

## A computer's role in revealing and influencing learners' understanding

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### ABSTRACT

The beliefs and "misconceptions" held by learners of elementary mathematics are well researched and documented. Theories about how these ideas change through experience and discussion with others have been developed over many years, and several theories highlight the role of cognitive conflict.

The paper will discuss ways currently being explored of using computer based modelling systems as a neutral but authoritative participant in a process by which cognitive conflict is generated and resolved by learners. The research highlights the computer's ability to reveal learners' mathematical beliefs and thought processes in a dynamic and visual way which can support articulation and challenge by the teacher and by peers. This feature may help improve our knowledge of how cognitive restructuring occurs, and ways of recording evidence about the change process, rather than just measuring the extent of change, will be discussed.

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### Introduction

Early ideas concerning the role of the computer in the process of teaching and learning were drawn from the behaviourist-influenced programmed learning movement. Subsequently, influenced by attacks on such ideas by Papert (1980) and others, by the increasing pervasiveness of software such as word processors and spreadsheets, and by the growing influence of Piaget and Vygotsky on educational developments, the predominant paradigm for educational computing has a much more constructivist feel to it.

Current advice to teachers emphasises that the computer's role is to help learners develop their understanding of the matter being taught, rather than attempting to transmit knowledge directly (NCET (1994a), for example). A computer package can allow students to express their own ideas, test them out and receive feedback; if they do this in groups of two or three, the computer provides better support for a dialogue about the concepts than written work does, and if the teacher or more expert peer works with them, the process of 'scaffolding' their learning is facilitated. There is a significant gap between theory and

practice, however; many mathematics teachers do not use computers in their teaching to a worthwhile extent (Tanner, 1992), and of those who do, their role in classroom alongside students and computers may not exploit the potential strengths of IT (Kennewell, 1993). Teachers face the additional difficulty that many pupils do not consider that mathematics is something that is open to discussion and negotiation (Joffe & Foxman, 1989).

It is helpful for teachers, therefore, to examine how the computer environment can enable learners to articulate their thoughts. There are two main ways in which this is achieved: by requiring mathematical processes to be carried out quite publicly on the screen; and by stimulating dialogue between learners, to which the teacher can listen in and contribute. The computer acts as an authoritative test-bed for pupils' ideas as they grapple with important changes in their thinking.

These features also provide a fruitful environment for the researcher in mathematics education, and we can consider the value of asking pupils to use computers in their work, not primarily because we believe that IT will enhance their learning, but because the process will reveal aspects of their thinking which are hard to discover in other ways. I shall explore first some issues concerning the type of thinking and learning which is the most interesting, and yet traditionally the hardest to influence or even to discover.

#### The process of conceptual change

Whilst recognising that there are many facets to the learning process, and many approaches to explaining them, we are focussing principally on the phenomenon of conceptual change in individual students, particularly at the level discussed by Strike and Posner (1985). This process involves changes to one of the frameworks used by the learner to interpret phenomena. Beliefs about aspects of the world which suffice for everyday life, and for the early years of schooling, are often too naive to support more advanced ideas. In mathematics, for example, a change from arithmetic to algebraic thinking is required (Herscovics & Linchevski, 1994). In Piaget's terms, these changes are accommodations of concepts to the learners' experiences; in Norman's

(1978) terms, a restructuring of concepts. However, such changes are not likely to be made naturally by children: they are brought about intentionally by the planned curriculum provided by the educational system. If we are to improve the quality of learning, teachers must help pupils to undergo conceptual change at appropriate times.

Strike and Posner (1985, p216) list a number of conditions for conceptual change:

- there must be dissatisfaction with existing conceptions;
- the new conception must be at least minimally understood

it must be initially plausible;  
it must be capable of greater explanatory power.

The process of change does not happen instantly. There may be particular events which are catalytic in nature, however, such as when students experience conflict between observed phenomena and their beliefs. Such events may happen by chance, but effective teaching should ensure that learners meet appropriate cognitive challenges regularly, in order to facilitate dissatisfaction with current conceptions and a need for greater explanatory power. These challenges should be interspersed with opportunities to explore in advance (so as to facilitate minimal understanding and the plausibility of new ideas) and to consolidate concepts subsequent to a period of change. Indeed, Hillel (1992, p209) identifies three types of problem set by teachers of mathematics:

- a. Problems to reinforce some newly learnt techniques and concepts;
- b. Problems to challenge students;
- c. Problems to develop some new mathematical knowledge.

