Assessing
and reporting mathematical problem solving

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Since the 1950s the problem solving abilities of students have become more and more the major focus of mathematics programs in at least the Western world. During the 1980s this emphasis on problem solving as the core of any school mathematics program has become the basic theme of mathematics curriculum renovations in UK, Europe, North and South America and Australia. For examples of this movement we need only look at government and educational community action following the publishing of the recommendations of the Cockcroft Report (1982) in the UK and in the USA following the influential statement published by the National Council of Teachers of Mathematics during 1989.

From the beginning, the teaching of problem solving in school mathematics has been beset by assessment difficulties. Large scale projects, such as the National Assessment of Educational Progress in the US, which are interested in more than a simple counting of right and wrong answers, mirror the major problems for assessment in this area – items derived from worthwhile problem situations have high error rates and it is seldom possible to track down the sources of the errors. For example, without expensive and skilful individual diagnostic procedures being instituted, it is not possible to tell whether an incorrect response is due to:

- inability to understand the information;
- lack of the prerequisite mathematical skills;
- use of a level of reasoning inadequate for the task set.

In addition, the increasing focus on problem solving as the keystone of a mathematics curriculum has paralleled a changing view on the place of assessment in a curriculum. Usefulness is now more likely to be judged on the basis of whether it can contribute to an explanation of the reasoning process involved. It is expected to assist in the diagnosis of a difficulty or to suggest another step in a teaching and learning procedure.

What would appear to be of most importance in school mathematics assessment is that problems be drawn from situations which the students are able to relate to, real life or not, and which require mathematical analysis.

In this paper two new approaches to the assessment of problem solving are presented. A unique feature of both is that their assessment results are connected directly to suggestions for further learning experiences, rather than simply reporting the current 'state' of the learner. The first assessment instrument is the Collis–Romberg Mathematical Problem Solving Profiles developed by Kevin Collis (University of Tasmania) and Tom Romberg (University of Wisconsin at Madison). Their approach is based upon the use of the Structure of Observed Learning Outcomes (SOLO) taxonomy applied to various aspects of mathematics. The second approach is that taken by the Profiles of Problem Solving (POPS) developed by Kaye Stacey (University of Melbourne), Susie Groves (Deakin University) and Sid Bourke (University of Newcastle). In POPS several problem-solving skills are assessed in relation to problems in context. Both assessment approaches have merit and are highly complementary to one another. A teacher who used both would be in an extremely strong position whern it came to planning further learning experiences for their students.

The Collis–Romberg Mathematical Problem Solving Profiles

The format of the items in these Profiles is derived directly from what Cureton (1965) called ‘superitems’ which he had based on one traditional style of test which had been around for a long time. In some traditional tests, items come in groups, e.g. paragraph-reading tests with several questions on each paragraph, or table-reading tests with several items on each table. The problem situations, or stems, (paragraphs and tables, for example) contain considerable information about the situation. These sets of questions Cureton called ‘superitems’.Problem solving items need to yield information about the level of a student’s reasoning with regard to each problem situation. To cater for this important aspect of assessment the SOLO (Structure of Observed Learning Outcomes) taxonomy (Biggs & Collis, 1982) was used as the blueprint for the development of superitems with questions at four of the five structural levels of the concrete symbolic mode. The reasons for using this taxonomy are that the levels of reasoning reflect the current neo-Piagetian view of skill development. Thus, rather than simply categorising errors and inferring how reasoning was carried out, the responses can be aggregated with respect to this framework.

This far, we have described the SOLO taxonomy in general terms.The SOLO taxonomy categorizes people's thinking by their use of the relevant data in a problem. The categories and their definitions are as below.

Unistructural: Use of one obvious piece of information from the problem stem. ≈9 years of age.

Uni-Multistructural: Transitional stage between the previous and the next level. Students appear to be sometimes at the Unistructural level, and at other times at the Multistructural level. ≈11 years of age.

Multistructural: Use of two or more separate pieces of information contained in the stem. ≈13 years of age.

Relational: Use of an integrated understanding of two or more pieces of information contained in the stem. ≈17 years of age.

Extended Abstract: Use of an abstract general principle or hypothesis which is derived from, or suggested by, the information in the stem. ≈17+ years.

