A METHOD OF ANALYSIS OF MULTIMODAL FUNCTIONING IN ELEMENTARY MATHEMATICS (A paper prepared for AARE Annual Conference, Deakin University, Nov. 1992)

Introduction

During the last decade the role played by intuitive and imagery based processes in the development of mathematical understanding and hence in the need for corresponding instructional practices has been highlighted by a number of researchers (e.g., Presmeg 1986; Maher & Alston, 1989; Moses, 1982; Yancey, Thompson & Yancey, 1989 and Wheatley, 1991). In a recent paper, Collis, Watson and Campbell (1991) found that ikonic functioning played a major role in the solving of novel mathematical problems by adolescents in the top streams for mathematics. Such ikonic functioning was associated with high levels of response on scales measuring vividness of imagery. In addition Davis (1984) comments that students are passing abstract mathematics courses without developing "an intuitive 'gut feeling' for the subject" (p.373), and he argues the need for a revision of overly-abstract curricula. Moreover, Shaw and DeMers (1986) showed a link between imagery ability and creativity in high-IQ children. These studies suggest that the development of mathematically competent students involves not only the traditional focus on computational and logical problem solving abilities, but also the development of associated ikonic skills. While there has been an increasing interest in the intuitions associated with mathematical understanding, their interaction with logical reasoning abilities, has not received much study. A theoretical model which provides a starting point for a consideration of this interaction is provided by Biggs and Collis (1991). Biggs and Collis (1991) and Collis and Biggs (1991) argue that much of our thinking in the area of problem-solving is multimodal. In particular, when school students are presented with a novel mathematical problem, they have available three modes of intellectual functioning which they can bring to bear to seek a solution: sensorimotor, ikonic and concrete symbolic. The first of these is usually not of great significance in school-based problem solving and will not be discussed here. The second is highly developed by the time students reach school and has reached this high level largely without school help. The third is very dependent on school-based teaching, and is exemplified by the learning of the symbolic systems of reading, writing and arithmetic. The study by Collis et al.(1991) mentioned above teased out the use high achieving students made of the ikonic (IK) and concrete symbolic (CS) modes by using a variation of Haylock's Think-Board (Haylock, 1984) as shown in Figure 1 and which may be described as follows. The broken line divides the board into two regions, the upper region represents concrete symbolic (CS) responses while the lower half represents ikonic (IK) responses. Each of these regions is divided into three smaller regions by the diagonals, these smaller regions represent subsets of the CS and IK regions of which they form a part. Figure 1. The Adapted Think Board   The students' responses were categorised according to the following criteria.   Structure Recognition: The recognition of a CS structure in a problem as similar to a previous example, e.g., "This problem will be solved using the same mathematical ideas that I used in the last one." Symbols: The recognition of an appropriate CS procedure to solve the problem, e.g., "This should be able to be solved using simultaneous equations".   Diagrams: The use of a CS type diagram, e.g., Venn Diagram, Graph, etc to solve the problem. Images: Reporting visual images related to the problem, e.g., "I can see the pieces of the cube with one side painted." Reality, Beliefs etc: The use of real world experience etc which

appears to have some 'practical' relationship to the problem, e.g., "Everyone knows that you can't run a car without petrol!!" 'Aha' Experience: A sudden, apparently unbidden, 'insight', into the structure of the problem, usually visual, e.g., "Oh I see what I have to do" or "I see what it's getting at."   As the earlier data were reanalysed and a larger sample of interviews with different mathematical problems, particularly school-based problems, was carried out, difficulties occurred when categorising responses at the boundary between IK and CS functioning. In other words it was often a difficulty to decide whether a response belonged to category 1 or category 6 and similarly category 3 or category 4 because the responses were not purely one or the other. For example an item involving finding the area of the walls of a typical drawing of a house, in which some walls were shown in the diagram and their opposite numbers were not, certainly involved knowing that the invisible walls existed and imaging them as of the same shape as the ones that were visible. The ikonic part of this task is obvious but to decide that the shapes were equivalent, that the given dimensions are the same and that they need to be accounted for would appear to be a learned concrete symbolic concept.   This latter notion is supported in an earlier paper by Campbell, Watson & Collis (in press) on the measurement of volume in which it was shown that, although children know intuitively (IK mode) that a solid figure made up of individual cubes must have supporting invisible sides, they do not see the need to account for them in counting the number of cubes (CS mode) forming the larger solid. Indirect supporting evidence is also given by cross-cultural studies carried out in New Guinea, South Africa and Australia (e.g., Harris, 1991) which show that identifying shapes such as rectangles and triangles is a cultural skill in what we call the concrete symbolic mode. The difficulty of classifying responses on the boundaries was catered for in the previous paper (Collis et al., 1991) by using a broken line between the categories and marking responses closer to or further away from the broken dividing line according to a judgement as to the weighting of each mode in the response. However, the investigator was forced to make a choice between the two modes on the basis of which mode was most involved and this was a possible source of unreliability that needed to be avoided because of the likely frequency of responses falling into these regions, particularly with school based problems. This paper reports the investigators' attempt to resolve the difficulties raised by the earlier method of classification as part of the overall investigation of adolescents' cognitions during both school-based and novel problem solving exercises.

