# Quality mathematics teaching: Consensus and contradiction\par

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Abstract

This paper arises from an attempt to survey professional educators to determine whether there is a common view of what constitutes quality teaching. Selected results from the survey are presented which highlight some apparent contradictions between commonly espoused views on learning and what experienced teacher educators believe about teaching. It is argued that if we are to accept that doing mathematics involves learners' activity we need to reconceptualise both teaching and mathematics education.

The Interim Report of the Review of Educational Research in Australia listed seven priority areas. Teaching was not among them. This highlights a separation which has developed between research on learning and research on teaching; and there is also a distinct division apparent in the literature. The literature on learning is more coherent than the literature on teaching: there are emerging trends among views of educators about how children and adults learn, but less development of what this means for teaching.

One view of learning which is gaining broad acceptance amongst educators is that of students constructing mathematical knowledge according to their personal experiences and prior understandings. This paper presents a discussion of some implications of this view for the teaching of mathematics, then relates these to the results of a survey of mathematics educators which was aimed at ascertaining if there is some agreement regarding the elements of quality teaching. There seem to be some incompatibilities between current, espoused views on learning and the perceptions mathematics educators have of quality teaching.

## Learning as construction

Present rhetoric in institutions involved with mathematics education for teachers, in government departments of education and in schools supports adaptation of curriculum content to fit with the prior experiences and understandings of students. For instance, the Australian Education Council (1990) claims that

It is now widely accepted that learning is best thought of as an active and productive process on the part of the learner. There are several important implications of accepting this simple proposition such as,\tab \line \'a5 Learners construct their own meanings from, and for, the ideas, objects and events they \tab experience;\tab \line \'a5
Learning happens when existing conceptions are challenged;\tab \line \'a5
Learning requires action and reflection on the part of the learner;\tab

\line \'a5 Learning involves taking risks. (pp. 16-17)

Current emphases on learners actively "constructing" their own meanings in mathematics education stems from a set{\fs18\up6 \chftn {\footnote

We cannot make a claim that there is one form of constructivism, for this would be self-contradictory. Different forms of constructivism have been outlined, including interpretations which have been classified as "trivial" constructivism, "radical" constructivism and "social constructivism" (see Ernest, 1991, 1992, and Ackerman, in press for comparisons and further discussion).}} of epistemological theories and philosophies with the common tenet that facts are made by people and their way of experiencing (von Glasersfeld, 1983a) through the building and re-building of knowledge within individual minds. Labinowicz (1985) summarises this position when he writes

Rather than passively copying knowledge that exists "out there", we

actively construct our knowledge of the world internally through conscious interaction with the environment. Each person's understanding is like a personal painting resulting from one's own interpretation and synthesis of reality. It is not a photograph of reality. It is a person's internal network of ideas that interacts with reality in a mutual transformation. (p. 5)

It is important to note that constructivism is not the domain of particular educational theorists. It is a developing field of epistemology which has been popular in psychology, cybernetics and sociology; a field which has a long history. Plato debated the notion of subjective realities with Protagorus who claimed that man is the measure of all things. Vico (1710) believed that "facts" are made by us and our way of experiencing, rather than being given by an independently existing objective world. Piaget (1937) summarised his position{\b }regarding the construction of knowledge neatly when he wrote that "intelligence organises the world by organising itself" (p. 311). Vygotsky's (1986) theory of internalization suggests that "An operation that initially represents an external activity is reconstructed and begins to occur internally" (pp. 56-57). Over the past fifteen years, von Glasersfeld (e.g. 1983a, 1987, 1992) has steadily developed the idea of active{\fs18\up6 \chftn {\footnote \pard\plain

"Activity" is not necessarily the carrying out physical actions, but implicit mental activity: imagining, generalising, hypothesising, mentally representing, fitting new knowledge with old and adapting prior understandings to accommodate new experience are all actions.}}, interpretive knowledge construction by learners. He claims that knowledge can be seen as

something that the organism builds up in the attempt to order the amorphous flow of experience by establishing repeatable experiences and relatively reliable relations between them. The possibilities of constructing such an order are determined and perpetually constrained by the preceding steps in the construction. (von Glasersfeld, 1983a, p.39)

Serious epistemological questions left unanswered in the constructivism debate relate to the existence of, and nature of, knowledge which has been developed over time within a culture: Can this knowledge exist outside of the minds of knowers? However, constructivism also raises a key question for pedagogy: If meaning is constructed within individual minds, what are the effective ways in which learners can come to know?

