Modes of representation of ideas, computers and learning styles in k-6 mathematics

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Constructivist theories of mathematics learning suggest that people personally negotiate meaning by creating different mental representations of mathematical knowledge. They may then progress towards relational understandings of mathematical ideas by making connections between the different modes of representation. Partnerships between concrete materials and computer software provide mutual benefits and make learning more powerful by the integration of concrete, pictorial and symbolic modes of representation.

This paper outlines a classroom study involving the use of the computer in linking the different modes of representation of mathematical knowledge. Three groups of students were formed, receiving either concrete, pictorial or symbolic treatments of whole number and decimal numeration. Aspects of individual student learning style were also identified and utilised in the analysis of results to determine the efficacy of different teaching approaches and of matching individual learning styles with specific types of teaching.

Concrete materials and modes of representation in mathematics education

Fundamental to contemporary curriculum documents such as the NSW K-6 Mathematics Syllabus (1989) is an acceptance of a theory of learning in which students are first introduced to mathematical concepts through concrete materials. Concrete materials, or "objects which represent mathematical ideas that can be abstracted through physical involvement with the objects" (Young 1983, in Kennedy 1986, p.6) are believed to be a crucial first step along a path leading towars the relational understanding of mathematical ideas. Students can extend their understandings of concepts by transforming concrete materials into semi-concrete representations such as pictures, images or diagrams, before finally internalising the concepts into abstract thought by means of symbols. Ideas in the process of construction can be represented by these "modes of representation" (Bruner 1966), whether enactive (involving manipulations), iconic (pictures and diagrams) or symbolic (words and symbols).

Teachers have traditionally sought to bridge the gap between concrete and abstract by using pictures of objects, textbook illustrations, models, the overhead projector, drawings and demonstrations during their work with students. (Heddens 1986)

A more recent reference to this strategy is provided by the NSW K-6 Mathematics Syllabus which acknowledges significantly that the computer

may provide a means of linking physical, pictorial or symbolic representations, though without replacing the need to use manipulative materials.
"Computers can also provide a link between concrete manipulation and abstract processing." (p.37).

The effectiveness of concrete materials

The value of using a concrete approach with students is demonstrated in research which suggests that mathematics achievement is enhanced by the use of concrete materials across a variety of topics, and at every grade, achievement and ability level.(Suydam 1984; Sowell 1989)

Research regarding the efficacy of concrete materials is, however, not entirely unequivocal (Lesh, Post and Behr, 1987; Kaput 1987).

Perceived control and management problems, demonstrations by teachers of mathematical ideas in which students act as passive observers, difficulties in representing large numbers and lack of variety in representations are all factors which can limit the success of concrete approaches. A more critical type of management problem is that sometimes the connections between the actions on the manipulatives such as Base 10 blocks and the actions of representation by formal mathematical notation are often masked because the cognitive load imposed during activity with the blocks is actually too great. (Kaput 1989)

Given the preceding findings it is clear that any device which could overcome difficulties associated with developing appropriate mathematical understandings would be a very powerful one.

The computer as a link between concrete and symbolic modes

It is held that the computer environment can largely overcome many of the difficulties previously associated with the use of concrete materials by the provision of a unique and dynamic link between the concrete and the symbolic. Ideas can be represented by pictures and mathematical symbols which are simultaneously transformed by mouse-driven manipulations enabling students to deepen their knowledge structures.

A new generation of computer software has became available in the last decade to help students make the transition from the concrete to symbolic ways of representing knowledge and to allow studies of the integration of concrete materials and software. (Berlin and White 1986; Newton 1988; Brown 1990; Perl 1990; and Thompson 1992)

Berlin and White (1986) investigated the effects of combining interactive computer simulations and concrete activities on the development of abstract thinking. The influence that concrete manipulations and computer activities had on the students' transitions from concrete understandings to abstract thinking was unclear. Evidence suggested that the concrete and computer activities had different effects on students according to their socio-cultural background and their gender, with some students doing better on tasks using concrete objects while others did better on those which involved computer use. In explanation of the differences in achievement according to the different teaching treatments, Berlin and White concluded that the preferred learning styles of the students acted to hamper the findings of their study. It was concluded that the topic needed much further investigation.