Procedure

Sample: Fifty Grade 10 students from two public High Schools in upper socio-economic class suburbs of Hobart, Tasmania were interviewed. Girls and boys were evenly represented. Twenty-four Advanced, 15 Level 3, and 11 Level 2 mathematics students were interviewed using a standard set of problems.

Items: The items used in the interview (See Appendix) were two 'school- based' problems (Drink-Driving and House) and two 'novel' problems (Hungry Man and Painted Cube). The school-based problems were designed to test students knowledge of mensuration (House problem) and mathematical techniques such as simultaneous equations, ratios or graphs (Drink-Driving problem) and allow students to apply this school-based knowledge during problem solving. The Drink-Driving problem could be solved by using various strategies ranging from simultaneous equations to graphs, to a method of using ratios. The school-based House problem could only be successfully completed using a specific set of mathematical techniques related to surface area and volume measurement. The novel problems were ones not typical of school-based instruction and so their solution had a less straight forward relationship with learned mathematical procedures. For example, the Hungry Man problem could be solved using equations involving mathematical operations such as ratios or proportions, or by working backwards and imaging the sequence of events. The Painted Cube problem required direct visualisation rather than the use of learned mathematical strategies. For the purposes of this paper responses to part (i) of the House and Painted Cube problems, parts (i) and (ii) of the Drink-Driving problem, and the Hungry Man problem will be analysed to illustrate the technique used for assessment.

Interview Procedure: All interviews were conducted in a quiet room with the interviewer seated adjacent to the subject. Before each interview subjects were given a brief overview of the testing procedures and were told that the interviews were not to be used as part of their course assessment but only for the purposes of the research project. Students were given a folder containing the set of four problems, the order of which were randomly allocated according to a latin-square design. Prior to the administration of the four problems subjects were required to work through four introductory problems that were related to the mathematical techniques required within the problem set (e.g., simultaneous equations (Drink-Driving), area of a triangle (House), pythagoras theorem (House)). These introductory problems served as a preliminary assessment of the student's mathematical knowledge and ensured that each student was equipped with the necessary mathematical knowledge to solve each problem, without directly associating an operation with its respective problem. Students were then allowed to solve each problem at their own pace. They were also required to show working where necessary and to explain verbally their thoughts during the solving of each problem. Students could leave a problem at any time if they so desired. Interviews were tape-recorded and the time taken to solve each problem was recorded. Transcripts for each interview were then obtained and these assessed and scored according to the scoring criteria.

Scoring Criteria: Complete transcripts obtained from each interview were dissected into separate responses. The start of a new response was considered to occur if there was an obvious pause after the last response, or if there was an obvious change in the strategy used by the subject during the course of his/her problem solving. Separate responses were then assessed and placed into one of three categories ranging from a purely ikonic response (IK)(e.g., pure image, emotional response) to a purely concrete symbolic response (CS)(e.g., usually a learned mathematical technique), with responses that contained some interaction between IK and CS strategies being assigned to an IK/CS interaction category. The IK/CS category was further divided into visual and a non-visual subcategories. Visual IK/CS responses corresponded to a demonstrated use of imaging in the CS mode (e.g., ability to visualise culturally defined shapes such as rectangles) and diagrams (e.g., graphical interpretation of the data). In these cases, imaging and diagrams were considered to be used within a CS-related framework to visualise particular aspects of a problem. This visualisation process (an IK-loaded strategy) actively assisted in the problem-solving process. Since the ability to graph data and use available information to develop a high-level of visualisation (e.g., construction of a 3D representation of a 3x3x3cm cube) relies heavily upon direct school-based learning, an interaction between CS and IK-type strategies was assumed. A non-visual IK/CS interaction classification was assigned to responses that were largely IK-type beliefs formulated upon school-based information derived from the question (e.g., using numbers in a qualitative assessment); in other words, responses that were predominantly intuitively-driven based upon the information provided by the question. A pure IK response could be either visually-loaded or non-visual. This category was therefore split into visual and non-visual IK subcategories. A visual IK response corresponded to