Some implications for teaching

There is a growing recognition{\b }amongst mathematics educators that students need to develop their own understandings of concepts. But for teachers to behave in classrooms in ways that encourage and acknowledge individual understandings, there are several possible dilemmas.

First, there is a potential problem because of the common view of the nature of mathematics as a shared reality{\fs18\up6 \chftn {\footnote

Even if one recognises that this would be a socially-constructed form of "common" understandings which have gained cultural capital, there is a strong belief that such knowledge is "out there". It is encoded in our objects (books and other media) and acknowledged during human interaction so often that it has become reified as part of our objectified environment - a heritage to be passed on to students.

Mathematical knowledge is generally viewed as the "self-evident, taken for granted, practical and intellectual activities that have been institutionalised by a community of knowers" (Cobb, Yackel and Wood, 1988, p.100). But the fundamental tenet of constructivism - that knowledge does not exist without cognitive activity - is incompatible with the concept of an institutionalised, but external body of knowledge.

The institutionalisation of education generally is based on the perception

that there is a codified body of knowledge which needs to be transmitted from one generation to the next. Constructivism, on the other hand, views knowledge as "learner's activity". (Wheatley, 1991, p. 11)

A second dilemma is that those aspects of knowledge which have been taken as shared within groups and cultures, and indeed between cultures, can never fit with the experience of every student. If we are to develop a constructivist philosophy, the concept of mathematical "truth" must therefore be replaced with subjective and individual "viability" (Wheatley, 1991, p. 10). But given the popular view of mathematics as a reified body of knowledge and skills, it is difficult to accept that it can take on an alternative form, i.e. mathematics as a set of individual and temporary understandings, constructed through interaction with other people and through actions on the environment. If teaching is to be based on this "personal world" of each student, it becomes a teacher's responsibility to gauge the level of each child's understandings{\fs18\up6 \chftn {\footnote

The constructivist would claim that this can be "true" only in terms of our ways of seeing it. However, Kilpatrick (1987, p. 17) acknowledges that "Successful teaching, like successful communication, depend on having a good model of the other".}} and then to provide activities and ideas that are appropriate, yet challenging, as well as understandable within the child's developing schemata. This is a tall order, especially as the teacher can construct only a personal understanding of any child's knowledge (Lerman, 1989).

Thirdly, not many teachers view mathematics as dynamic, generative, fallible activity. On the contrary, the commonly accepted view of mathematics curriculum is of a regulated progression of well-processed content, packaged into controllable and familiar portions. This arrangement of knowledge into a sequence of skills and sub-skills (Wheatley, 1991) ensures maximum coverage and comforting predictability as well as relatively simple assessment material. To open up opportunities for individual curricula to be realised lessens teacher (and State) control over content. It also contradicts traditional classroom management approaches. But if we recognise that student learning involves choice (Gergen, 1985) and the active re-construction of received messages, what it is "to teach" is brought into question. While teachers can still set learning objectives, with constructivism there can be no assurances about what has been learned through imposition-based pedagogy (Bishop, 1985).

The development of personal autonomy in mathematics, a goal many teachers espouse, has the potential to conflict with broader cultural goals, with widely-accepted content objectives and with hegemonic relationships in classrooms. Constructivism implies a major shift in teaching from behaviourism to student-led development and to the sharing of mathematical processes as well as products. For instance, true negotiation of understandings would require that the use of standard algorithms could be based only on students' reaching agreement about these being meaningful solution processes.