Mirroring software

Perl (1990) defined a number of different models that make the synthesis of concrete materials and software possible. So called "mirroring software" involves the display of objects that look like corresponding manipulatives and are used in exactly the same way. A study by Thompson (1992) utilised such a mirroring program called Blocks Microworld designed by himself. In a classroom study, Thompson found that a group of learners using Dienes' Base 10 block computer simulations outperformed another group who used actual wooden blocks. Thompson concluded that the enhanced performance was due to the lessening of the cognitive load as students can make the link between concrete and numeric representation by performing parallel series of actions simultaneously.

THE STUDY

It was decided to further investigate the effects of combining interactive computer simulations and concrete activities on the development of abstract thinking in primary mathematics. Three different teaching treatments were devised and their outcomes evaluated to determine their efficacy, or whether Thompson's findings could be replicated. The groups of students involved received one of the following treatments constituting either: a concrete approach using Base 10 blocks; a traditional abstract approach using teacher language and use of symbols via the chalkboard; or computer simulations using Blocks Microworld . Each group of students was taught in turn by the same teacher and lessons ran successively over 10 daily sessions of 40 minutes. There were strict protocols regarding the content of each particular session and the type of instruction which was used.

The content for the 10 lessons was in the area of numeration with whole numbers and decimals. A pre-test and a post test was administered to all subjects.

The efficacy of using a learning style instrument in assigning individuals to certain groups receiving different teaching styles was also evaluated. By attempting to control the variable of learning style it was anticipated that many of the difficulties associated with the findings of the Berlin and White study could be overcome.

The particular learning style instrument chosen was the Perceptual Response Subscale of the NASSP Learning Style Profile (1986). This was administered in the form of an oral quiz involving an initial reaction to information as visual response, auditory response or emotive (kinesthetic) response. From this it is possible to identify the preference of learners for auditory or visual percepts or whether they react physiologically to information, indicating they prefer kinesthetic learning.

A total of 294 students across the school were tested for their learning style preference, with 36 students being identified as showing an extreme preference for a particular learning style. These students, ranging in age from 9 to 12 years, became the subjects of the study. Equal numbers of students of each preferred learning style were randomly assigned across the 3 teaching treatment groups so that 9 sub-groups were defined:

Group 1 - kinesthetic learners receiving concrete only activities.
Group 2 - kinesthetic learners receiving traditional instruction.
Group 3 - kinesthetic learners receiving computer only activities.
Group 4 - auditory learners receiving concrete only activities.
Group 5 - auditory learners receiving traditional instruction.
Group 6 - auditory learners receiving computer only activities.
Group 7 - visual learners receiving concrete only activities.
Group 8 - visual learners receiving traditional instruction.
Group 9 - visual learners receiving computer only activities.

The 3x3 factorial design assessed the effects of the methods of instruction and learning style. It was hypothesised that groups of students receiving instruction which closely matched their learning style would outperform their peers whose styles were mismatched.

RESULTS

A 3x3 factorial analysis of variance was carried out (pre-test, post-test by gender, style and treatment). No statistically significant differences were found at the .05 probability level. (see Tables 1 and 2). There was no difference in performance of the concrete materials group, the computer group or the group which received traditional instruction according to the measures administered. The findings of the earlier

Table 1 ANOVA
by pretest, gender,style and treatment

Table 2 ANOVA
by post-test, gender,style and treatment

Thompson study were not supported by this study - learners using Dienes' Base 10 block simulations did not necessarily outperform those using the actual blocks or those receiving traditional instruction.

In relation to the expressed learning style preferences no significant differences in performance were identified. The hypothesis that learners would perform best with the teaching treatment that corresponded to their learning style did not eventuate.

DISCUSSION

The findings of this study do not support the proposition that the best results are achieved when computer simulations are employed in the learning of certain mathematical concepts or when student learning style preferences are matched with instructional methods.

A number of reasons may be advanced to explain these findings:

1. Although the learning style preferences identified by the NASSP instrument were strongly defined in the subjects chosen, secondary response characteristics still exist and may be activated. In fact, unless they are sensorily impaired, students are using a combination of modalities in processing information and are thus never entirely auditory, visual or kinesthetic learners at any point in time. This factor could serve to mask the findings regarding the efficacy of the different teaching approaches.

2. It may not be accurate to categorise a child as a having a particular learning style preference, innately established within that individual. Rather, it may be more appropriate to define a number of contexts, or learning tasks in which particular behaviours are manifested, indicating certain learning style preferences which vary according to the particular context. Collis and Biggs (1991) argue that much complex problem solving is in fact multimodal, and that moving between different representations of ideas provides depth and richness of thinking. The dream of many teachers of matching teaching styles to individual learning styles may well be a false one. At the least, teachers should be cautious about regarding a single teaching style as always most effective for certain student learning styles.