a pure visualisation of objects, people and so on, that did not directly relate to CS-strategies in any way (e.g., Painted Cube- " see the red paint"; Hungry Man- "see the men asleep; see the apples"; Drink-Driving- "see a picture of Mark, Wayne or the accident"; House- "see a particular coloured house etc."). A non-visual IK response corresponded to a belief or an emotion that was not related to school-based knowledge and did not actively serve to assist in the problem solving task (e.g., "I hate fractions..."). A pure CS status was assigned to responses that showed the use of CS-type strategies which did not involve ikonic support (e.g., formulae; direct mathematical calculation), demonstrated the use of mathematically-applied operations only, to assess the available information, or simply involved the quantitative manipulation of numbers. Results The following examples compare different strategies that students adopt during a problem solving task. Each of the four problems is used to demonstrate how response patterns can differ between students during the course of problem solving.

1. Hungry Man problem
Interview No. : 45 Problem : Hungry Man
IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9Response 10Leaves problem after 5:34 mins

Interview No. : 33 Problem : Hungry Man
IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9Time taken: 1:12 mins Answer: Correct
For the Hungry Man problem, student 33 started with a non-visual IK/CS response that was associated with a rapid assessment of the problem-type, and then considered the use of visual strategies. This response was immediately followed by a series of effective CS-based responses that eventuated in a correct solution to this problem within minimal time. In contrast, although student 45 started with a non-visual IK/CS assessment of the problem, this response was followed by a number of inadequate CS-based strategies and then non-visual IK/CS-based strategies that failed to solve the problem after a much longer period of time. The responses of student 45 are in clear contrast to those of student 33. After a brief analysis of the problem using IK/CS strategies student 33 immediately proceeds to the solution via clearly defined CS procedures. On the other hand, student 45 can not seem to get a grip on the essence of the problem and has many IK/CS responses which show her confusion. .PA

2. Painted Cube problem
Interview No. : 33   Problem :   Painted Cube (i) IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9(i) Time taken: 2:00 mins Answer: Correct
Interview No. : 50   Problem :   Painted Cube (i) IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9Response 10Response 11Response 12Response 13Response 14Response 15Response 16Response 17Response 18Response 19Response 20Response 21Response 22Response 23Response 24Response 25Response 26Response 27Response 28(i) Leaves problem after 9:04 mins   Student 33 used visual images of the cube, here classified as IK/CS strategies, to solve this problem within a short period of time. In contrast, student 50 made no attempt to visualise the cube and demonstrated an ineffective alternation between pure CS and non-visual IK/CS-strategies when solving the problem. She discontinued the attempt after nine minutes. 3. Drink-Driving problem Interview No. :   45   Problem : Drink-Driving (i)+(ii) IKIK/CSCSVisualNon-VisualVisual

(Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9Response 10Response 11Response 12Response 13Response 14Response 15Response 16Response 1Response 2Response 3Response 4Response 5Response 6(i) Time taken: 3:36 mins Answer: Correct (ii) Time taken: 2:20 mins Answer: Correct Interview No. :   33   Problem: Drink-Driving (i)+(ii) IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 1(i) Time taken: 3:34 mins Answer: Correct (ii) Time taken: 27 secs Answer: Correct   The examples above demonstrate two different approaches to solving the Drink-Driving problem. Student 45 responded predominantly within a pure CS-based mode with occasional changes to non-visual IK/CS interactive strategies during the course of working on the problem. Student 33 in contrast, operated largely within an interactive IK/CS mode using visually-based methods to arrive at CS-based conclusions. These visual methods included graphical representations of the data that allowed this student to complete the problem successfully within a short period of time. 4. House problem Interview No. :   18 Problem: House (i) IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Response 3Response 4Response 5Response 6Response 7Response 8Response 9Response 10Response 11Response 12Response 13Response 14Response 15Response 16Response 17Response 18Response 19Response 20Response 21Response 22Response 23Response 24Response 25Response 26Response 27Response 28Response 29Response 30Time taken: 5:08 mins Answer: Correct

Interview No. : 50 Problem : House(i)
IKIK/CSCSVisualNon-VisualVisual (Imge/Dgrm)Non-VisualResponse 1Response 2Time taken: 45 secs Answer: Incorrect For the House problem to be solved successfully the student first needed to observe the diagram carefully and analyse every surface of the house, whether drawn explicitly or hidden, and second, to calculate each surface area using specific mathematical procedures. The examples above demonstrate two different levels of such discrimination whereby student 18, who correctly completed the problem, showed a pattern of responses that alternated between a visual IK/CS mode (i.e., discrimination of shapes on the diagram as well as ability to recognise hidden sides to the house) and a pure CS-based mode (i.e., specific calculation of surface area for each shape). In other words, this subject was able to recognise the steps needed to calculate the total surface area of the house. In comparison, while student 50 demonstrated an alternation between visual IK/CS and CS modes, she did not appear to recognise or understand the concept of surface area and showed a marked reduction in the use of these strategies.