In summary, the ramifications of accepting that knowledge is a unique syntheses of understanding which results from a learner's activity are vast. Most importantly, we can no longer assume that the efficient imposition of curriculum content epitomises good teaching.

Quality teaching: opinions of educators

The results which are presented in this paper are taken from part of a broader survey which was designed to seek the views of teacher educators on a range of the components of classroom teaching. The survey arose from earlier studies of views of student teachers where Mousley and Clements (1990) and Mousley, Sullivan and Clements (1991) sought to determine perceptions of mathematics classes during practicum sessions. They found that the student teachers' perceptions of teaching were the antithesis of current theory and of behaviours espoused in teacher education programs.

Over a range of schools, year levels and geographical positions, the students' reported curricula driven by texts and tests, of teacher

dominance of discourse and thinking, and of little attempt to make mathematics relevant to children's lives or to individual understandings.

There seem to be two plausible explanations for these perceptions. On the one hand, it may have been that the observations made by the teacher education students are an accurate portrayal of current teaching in mathematics. On the other hand, it is possible that these teacher education students may have observed some quality mathematics teaching but may not have identified the features of such teaching because of the subtle and sophisticated ways in which it differs from mediocre teaching. In any case, it seemed clear that student teachers were not having the types of experiences in schools which would assist them to develop educational philosophies, learning environments and teaching strategies to support knowledge construction.

Further discussions amongst the researchers led to a querying of whether the factors on the students' questionnaire would be accepted by the wider mathematics education community as aspects of quality teaching in today's climate of educational reform. Thus a survey was prepared to seek views on characteristics which are most important for quality teaching. The survey, which had mainly fixed format items, and some open response items, was first trialled by twelve teacher educators who were asked to complete the questionnaire with an observer present. The educators were asked to think aloud while responding to the items to determine the appropriateness of questionnaire items and wording. All comments and queries were recorded, then after appropriate revision the process was repeated with a further four teacher educators completing the survey and being interviewed.

The revised survey was mailed to three groups. There were 40 survey responses from graduate students in mathematics education (100% return), 56 from Victorian teacher educators (80% return), and 29 from American teacher educators (40% return). These groups were selected because they represent an informed view of current issues in teaching and learning. The graduate students were all experienced teachers who had elected to completed higher degrees studies in mathematics education. The Victorian teacher educators are members of a group which meets quarterly to discuss issues related to mathematics teacher education, and their survey forms were supplemented by a few responses from experienced mathematics education consultants. The U.S. teacher educators were drawn from the membership list of the International Group for Psychology in Mathematics (PME-NA) group.

In the structured part of the instrument, there were nine groups of descriptors. These clustered around the factors {\i teaching environment}; {\i aims}; {\i content}; {\i presentation}; {\i class activities}; {\i questions}; {\i aids}; {\i assessment} and {\i closure}. Each single descriptor was presented on a bipolar, connected with another by a line.

The respondents were first asked to mark on each line where a teacher ought aim in order to facilitate quality mathematics lessons. These marks were then converted to a scale from 1 to 7 so that comparison of "scores" could be made.
In the following tables, the overall mean of the placement of emphasis on each bipolar is marked with an "X", and the actual mean and standard deviation are shown.

After each group, the respondents were also asked to indicate which of the descriptors they considered the most important feature of quality lessons, and also to indicate the second and third most important. For each group, there was also an opportunity for respondents to add their own descriptors.

After completing all nine groups, respondents were asked to indicate which of the descriptors they considered to be in the 5-10 descriptors most important overall.

Views of educators on the aims of lessons

For this paper, only the results related to aims, content and presentation

- and how these might relate to the construction of mathematics concepts - are discussed.

Negotiating lesson content

The results for four descriptors related to aims of the lesson are presented in Table 1. The stem used to introduce this group of descriptors was "In the lesson ...".

In this set of responses, as with the following results, the main observation was of the diversity of replies. This is presumably a feature of the breadth of the teaching task itself. The diversity of responses may also be due to the different interpretations given to the terms used within the groups. Nevertheless it is interesting to describe whatever commonality or consensus does exist. Only results where there were significant numbers choosing a descriptor are presented.