For specific content areas it is clear that account must be taken of the particular idiosyncrasies peculiar to that area. For school mathematics the target mode for learning is the concrete symbolic (see Collis & Biggs, 1991). The items here were designed to assess a student’s level of problem solving ability in school mathematics by asking a series of questions about a problem in such a way that each succeeding correct response would require a more sophisticated use of the information given than its predecessor. This increase in sophistication was planned to parallel the increasing complexity of structure noted in the SOLO categories.

The Collis-Romberg Mathematical Problem Solving Profiles consist of four main elements:

- Profiles assessment items;
- profiles diagnostic forms for recording individual and group

information;

- guidelines for using the Collis-Romberg Mathematical Problem Solving Profiles for diagnostic assessment;
- suggestions for further teaching based on the student's current level of understanding.

The Profiles are designed to measure a student's performance on five aspects of mathematics which form a foundation for problem solving. These aspects are:

Algebra
Chance and data.
Measurement
Number
Space

The measures obtained on these topics are interpreted using both the SOLO taxonomy and the number of correct responses through the use of individual diagnostic forms.

This idea is probably best understood by considering an example. For the following item it is intended that in order to obtain a correct answer, the student would need to process the information in the stem in at least the ways described below:

This is a machine that changes numbers. It adds the number you put in three times and then adds 2 more. So if you put in 4, it puts out 14.

U If 14 is put out, what number was put in?

(answer: 4): One piece of information is used, one closure is required, and the information is obtainable from either the last sentence in the stem or the diagram - unistructural response level.

M If we put in a 5, what number will the machine put out?

(answer: 17): All the information is used in a sequence of discrete closures; the stem is seen as a set of instructions to be followed in order - multistructural response level.

R If we got out a 41, what number was put in?

(answer: 13): All the information is used, but in addition the student has to extract the 'principle' involved in the problem well enough to be able to use it in reverse; the student needs an overview of the instructions in the stem to carry out the appropriate operations - relational response level.

E If x is the number that comes out of the machine when the number y is put in, write down a formula that will give us the value of y whatever the value of x.

(answer: y = x - 2) The student has to extract the abstract general principle from the information and write it in its abstract form, which involves dismissing distracting cues, perhaps forming hypotheses and testing them, and zeroing in on the relationships involved - extended abstract response level.

To facilitate the classroom use of the Profiles the complexities of the SOLO taxonomy need to be overcome. Once a Collis–Romberg Mathematical Problem Solving Profile has been administered correcting has been made quite simple through the provision of answer keys. In cases where a student's answer is unclear, teachers should of course refer to the working of the question for guidance, and if necessary, to the student.

Once the Profile problems have been corrected, the Profile itself may be prepared. Unlike achievement assessments, the Profiles are constructed from more than just the number of correct or incorrect responses made by the student. Though this means, of course, that there is more to do, the information gained through the full analysis is a more than sufficient recompense for the effort.

It should be remembered that the Profiles classify a student's response on particular topics first, and then aggregate this information to give an overall level of functioning. Both pieces of information are important, but it is the identification of particular strengths and weaknesses that makes the Profiles so valuable in the problem solving classroom.

Appendix A contains a prepared sample Profile. These are the steps involved in preparing a Profile:

- The student's responses are checked, with ticks marking correct answers.
- The number of ticks for each row is totalled and entered in the box. at the right of the row. The topic names are written at the end of each row. A lower row total indicates a topic of more concern for that student.
- The next part of preparing a Profile is to ascertain SOLO classifications. This is found simply by shading the box where the last tick for each row is found. Now total the shaded squares for each column and enter the totals in the box below each column. The highest of these totals gives the over-all SOLO level for this student.

Where two columns have the same total, you should use other relevant information about that student to decide whether to allot them to the higher or lower category. In the case where no other information is available, caution would suggest that they be placed in the lower category until further evidence is available.

- The last part of preparing a Profile is to locate the lowest row total to ascertain the topic of most concern for this student.

The topic names are written at the end of each row.

Note that by joining the individual topic SOLO levels, it is possible to produce a visual impression of the student's over-all performance. Such impressions are a valuable adjunct to the SOLO performance level especially when a student's level of performance is at odds with other performance information. In all cases, however, the priority topics for each student need to be investigated, and the future learning pages are designed to give a start to this.