Summary and Conclusions

From the examples given it is evident that during school-based and novel problem solving tasks students sometimes adopt strategies that require a close association between both IK and CS modes of processing. That is, there is an interactive relationship between IK and CS modes. This relationship is particularly clear when CS-strategies are accompanied by IK-based techniques such as diagrams or visual images that have a learned mathematical base (e.g., the ability to dissect visually a 3x3x3cm cube to locate each of its 27 smaller cubes; interpreting data in terms of a graphical representation). Here, the IK-loaded strategies serve to support effective implementation of CS strategies. Such examples therefore illustrate the need to specify those stages during which both CS and IK modes of processing are utilised during a problem-solving task. This paper therefore highlights the modification and refinement of past scoring procedures by including a separate category that defines the interaction between IK and CS strategies. Furthermore, it acknowledges the different types of IK support available, those being within either a visual (i.e., images, diagrams) or non-visual context. Additional modifications to the scoring criteria include either visual or non-visual pure IK strategies, while CS strategies are represented in a pure form. Comparison of the strategies used by students to solve school-based and novel problem-solving tasks revealed differences in the sequence of response patterns. In general, effective problem-solvers appeared to show a clearly defined pattern of responses with effective assessment of the problem-type being followed by an uninterrupted sequence of responses that led to a correct solution. For example, the most effective solution to the novel Hungry Man problem involved a clearly defined set of CS procedures, whereas an ineffective solution was associated with an increased frequency of non-visual IK/CS responses. Alternatively, a school-based problem such as the House problem required careful assessment of the diagram, to identify each shape in order to proceed with the use of specific mathematical procedures and calculations (i.e., a visual IK-loaded strategy associated with learned mathematical shapes). From the examples given, it was evident that the most efficient problem-solver showed a pattern of responses that alternated between visual IK/CS (i.e., diagrammatical discrimination of mathematically-learned shapes as well as the ability to recognise unseen sections to the house) and CS modes (i.e., following up the visual discrimination of each surface with a specific calculation for surface area). Here, an increased frequency of IK/CS responses within a visual context served as support for the implementation of CS strategies. The weaker problem-solver however did not appear to understand the concept of surface area and showed a pattern of responses in which the alternate use of visual IK/CS and CS strategies was markedly reduced.

In conclusion, there is a need to recognise and acknowledge the interactive relationship between IK and CS modes of processing, where IK-based strategies appear actively to support the implementation of CS-based strategies. Furthermore, this analysis of individual responses from a small sample of Grade 10 mathematics students suggests that neither school-based nor novel problem solving tasks are limited to a set response sequence. Some response sequences however appear to be more effective than others and this depends upon the structure of the problem.

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Appendix
DRINK-DRIVING PROBLEM

The blood alcohol readings of two drivers were recorded the morning after an accident. The readings were: Mark Wayne 6 hours after accident: 5 units 5 hours after accident: 7.5 units 8 hours after accident: 2 units 9 hours after accident: 5.5 units Assuming a linear relationship answer the following: (i) Who had the highest reading at the time of the accident?

(ii) When are their readings the same?

HOUSE PROBLEM

For the house shown above, given that it has a rectangular base, find the following: (i) the Surface Area.

HUNGRY MAN PROBLEM
Three tired and hungry men went to sleep with a bag of apples. One man awoke, ate F(1,3) of the apples, and then went back to sleep. Later a second man awoke, and ate F(1,3) of the

remaining apples, and then went back to sleep. Finally, the third man awoke and ate F(1,3) of the remaining apples. When he finished there were 8 apples left in the bag. How many apples were there originally?

PAINTED CUBE PROBLEM

(i) A cube that is 3cm by 3cm by 3cm was dipped in a bucket of red paint so that all of the outside was covered with paint. After the paint dried, the cube was cut into 27 smaller cubes, each measuring 1cm on each edge. Some of the smaller cubes had paint on 3 faces, some on the 2 faces, some on only 1 face, and some had no paint on them at all. Find out how many of each kind of smaller cube there are.