards richness: The cubes did not prove very efficient in demonstrating an index law such as 26 ÷ 22 = 24. The method tried in such a case was to build a model of 26 and divide it evenly into 4 pieces, each of which could have been recognised as 24.

Dempster (1981), after exhaustive examination of many studies, was able to claim that "memory span is indicative of overall intellectual ability" (p. 65). Boulton-Lewis used digit span "to determine short-term memory" (1987, p.335). Moreover, Halford (1993) pointed to the potential disadvantage that some analogues "can actually increase the learning or memory load" (p. 220). The hypothesis that those with larger digit spans would manage the concrete model more effectively than those with shorter spans was not supported by the data from School B: Correlations between digit span and score gains from Pretest to Posttest were almost all non-significant for various aspects of the understanding of indices.

The evidence points to the possibility that the concrete analogue used for indices was neither redundant (Sweller, 1993) nor a distraction (Halford, 1993). Figures 3, 4 and 5, for instance, indicate that considerable learning did take place with the aid of the unit cubes, despite the time needed to manipulate them and despite the limited richness of the analogue for clarifying higher-order relationships.

School C: Factorisation and Expansion. In School C the situation could have been regarded as the worst scenario possible for a concrete analogue. The class had a poor attitude to mathematics and were low performers. The area model itself provided several challenges. It seemed like the case of a brain capacity overload, with fresh algebraic concepts to learn and a fairly difficult model to cope with at the same time: The limitations of analogues exemplified! (See Halford and Boulton-Lewis (1992). Some of the difficulties with the area model were: the cut-out squares and rectangles were labelled with lengths marked 1, x, y, or z. The students were shown that the letters were not whole number multiples of the unit length being used. Some tried using approximate numerical values in place of the letters. When you take a rectangle of dimensions 1 by y, the length of two sides is y and the area is also equal to y. This was explained, but it is not an easy concept. Several students made the predictable error of describing the length of a rectangle by incorrect algebraic expressions. For instance, if two rectangles had been pushed together to give a length of y + z, they would write yz as the length. They needed to be reminded that mathematics is for real, and that the model was not make-believe. The letter x was easily confused with the multiplication sign and z with the digit 2. Other letters would be usesd next time around. The first period of the four for intervention teaching was spent trying to iron out some of the difficulties rather than making much progress with factor problems. The next period the students were instructed to write the area calculations on each piece of cut-out paper squares and rectangles. For instance,

A = 1 x y = yor A = y x z = yz.

This strategy reduced the unproductive effort needed "to attend to several sources of information simultaneously" (Sweller, Chandler, Tierney & Cooper, 1990, p. 189).

The outcomes are not as dismal as the circumstances might have predicted. The Concrete group did show signs of learning, and even surprised their class teacher at times. Of the 20 students

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interviewed, 70% said that they really enjoyed using the area shapes and another 10% said they enjoyed them some of the time. Also 70% said that the materials helped them learn, with another 20% saying they helpd sometimes. Efforts to link the digit span result to performance were not successful using correlations with learning gains, analyses of variances, or examining the extreme scores on performance variables. The classes, in fact, recorded no significant differences on digit spans.

Concluding Remarks

This type of data collection needs to be extended. The writer hopes to do this in the near future and to include the topic of first degree equations as well as those treated so far. Much is still to be learnt about the mental processes involved in the learning of early algebra and the place of appropriate concrete analogues in this learning. Measures of digit span could well be augmented by measures of preferred thinking styles and preferred perception modes, as these could have some covariant influence on individual's learning - either with or without the assistance of concrete analogues.
In observing and/or replaying tapes of the lessons, it was noted that the Concrete Approach classes class spent more time on discussions (while working with concrete materials), whereas Traditional approach classes spent more time on writing answers. Leinhardt (1988, p. 141) drew attention to a potential role of concrete analogues which could have been relevant here:

We need to explore more elegant ways of building consistent concrete representations that can serve as both an explanatory and exploratory system for children and to give them language tools for talking about such systems.

Attitudinal gains were no surprise: Generally favourable reactions from teachers and students to the use of manipulatives for algebra were reported in Quinlan et al. (1993) following four years of action research. As Collis and Biggs put it, “It seems that a well organised inter-modal strategy influences children’s attitude to, as well as their comprehension of, the content being taught" (1991, p. 202).

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