3. The learning style instrument administered, though purportedly reliable and valid, may have a number of weaknesses. It's accuracy depends on subjects understanding what is required to do the test and being prepared to reveal the knowledge required. The fact that a student's emotive response to a test item indicates a preference for a kinesthetic mode of learning may well be questionable. It may be more

useful in future to use observational methods and interviews to establish the learning style preferences of individual students.

4. The quality of the teaching across the 3 groups in terms of making meaningful representations for students may be the predominant factor rather than the advantages of any one teaching method and the tools it uses.

5. The alleged continuum from concrete to abstract representations may not exist. Many constructivist theorists deny the existence of the continuum, in favour of students representing knowledge idiosyncratically, according to their needs and dispensations. In a summary of research assessing the effectiveness of radical constructivist ideas in mathematics education. Ellerton and Clements (1992) refer to critical findings of Cobb (1990).

"Students do not learn mathematics by internalising it from objects, pictures or the like. Mathematics is not a property of learning materials, structured or otherwise." (p.8)

If this is the case then students should be exposed to a variety of representations so that the appropriate ideas can be constructed. "Reconstructions and verbalisation of mathematical ideas and solutions should be encouraged." (p.7)

These are matters that are worthy of further investigation.

BIBLIOGRAPHY

Berlin, D. and White, A. (!986) Computer simulations and the transition from concrete manipulation of objects to abstract thinking in elementary school mathematics, School Science and Mathematics: Vol. 86(6), October 1986, pp. 468-479

Brown, S. (1990) Integrating manipulatives and computers in problem-solving experiences, The Arithmetic Teacher: October 1990, pp.8-10

Bruner, J.S. (1966) Towards a Theory of Instruction. New York: W.W. Norton

Campbell, P. (1988) Microcomputers in the primary mathematics classroom , The Arithmetic Teacher: February 1988, pp. 22-29

Cobb P., Yackel, E. and Wood, T. (1992) A constructivist alternative to the representational view of mind in mathematics education, Journal for Research in Mathematics Education: Vol. 23, No. 1, pp. 2-33

Collis, K.F and Biggs, J.B (1991) Developmental determinants of qualitative aspects of school learning. In G.Evans (Ed.), Learning and teaching cognitive skills (pp.185-207) Melbourne: ACER

Dienes, Z. (1960) Building up mathematics, London: Hutchison Educational Ltd.

Ellerton, N.F. and Clements, M.A. (1992) Some pluses and minuses of radical constructivism in mathematics education, Mathematics Education Research Journal, Vol.4, No.2, 1992, (pp 1.-22)

Heddens, J. (1986) Bridging the gap between the concrete and the abstract, The Arithmetic Teacher: February,1986. pp.14-17

Kaput,J. (1989) Linking representations in the symbol system of algebra. In C. Kieran and S.Wagner (Eds.), A research agenda for the learning and teaching of algebra. Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum.

Kennedy, L. (1986) A Rationale , The Arithmetic Teacher: February 1986, pp.6-7

Lesh, R., Post, T. and Behr, M. (1987) Dienes revisited: Multiple embodiments in computer environments. In I. Wirzup and R. Streit (Eds) Developments in school mathematics around the world. Reston, VA: National Council of Teachers of Mathematics

National Association of Secondary School Principals, (1986): Learning Style Profile: Reston, VA: National Association of Secondary School Principals.

Newton, J. (1988) From Pattern-block play to Logo programming, The Arithmetic Teacher: May 1988, pp.6-9

NSW Department of School Education (1989) Mathematics K-6 Syllabus.

Perl, T. (1990) Manipulatives and the computer: a powerful partnership for learners of all ages, Classroom Computer Learning: March 1990, pp. 20-29

Sowell, E. (1989) Effects of manipulative materials in mathematics instruction, Journal for Research in Mathematics Education: Vol. 20, No. 5, pp. 498-505

Suydam, M. (1984) Research report: Manipulative materials, The Arithmetic Teacher: May 1984, p.27

Thompson, P. (1992): Notations, conventions, and constraints: contributions to effective uses of concrete materials in elementary mathematics, Journal for Research in Mathematics Education: Vol. 23, No. 2, pp.123-147

Sources of cited software

Blocks Microworld

Intellimation Library for the Macintosh PO Box 1530 Santa Barbara, CA 93116-1530