However, most formal teaching of mathematics comprises the answering of questions which the students have not asked and the explanation of phenomena which they have not even noticed - the only need for explanation which the students feel is based on their expectation that they will shortly be set an exercise. West & Pines (1985) thus see the development of two strands of learning for students: that from their intuitive knowledge of the world, and that from their formal teaching. Genuine conceptual learning results from the integration of these two strands.

#### Probing and influencing understanding

In exploring the question "how do we know that learning has happened", Passey and Ridgway (1991) identify two approaches. One is to look for changes in behaviour, the other to look for change in some mental state. If we adopt the latter view, however, assessment is problematical. As Eisenberg (1991) points out, "it is only possible to infer the cognitive structure of students' conceptualizations through the concept images they invoke in written and spoken communications". Techniques such as concept mapping, prediction-observation-explanation (POE), and interviews about events, instances and concepts (IAI), have been fruitful in exploring the nature of learners' concepts and beliefs (White and Gunstone, 1992). Such techniques allow us to access students' conceptual structures indirectly, either by using the students themselves to provide a representation on paper, as in the case of concept mapping, or more subtly by observing responses to situations and to questions about them (POE and IAI).

In order to identify whether conceptual change has occurred, over a period, we can adopt a pre- and post-test design, where the testing involves either traditional written answers to questions, or one of the

phenomenographical techniques. It is difficult, of course, ever to observe a phenomenon without changing it in some way, and this is

particularly true in the field of cognitive development. The more valid an instrument we design to probe understanding, the more it will stimulate the sort of cognitive activity which will lead to conceptual change or development before our "treatment". Indeed, as White and Gunstone (1992) note: "good testing devices are good teaching devices" (p39).

There may be qualitative indicators of when the process of change is occurring, which can provide evidence additional to our measures of what change has occurred. Comments by students about their thinking may be received or overheard, or they may be explicitly sought by the teacher. Bell (1993) reports a student articulating a change in belief about the nature of mathematical problem-solving, and describes general characteristics of student behaviour during planned cognitive conflict.

Further work is needed on an ethnographic approach to the exploration of informal methods and intuitive conceptions as part of "classroom culture". In this culture, tasks are usually achieved in the way which is easiest or most interesting for the students rather than as the teacher intended (Kennewell, 1993). Research and teaching are often affected by failure to take account of this factor.

#### Confronting cognitive conflict

Such changes are hard to bring about, however. There is much evidence from the literature of alternative conceptions in science to show that learners usually avoid making a necessary conceptual change, even when faced with situations which conflict with their current beliefs. They claim that they have made a mistake with the experiment, or with the data, or they rationalise what they have seen by adding a special case to their current conceptual structure. For example, students who believe that heavy objects fall faster than light ones will use the negligible variations caused by air resistance as evidence to support their beliefs (Champagne et al., 1985). Similar phenomena are observed in mathematics. If, for instance, they have constructed a belief (based on years of calculating with natural numbers) that "multiplying makes things bigger", they will continue to suggest that  $0.2 \times 0.3 = 0.6$  despite being taught methods which give the result  $0.06$ .

Bell and various colleagues (1981, 1993) based at the Shell Centre for Mathematical Education have successfully devised activities for stimulating and confronting cognitive conflict under the label of "diagnostic teaching" (or "conflict teaching" (Underhill, 1991)). This involves appropriate exploratory work in the topic first, followed by conflict/discussion lessons where pupils are given carefully designed tasks together with encouragement to voice their ideas and question each others' assumptions with the teachers' support. These tasks might

be:

challenges eg What do you multiply 5 by to get the answer 2?;  
predict/check tasks eg What do you think that 2 divided by 5 will be?  
Check on the calculator;  
requests to identify and explain an error made by someone else, for  
example  $2 \text{ divided } 5 = 2.5$ ;  
games eg finding matching pairs amongst cards containing sums such as  
 $0.3 \times 0.4$  and  $3 \times 0.04$ ;  
making up questions (in or out of context) which produce a given answer  
such as 0.15.