Many of the suggested activities come from the Mathematics Curriculum and Teaching Program (Lovitt and Clarke, 1988). These are labelled as MCTP together with the activity name and the relevant page number. Our sample student Mary, has been clasified as a unistructural reasoner, with weaknesses in Algebra and Measurement. The Collis-Romberg Teacher's Manual makes these suggestions for Mary's future learning:

ALGEBRA

Growth in this area can be provided by greater experiences with unknown quantities, the use of variables in equations and inequalities, representative symbols, functions, and formulae.

Here are some suitable activities.

* Have group discussions of teacher produced examples of algebra.
* Solve teacher–guided student–generated problems with small groups or pairs, discussing hows and whys and examining alternatives.
* Play with symbol-only input-output machines developing written equations for solutions.
* Use class developed nonsense situations. Students create symbol problems using agreed situations.

* Display pattern designs and have students repeat pattern relations to a different pattern design.
* Compare graphic representations of different fractions on number lines.
* Extend greater than and less than ideas into recipe or quantity representation.
* Create a code using symbols of letters instead of numbers. Solve a few 'letter' problems, i.e. if a = 6, b = 2, c = 1, then what is cab etc.
* Reverse the preceding experience and have numbers represent letters.
* Play games of making change. List a given number of purchases and their prices and ask if $20 will cover the cost.
* Decide how many goals are needed for allowing a team to win a game with a given score discrepancy. Discuss the alternative of the losing team scoring a few more goals (based on 'Sum Stories Game' page 264, MCTP).

MEASUREMENT

You might try these activities to strengthen measurement concepts at the Unistructural level.

* Have students record the length of a room with informal units, e.g. individual hand spans. Note the discrepancies; discuss the need for communication with other people, etc.
* Have a scavenger hunt with a small group looking for objects in the room which match given measurements or have students measure a list of items. Discuss.
* Have a student draw a picture of a problem relevant to them (i.e. a fence for their dog) and give dimensions of two sides. Ask a partner for the length of materials needed if it is a square, or if it is rectangular.
* Graph 100 'Smarties' by colours (50 red, 30 blue, 20 green). Ask

questions related to the chart (i.e.of the 100, how many red 'Smarties' are there? What percentage is blue?).

* Play a hide and seek game where eight hidden students have a number of either positive or negative value. Searching students find a hidden student who then signs the searcher's card. After ten minutes blow a whistle. Tally the value of each of the searcher's signature cards using the value assigned to each hidden member. Don't tell the searchers or hiders there is a negative value possibility until the end. (To simplify, don't get into negative numbers. Assign only positive numbers or zero for values).
* Play card games using number cards only.
* Play maths lotto (page 112, MCTP).
* Use a simple random number machine (lotto machine not a computer program) to invent games, record results, etc. Give every child a number and see the pattern of numbers when allocated 1, 2, 3, ...n at a time.

A variety of class organisations and recording methods can and should, be used with each of the above suggestions.

Profiles of Problem Solving

This profile is global in the sense that it draws on various areas of the mathematics curriculum and assesses several broad aspects of problem solving rather than focussing of specific skills or strategies. It has been designed for convenience of administration, being in paper and pencil format to be used by a whole class in about 30 minutes. Whilst one would agree with Charles and Lester (1982) that 'many of the goals suggested for teaching problem solving are not amenable to assessment through traditional pencil and paper formats' (p.32), some goals are amenable to variants of traditional assessment methods and a subset of these is assessed by POPS.

Because of the many different skills and abilities involved in problem solving, a comprehensive assessment of a child's problem solving performance requires data from a variety of forms of assessment including formal and informal observations of the child at work, practical tasks, and projects extending over several days. Teachers are urged to view POPS as just one of a range of complementary methods of assessment.

Schoen and Oehmke (1980) used word problems as a basis for multiple-choice items to assess specific problem solving skills, such as the ability to understand a problem statement and the ability to generalise the solution of one problem to another situation. Similarly the American National Assessment of Educational Progress (NAEP) mathematics assessments (see Carpenter, Corbitt, Kepner, Lindquist and Reys, 1980) included, for example, word problems with insufficient or extraneous data to assess children's success in 'understanding the problem', Polya's first stage of problem solving (Polya, 1957). In POPS, children are presented with tasks involving each of a range of strategies (such as finding and using patterns, working systematically and visualising) in a variety of mathematical contexts (such as numerical, geometric and measurement).

Problem solving tasks are therefore characterised by these features:

- The tasks should be unfamiliar in an essential way – children should be confronted with goals which cannot be reached by one-step routine application of knowledge that they have been taught. Word problems such as a teacher might use as exercises in the teaching of mathematical content, are not regarded as unfamiliar in this sense, even though many children find them difficult.