Table 1: Descriptors related to the lessons

Ninety two of the respondents chose "there was a clear purpose" as the most important descriptor of this section, with a further 16 rating it a second most important. In contrast, only 10 respondents rated "negotiation of aims" as most important for the section, 19 respondents chose "the pupils were aware of the aims" and only 5 chose "the aims were negotiated".

Likewise, while 56 respondents rated "there was a clear purpose" in the 5-10 most important descriptors overall (and it was easily the most commonly chosen descriptor overall out of a list of 78) but the other three descriptors suggesting the involvement of students in setting aims gained very little support overall.

This suggests a contradiction: one would think that it is not possible to achieve a clear purpose and yet to allow disparate aims to be negotiated. A "clear purpose" is somewhat indicative of decisions being made by the teacher on the direction of the lesson, and this seems to be backed up by the fact that the majority of respondents did not think it is so important for the students to be aware of the aims.

Negotiation of lesson aims, content and activities seems consistent with the view of learning described above, because it would allow for different backgrounds of experience as well as individually meaningful activity. However, acceptance of this theoretical stance {\ul alongside} the expectation of clear purposes for a lesson provides an interesting challenge for teacher educators. The two positions are not necessarily incompatible, but it is clear that we need to explore how teachers may open up opportunities for more meaningful dialogues with students about lesson aims.

Within this same group of descriptors, it is interesting to examine the spread of responses on the pair of descriptors forming the "negotiated" to "imposed" continuum. Table 2 presents the percentage of respondents giving high-on-negotiation, medium-on-negotiation and low-on-negotiation as factors of a quality lesson. The responses are presented in three categories, graduate students, Victorian mathematics educators, and U.S. mathematics educators.

Again, there is a spread of responses. While some rated negotiation highly, others clearly felt that a mix of negotiation and imposition is more appropriate. There is no significant statistical difference between the groups of respondents.

Table 2: Distribution of responses (%) on the negotiated/imposed descriptor

Table 3 shows a break-down of results according to the background of the respondents. A rating of 1 indicates that the respondent felt strongly that aims should be negotiated while a rating of 7 favours imposed aims. There was also no clear trend related to the background of the respondents.

Table 3: Rating of categories of respondents on the negotiated/imposed descriptor

Another aspect of lessons which exposed the respondents' views on the teaching tasks is the priority they give to lesson content. The 11 descriptors related to lesson content are presented in Table 4. The stem used to introduce this group was "The content of the lesson ...".

Table 4: Descriptors related to the lesson content

Lesson content "was linked to pupils' experience" was selected as most important from this group by 28 of the respondents, and within the top three by 62. Twenty-four considered "was linked to real life" to be most important, 54 rated "provided some challenge" within the top three, as did 42 for " was mathematically important".

Descriptors from this section were marked within the 5-10 most important descriptors overall by significant numbers of respondents. These were "was linked to pupils' experience" (36), "provided some challenge" (31), "was mathematically important" (30), "was linked to real life" (27), and "was relevant" (21). As for the first set of descriptors, this set showed no statistically significant difference between the groups of respondents, either by country or by current professional role.

These findings, taken together with the first set, seem to represent a certain view of teaching mathematics. The combination seems to require a teacher who will decide the lesson direction and focus, but be sensitive to the needs and experience of the students.

Views on the lesson presentation

One critical feature of classroom teaching is the role taken by the teacher in the presentation components of lessons. The responses on the 13 descriptors related to presentation of lessons are shown in Table 6. The stem used to introduce this group was "The presentation by the teacher

Table 6: Descriptors related to the teacher's presentation

"Facilitated learning" was selected as the most important descriptor in this group by 35 respondents, and within the top three by 74. Other frequently chosen descriptors within the three most important were "was meaningful" (54), "encouraged pupils to be independent"(54), and "was interesting" (31).