Observing the use of tasks designed to promote conceptual change  
reveals much about pupils' understanding, regardless of whether change  
takes place or not. We can use such tasks at different levels: for  
diagnosing particular pupils' difficulties (which may identify needs  
for remedial work with scope far beyond the original topic

investigated); for evaluating methods of teaching the topic; or for  
formal research which aims to establish or test theories about  
learning. A key feature of the method is that learners are placed in  
situations in which they are required to make their ideas and beliefs  
explicit, exposing them to challenge in a supportive situation. Their  
ideas may be carefully challenged by the teacher through prompts and  
questions which will help them to adapt their thinking, or through  
argument with their peers who may be equally, but differently, wrong.  
This is a scenario in which the computer seems to be able to play a  
useful part (Light, 1993).

Using computers to help probe understanding and promote conceptual  
change

There are many ways in which computers may aid learning, and I shall  
not attempt to catalogue them here. De Corte lists a number of  
teaching methods for which IT "provides possibilities for optimum  
realization and application". One of these methods is articulation,  
which "refers to any technique that helps students to spell out and  
make explicit their knowledge and problem-solving procedures" (de  
Corte, 1990, 83). Fraser (1988) identifies four ways in which it is  
possible to 'converse' with a computer:

1. by programming it
2. using 'tools' such as graph plotters, spreadsheets
3. interaction with a simulation or similar catalyst
4. in response to a program acting as 'tutor'.

The use of Logo as a computer language which learners can use to solve  
problems and carry out useful investigations in mathematics is well  
established, (for example: Noss, 1986; Hoyles & Sutherland, 1989) and  
the construction of computer models using Logo or spreadsheets has been  
found to achieve just the sort of effects we require in revealing

learners' thinking and arbitrating during socio-cognitive conflict (Sutherland, 1989; Hoyles, Sutherland & Healy, 1991). Furthermore, Hoyles et Al. identified that group work with computer models aids the 'distancing' (or 'decentring') that students need to experience in order to make the important step from the spotting of a pattern to the articulation of a general rule.

The use of generic tools has provided the focus for most recent work, and of particular interest is the help that computer modelling tools can give students in learning through representing their own mental model and exploring its consequences (Bliss & Ogborn, 1989). Modelling tools require students to articulate their thinking in order to make the computer work for them, and they provide immediate feedback concerning the representation of students' ideas at a syntactic level, even to the extent of offering explanation concerning errors and ambiguities (Miller et Al., 1993). Pirie (1991, p159) highlights the difference between what students can express verbally and the thinking which has actually taken place; indeed, "Pupils verbalising their mathematical thinking may be found to form their possibly erroneous mathematical beliefs by the force of their own explanatory words." The computer modelling environment can help to overcome the difficulties which students have in writing down their mathematical ideas, and to reduce the limitations of the oral communication which is normally the only alternative.

The availability of computers enables us to improve our research instruments in mathematics education. In the study of scientific learning, the researcher can easily show the student something, ask them to predict beforehand what will happen under certain conditions, let them observe what happens, and ask them to explain it afterwards.

However, such probes have been difficult to implement where procedural mathematics is involved (though the Assessment of Performance Unit has devised many practical tasks which are effective in revealing students' thinking (Joffe & Foxman, 1989)). With a formula to be evaluated or a shape to be transformed, the only substitute for observing something happening would be for the student to observe the researcher carrying out a procedure, and it is less likely that the student will feel a need to explain what has happened. The dynamic way in which students interact with computer models has an effect on the nature of the POE investigations which can be carried out. The computer provides a version of 'real' mathematics in an interactive form, and makes the design and use of POE situations much more effective.

Interviews about instances and events can also be extended. As well as showing the student two numbers or shapes, and asking questions about the similarities and differences between them, the process of change from one to the other (and back again), can be explored - and explored repeatedly with the same or different values of variables. In

addition, new terminology and notation may be available from computer work (Sutherland, 1989) of which students feel more ownership than traditional mathematical language, and can thus be used more freely to articulate their thoughts.