- The tasks should be concerned with familiar situations, potentially arising in the every-day (work or play) life of the child, where simple mathematical knowledge and mathematical thought processes are useful. For example, taking everyday usefulness as the criterion, the items should have a more obvious face validity than items in an intelligence test.
- At least in contrast to the usual tasks of school mathematics, the tasks should be relatively long, requiring some time for thinking and for the combination of two or more factors.

The mathematics content areas required in the items are the whole number algorithms up to multiplication by a multiple of 10, estimation, addition of money, recognition of units of volume and weight. As has been frequently observed in their studies (see for example, Hart, 1981: 212; Bourke, 1979: 273), many children do not use the algorithms they have been taught in school in tasks presented outside the normal school format. Division, for example, is often replaced by trial multiplications, multiplication is often done by repeated addition – a child with little knowledge of formal school mathematics but with a good sense of number can do quite well. Two of the items in POPS become routine with algorithms learnt later in schooling.

In addition to measuring how successful children are in getting right answers to problems (the product), an assessment of mathematical problem solving should also give information about the processes for solving problems which the children employ. There are many different problem solving processes and also many different detailed schemes which list and categorize these processes (see, for example, Burton, 1980; Polya, 1957; Charles and Lester, 1982).

The process categories assessed by POPS are

Correctness of Answer (Was the answer correct?)
Method Used (Was the approach efficient?)

Accuracy (Were the calculations free of errors?)
Extracting Information (Was the problem understood?)
Quality of Explanation (Was the explanation clear?)

The requirements that students write an explanation of their answers is one of the features that sets POPS apart from conventional assessments of mathematical skills. Explanation is valued and measured because communications is seen as an important skill in itself and because it is an important indicator of cognitive development (see Collis and Romberg (1992)). More practically, adequate explanations are essential to give the teacher insight into the processes that have been used. As with the APU experience (Ruddock, 1983) it had been feared that upper primary level children would not write sufficient explanation to enable the marker to judge the processes used. Although this remains a concern, the administration method used by POPS (lock-step) is reasonably successful in overcoming it by repeated emphasis on the importance of providing explanations adequate for the making of judgements on broad categories of problem solving skills.

The lock-step method of administration is used because during development of POPS children asked fewer questions of the teachers and teachers felt that it reduced reading and comprehension difficulties. Analysis of the data suggested that all children scored better, especially on the last question. Girls, in particular, were advantaged. The method was favoured also because many of the children filled in the waiting time after finding an answer by explaining their thinking more fully.

Developing a profile that is to be used widely puts stringent demands on the wording and readability of the items and the specification of administration instructions. The wording of the questions and the situations presented were constantly adjusted to ensure that almost all children could understand the real situation, even though it was clear that a sizable proportion of them were not able to deduce from the situation the appropriate mathematical relationships.

The five categories of process measures were arrived at by combining into logically cohesive groups the individual elements of process which could be identified commonly in answers to the six items. Once process categories had been established, detailed marking guidelines were developed through trial marking and consultation. In particular, concern was expressed that teachers using POPS should be in no doubt as to scoring.

There are six assessment items, most of which have several parts.

Medicine: Students are asked to calculate how many days a bottle of medicine will last given the contents of the bottle, the dosage and the number of times it is to be taken daily as well as some irrelevant data.

Coin Machine: A vending machine which dispenses chocolates indicates the price (of the chocolate) and the denominations of the of coins that could be used.

Houses: An aerial photograph of a partially-regular section of a city is supplied. Students are asked to estimate the approximate number of houses pictured.

Light Weight: Pictures of three types of paper clips are shown, and a different quantity and total mass is given for each. Students are asked which type is the lightest.

Dice: Three orientations of a die are pictured and students are asked to describe a hidden face.

Ladders: Diagrams of matchstick ladders are shown and students are asked the number of matchsticks required to make longer ladders.

The development of the scoring scheme for POPS was informed by the responses given by the students during development. Although this has resulted in a complex scheme, the insights into students' problem-solving development provide more than adequate compensation for the time invested. Because development is being assessed partial credit is given. Scores for each process are recorded on the score sheet, then totalled across questions and the results transferred to the POPS Individual Profile form. A Group Profile may also be constructed to give an over-all picture of a whole class or group.