"Facilitated learning" was chosen among the 5-10 most important descriptors overall by 42 respondents, "encouraged pupils to be independent" by 35, "was interesting" by 22, and "allowed pupils to find out for themselves" by 19.

The two descriptors from this set for which the means were least extreme were that the presentation "was negotiated" and "had the timing determined by the pupils". These descriptors refer to the extent to which the teacher relinquishes control over the content and the mathematics. It seems that despite the rhetoric of current theory, the respondents do not give this a high priority.

When the majority views for this section are combined with those of the previous two, the composite picture for a quality lesson develops as a teacher-led facilitation of learning, where the students are engaged in meaningful and interesting activities which encourage them to be independent and to think for themselves.

## Finding a way to describe teaching

What has emerged are trends in the majority views on quality mathematics teaching. The first requirement seems to be a teacher who is in control of the teaching, who has a clear purpose for lessons, who facilitates learning and is interesting, but who encourages independence and allows pupils to think for themselves. The second factor seems to be lesson content which is challenging and mathematically important, but based on the children's experiences with emphasis on relevant, real-life experiences.

Clearly, a survey is no way to resolve, or even identify, specific issues relating to the incompatibility of constructivist philosophies with institutionalised practices. It should be recognised, however, that the respondents to the survey are all experienced educators whose work is mathematics education. They are professionals at the centre of both developments in theory and innovations in classroom practice as well as people who are responsible for reconciling the two. Mathematics educators need consistent and mutually supportive views of knowledge, learning and teaching. It seems that their responses are compatible with the constructivist views of learning, but not with the notions of mathematical knowledge being a production of the individual{\fs18\up6 \chftn {\footnote

This same tension -- between knowledge existing only in the mind of the knower and knowledge as a socially-developed part of a culture external to the individual -- has been evident in developments of epistemological theorising. While earlier literature (e.g. von Glasersfeld 1983a, 1983b) focused on the notion of knowledge being the product of individual organisation of experiential worlds, much recent theorising (e.g. Bauersfeld, 1988; Bauersfeld, Krummheuheuer & Voight, 1985; Ernest, 1991; Wertsch, 1991; Yackel, 1992) has examined the idea of knowledge as social mediation. }} or learners actively creating their own learning objectives.

## Conclusion

Because of the influence of theorists who have argued that gaining knowledge is an active process, pedagogy {\ul is} changing. A growing emphasis in primary schools on "real world{\fs18\up6 \chftn {\footnote
 Unfortunately, this is still not often the "real world" of the student.}}" mathematics, exploratory learning, solving problems in groups and the making of mathematical models are indicative of the impact of new beliefs

about learning. Similarly, the emphasis in secondary schools on modelling, investigations and projects is the result of changing pedagogical philosophy. But so long as teachers consider mathematics as a "body" of knowledge distinct from individual conceptual activity, teaching will remain a process of imposition.

We, as mathematics educators, need to consider ways of conceptualising knowledge, and hence re-conceptualising methods of pedagogy, so that the individual and the social are recognised to be indissolubly interconnected. We need to research ways of
conveying through everyday classroom practice a belief that individuals create social mathematics and also interpret it;
supporting the creation of communities of understanding (Johnson, 1987) that are based on individual experiences - rather than further refining how to teach "given" concepts;
using interpersonal processes to turn subjective mathematical knowledge into acceptable objective knowledge (Ernest,1991) without destroying the notion of autonomous learning;
creating processes for the negotiation of learning goals, with a recognition that students actively set and pursue these by selectively interacting with communications from others.

In summary, if we are to accept current theories about knowledge, we need to review notions of learning and teaching in the light of these

developments. While the starting point for research is most frequently {\ul how} students learn, perhaps this depends upon prior identification of {\ul what} they must learn. If mathematics is a body of objective knowledge (and the educators' views imply that at least some aspects of mathematics knowledge are objective), and if knowledge can be transmitted, then research can continue to be aimed at developing efficient ways to transmit understandings. But if mathematics is "learner's activity" (Wheatley, 1991) we need to reconceptualise our roles as teachers and as mathematics educators.

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