In the context of learning science through simulation, it has been noted that computer models:  
require children to make their implicit reasoning explicit;  
enable children to visualize the consequences of their reasoning, and provide an object for reflection and communication with others;  
provide pictorial representations and dynamic displays of models of phenomena which could form useful bridging analogies;  
(Driver & Scanlon, 1988, p30)

These findings were from work involving students using dynamic models, and I suggest that a dynamic modelling system such as Model Builder (AUME, 1991) would be a useful addition to the toolkit of software which should be available in the mathematics classroom. Teachers can set tasks in this environment which set up the conditions for conceptual change and induce appropriate cognitive conflict.

The computer has been found to be less threatening than paper as an aid in articulating ideas. It displays students' writing in a standard way, less personalised than hand writing, which makes the changing of initial ideas more acceptable. Teachers can talk to pupils in terms of "trying to make the computer understand" when they are engaging in computer modelling. They can point out that, since the computer is not intelligent, we may have to try various ways to make it do what we want. Failure is thus no longer a threat to pupils' self-esteem. Students also value the good presentation of their work that they can achieve with computer printout.

It is important in teaching and research that we can listen into students' conversations and observe their actions in order to monitor their thinking during a task. This process is easier in the computer environment, and also seems less threatening. Observers making field notes are soon either ignored or involved as participants as the computer activity engages the learners. Video cameras focussed on the screen are unobtrusive, yet they pick up most of the relevant material for subsequent analysis. Students can be asked to print out their work at particular stages to provide a permanent record of the development of their ideas, and we can even program the computer to record every key press to be replayed later to closely examine how they have tried out ideas and subsequently changed them.

The culture of mathematical learning may be enlarged by the computer in that it "offers a context in which mathematical formalisation is a necessary part of a system to be explored" (Noss, 1988, p260). It is

now possible, using calculators, graph plotters, symbolic algebra manipulators, spreadsheets and dynamic modelling systems, to create situations where students are asking the questions about formal mathematics that the teacher wishes to answer, and where students feel a need for the explanations which the teacher wishes to give.

The over-riding acceptability of computer work in student culture enables us to overcome many students' negative attitude to the cognitive effort involved in learning mathematics, and to stimulate a quality of thought which is beyond what the students normally achieve.

"My brain hurts" is a common quote after unaccustomed sustained cognitive effort. Students work together more effectively when using a computer, and this may facilitate the social construction of knowledge.

However, the feeling that computer work is impersonal may limit its contribution to conceptual change - students may feel they can ignore phenomena revealed by the computer, rationalising any conflict they experience in terms of differences between the 'real world' and the computer model. Indeed, the National Curriculum for England and Wales (DES/WO, 1990) requires that students be taught to analyse and evaluate computer models.

The prevalence of peer support during computer work - part of what O'Shea et Al. (1993) refer to as "computer culture" - may also facilitate learning, though some evidence that it does not apply where the computer program is easy to operate and students are able to concentrate fully on the subject matter (O'Shea et Al., 1993). In any case, the nature of the information passed around the computer class, and the ways in which students interact, are unlikely to promote conceptual change in individuals. What may be possible, however, is for the teacher to use the place of computer work in the 'common knowledge' of a class to help develop a shared understanding of concepts being taught, in ways described by Crook (1991).

#### Difficulties inherent in using computers

All teachers whose students have used computers in the classroom will be able to identify the "IT interference factor" (Birnbaum, 1989); this is the extent to which a process of applying IT to a task is inhibited by ignorance of how to use the IT package itself. But there is a further factor which makes research in the computer environment more difficult. The sort of difficulties concerning beliefs and concepts we have considered for science and mathematics also apply to computers themselves, and misconceptions about the computer and its software may obscure our observations of the concepts being studied. For example, when asked to complete a table of values on paper (see fig 1), then produce a similar table on a spreadsheet program using a formula for the second column, several pupils ignored the instruction concerning the formula. We have learned to look behind this apparent carelessness or laziness in order to seek the pupils' beliefs about the task or the tools available.

fig. 1 here

There are three possible explanations for this type of phenomenon:

1. Pupils focus on the task outcome only, and do not share the teachers' objectives concerning the process. They therefore follow the procedure which they perceive will lead to a correct result in the

quickest way possible. This is a common explanation of classroom events; see, for instance, Kennewell (1993).