The score for each part of each POPS question is entered in the appropriate space on the Student Score Sheet, and when all scoring has been completed, the totals for each of the process categories calculated. It is these totals which will form the basis for the POPS Individual Profiles and subsequent interpretations.

Constructing a POPS Individual Profile is very simple. (A sample Profile form is included in Appendix B). One circles the student's score for each of the processes. Those students who have a zero score for a process should be placed at the left-hand end of the continuum, and those who have a perfect score should be placed at the right-hand end. Any comments you may wish to record can be written in the appropriate comments box as you mark the scores on the profiles.

Now join these score-points down the page, from Correctness of Answer to Quality of Explanation, as shown below. This gives a visual impression of the student's development on these problem solving processes.

The profile generated by the method just described gives a visual guide to a student's current development in problem solving as measured on the five processes of the POPS instrument. As with any measurement there is always a need for caution and thus the POPS results must be interpreted carefully. The scores achieved by a student on any assessment must always be accepted as being representative of that student on that day for that particular assessment instrument. On another day a score could be different, even with the same assessment instrument. Re-using the same assessment instrument several times will not solve the problem of reliability of results, as a practice effect may occur. For a particular process a score alone is not sufficient to completely describe a student's development, so care must be taken to compare the POPS results with other relevant data such as the student's folios of work, observations, and so on.

Each of the process scales not only differs from the others in terms of the number of score points along it, but also in the spread of those scores. The position of each score point along a continuum is determined by the ease, or difficulty, of obtaining that score. For example, to obtain a score of 6 for Extracting Information, is easier than obtaining a score of 6 for Quality of Explanation.

Each Individual Profile is divided into three levels defined by shading. These levels are called Beginning, Developing, and Advanced. These levels define a student's development in a broader yet more useful way than scores, especially for communicating with non-specialists such as parents. The full description of these levels for each process continuum is described on the back of the Individual Profile. In the case where a student's score places their profile line near the boundary between two levels, it is advisable to be conservative and, if other performance data is unavailable treat that student as being at the lower level.

To assist terachers plan for the students assessed, POPS includes ideas for future learning. To give an idea of this section of the POPS package, reproduced below are some sample pages of the manual.

METHOD

This process aims to assess the strategies used by the student. The POPS items deliberately assess different strategies which represent only a sample of those available. Although it is not suggested that general problem-solving strategies be taught formally in the classroom, teachers should discuss them whenever the opportunity arises. Examples of particularly important strategies are: looking for patterns, working systematically, and recording what you do.

Teachers can:

- label strategies so that gradually a shared classroom vocabularly is established, based on experiences;
- choose problems which illustrate certain strategies well. Suggestions can be found in Teacher Tactics for Problem Solving (Stacey and Southwell, 1983).

Those children who had problems with Medicine could benefit from the likes of Find the Facts (from TOPS, Greenes et al, 1982) and similar activities which focus on the data necessary to solve a problem.

Coin Machine requires children to find all the possible combinations. Thus systematic listing is an important skill and there are several aspects of it which can be taught. The basic idea of holding one variable constant while varying others can also be found in Pam and Robert, Basketball Tournament, Perimeter Possibilities, Sums and Differences and Twenty Cents in Teacher Tactics for Problem Solving (Stacey and Southwell, 1983).

The Method score for Coin Machine requires a judgement about the use, or not, of a system. Examples of systems are:

- Number of 5¢ coins
  5¢ -- 20¢, 10¢;
  5¢ -- 10¢, 10¢, 10¢;
  None with two.
  5¢, 5¢, 5¢ -- 20¢;
  5¢, 5¢, 5¢ -- 10¢, 10¢;
  None with four (odd number needed);
  and so on.
- Hold the largest coin fixed
  20¢ -- 10¢, 5¢;
  20¢ -- 5¢, 5¢, 5¢;
  10¢ -- 10¢, 10¢, 5¢;
  10¢ -- 10¢, 5¢, 5¢, 5¢;
  10¢ -- 5¢, 5¢, 5¢, 5¢, 5¢;
  5¢ -- 5¢, 5¢, 5¢, 5¢, 5¢, 5¢.

There are many different strategies for estimation and children will need to use some of them to solve Houses. Two useful strategies are:

- round the numbers involved and employ some compensating rule to improve accuracy;
- use sampling to gauge the overall result.