2. Whilst familiar with formulas, the pupils do not have procedural skill in constructing them, and even in the computer environment with its tolerance of failure, they are not prepared to take risks with their mathematics.

3. Pupils have misconceptions concerning the role of the computer. Their experiences have mainly been of the computer setting them questions and telling them whether they are right or wrong, or perhaps much of their computer work has been mere copy typing with a word processor. In either case, their beliefs will inhibit their achievement of the mathematical learning objectives.

For many pupils, all of these factors will be influencing their actions. We must be aware of this when teaching or researching in a computer environment, and plan opportunities to reveal and confront the possible misconceptions if our teaching is to be effective and our research valid.

Furthermore, Balacheff (1986) has noted that students' concern when solving problems is "to be efficient, not rigorous; it is to produce a solution, not knowledge". This phenomenon seems more obvious in a computer environment; is this merely because it is easier to discover students' thinking about their goals in this situation, or is it because "reliance on computer feedback [is] the cause of an undue emphasis on 'producing solution' rather than 'producing knowledge' (Hillel, p206). It is important, therefore, that the teacher introduces opportunities for discussion on how a solution has been achieved and reflection on what has been learned instead of merely assessing the quality of the product.

## Conclusion

I am suggesting that we have undervalued the potential of computer work for revealing students' current understanding of concepts, for stimulating cognitive conflict, and for aiding the sort of interaction between the student and the computer, the peer or the teacher which leads to cognitive restructuring. Furthermore, we have not fully

exploited the use of the computer as a tool for research into the processes of learning per se.

It is important to investigate learning processes in the culture of real classrooms as well as in the clinical environment needed for POE and IAI techniques. There are limits to what external researchers can achieve, of course. When O'Shea et Al. (1993) investigated the use of computer models to meet the challenge of promoting conceptual change in the classroom, they were successful in achieving this aim, and also found learners to be more confident of correct explanations, and more articulate in their incorrect explanations of situations. However, it is clear that they were working in what Passey and Ridgway (1991) refer to as "'ideal classroom conditions', with an atypical volunteer teacher ... where generalisation of results to representative classrooms, where dilution and corruption of the designer's intentions are commonplace, is problematic". The increasing involvement of teachers in their own action research is important to overcome the limitations to the validity of findings obtained in artificial situations. Encouraging an action research approach to curriculum development will also help more teachers to integrate the use of IT into their teaching rather than adding it on.

Looking forward, we can see that the increase in availability of information technology, and the improvement in user interfaces for software, should reduce the IT interference factor. Teachers will need to develop sufficient confidence to let the pupils apply and develop their capability, however. The motivating value of computers does not seem to be diminishing; there is no evidence that the prevalence of games in home computer use is leading to a rejection of perhaps less exciting environments in school, though it may foster particular beliefs about the role of the computer which will need to be overcome. A more significant trend is that towards portable computers, and the benefits of individual ownership, even if only temporary, are considerable (NCET, 1994b). The small screens and less distinct displays inhibit the sharing and challenging of ideas which has been found essential in promoting conceptual change. It will therefore be essential to preserve desk-top computer facilities suitable for small group work in classrooms. Furthermore, in trying to exploit the cultural resonance which IT has for young people, we must beware the possible tensions and barriers to articulation in the classroom which may be caused by over-structuring the learning activities we devise for teaching and research.

Conceptual change needs an environment - a 'community of enquiry' - which is missing from most classrooms at present. The computer is having some effect in enabling teachers to open up their environment to hypothesis and exploration. It has been established that the teacher has a subtle but vital role in a computer learning environment, in terms of the planning of tasks and intervention during learning

activities to provide scaffolding and challenge (ref Hoyles & Sutherland, 1989; Noss, 1986). Without the computer, peer discussion in potential conflict situations may merely reinforce naive conceptions, and the teacher may be too ready to provide the 'right answer' in a way which will not influence the students' intuitive thinking. The computer can take on a role which has the mathematical authority of the teacher, yet is accepted as a partner in a mathematical 'conversation'.

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1 By 'real', I mean a representation of mathematics which mathematicians generally agree is what students should learn.