Teachers should ensure that their teaching of estimation includes estimates of quantities other than just numerical. The activity Hundreds and Thousands from Teacher Tactics for Problem Solving (Stacey and Southwell, 1983) and the Map of Australia activity from Mathematics Curriculum and

Teaching Project (Lovitt and Clarke, 1988) would be useful resources here. The problem Dice requires skills of visualization. When administering this item teachers may notice that some children get the correct answer very quickly. These children are probably mentally rotating the whole cube and because this process is hard to verbalize, these children often cannot explain what they have done. (It is for this reason that explanations are not required for this item). This mental rotation is a powerful ability and can be enhanced through practice. Suitable resources for this include Tangram Sets, Tangrams for the Overhead Projector, Tangram Template, Tangrams and Tangram Treasury, all of which use tangram puzzles as the medium for visual skills strengthening. A wider range of materials would include Symmestries with Pattern Block Designs, Visual Thinking Cards, Geometric Playthings, Build Your Own Polyhedra, Exploring with Polydrons, Puzzles in Space, and DIME materials. Practical exercises with real 3-D objects is essential and includes tinkering with building blocks, Lego and other similar materials. (Some believe that boys have this visual ability better developed than girls because they have had more experiences playing with construction toys).

It is also possible to use analytic techniques to supplement visual skills. For example it can be observed that on the die every sign is adjacent to four of the others and opposite the other one. The problem requires us to find what is opposite the tree, hence not adjacent to it. The views show that the four neighbours of the quartered circle are the cross and the molecule in the first view, and the arrow and the circle in the third view. This means that this is what is opposite the tree. Teachers should discuss these analytic methods as they try to build visualizing skills. Suitable problems from Teacher Tactics for Problem Solving (Stacey and Southwell, 1983) would be Square, Stacks and Train tunnel.

The Ladders problem assesses a student's ability to recognize and use a pattern in more and more difficult ways. Most students recognize the geometric pattern (although some will not) and also recognize that 3 more matches are needed for every new rung. They can use this for the 112 rung ladder, but not the 1000 rung ladder. This question requires a more powerful generalization. There are many problems which can be used to help students practise finding and using patterns. For example, from Teacher Tactics for Problem Solving (Stacey and Southwell, 1983) there are Alphabet Blocks, Village Streets, Handshakes, Magic Plant, Staircase, Paper Fold, and Diagonals.

QUALITY OF EXPLANATION

It is important to be able to explain mathematical ideas clearly and be able to communicate results. Another reason for the importance of explanation is that it forces students to review what they have done, providing an opportunity for checking their logic and calculations. Explaining mathematical ideas is difficult. It has been observed that as a general rule children will be more capable of doing than explaining what they do at all levels and at all stages of development. At first children will only be able to explain parts of what they do, but this can be improved upon. Children's explanations can be improved in a variety of ways:

- emphasize that, just as in writing, several drafts are often needed, so the same methods can be applied;
- gradually encourage children to explain 'why' as well as 'what' they did when they give an answer. Provide samples of what this might look like. For example, in Light Weight:

I found the weight of one paper clip by dividing the number of clips by the weight and I thought 1 remainder 1 was the smallest answer.

For the Medicine question explanations might be:

I multiplied 3 by 20 then divided 60 with 30.

or

I multiplied 3 by 20 to find the daily dose and then divided to find how many daily doses were in the bottle.

- children should have time to practise explaining. Try explaining over an imaginary telephone – geometric shapes could be described to a friend;
- let children explain to a friend how to solve a problem. The friend writes the instructions and then attempts to solve the problem;
- encourage children to use the units associated with the problem. For example 100 gm for 60 paper clips, rather than just a 100 for 60;
- have children make posters and write about their mathematics saying 'why' as well as 'how' for their classmates and friends;

The essence of explanation is to link the situation with the calculation, prior to solving the problem and then back again from solution to the original problem afterwards. Progress to good explanations may be slow as children have typically not experienced this aspect of mathematics before.

CLASSROOM RESOURCES

The Arithmetic Teacher. This journal contains articles on problem solving in each issue throughout the 1980s. 'Problem Solving: Tips for Teachers' is a regular feature during this period. A collection of these articles has been published and is listed under O'Daffer in the list below.

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APPENDIX A

Copies of the items contained in Appendix A may be obtained through inter-library loan from the Librarian, The Australian Council for Educxational Research, Post Office Box 210, Hawthorn, Victoria 3122.