Constructivism in mathematics education: Chaos in action

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Abstract
Order in classrooms is an attribute which many teachers strive for, but recent innovations have brought relative disorder. However, the work of Benoit Mandlebrot encourages us to think of a new kind of order - that which grows out of disorder. This paper draws on data collected in four classrooms where teachers are encouraging the construction of mathematical understandings through the use of student-initiated group activities. Classroom scenarios are used in the examination of strategies that teachers employ to link the resulting 'disorderly' curriculum with the more traditional, socially-valued curriculum.

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Constructivism

The current move away from attempts to merely transmit knowledge in both primary and secondary mathematics classrooms is based on the premise that students must construct their own understandings of mathematics - because useful knowledge cannot be absorbed ready-made from teachers without it being adapted to fit prior understandings of children. Since the sixties, there has been increased emphasis on innovations in mathematics education such as 'discovery learning', 'manipulation of objects' and 'real-world problem-solving'. Recently, a push towards cooperative group work in both primary and secondary mathematics classrooms has resulted from a belief that students can come to better understand mathematical concepts through discussion and group exploration of ideas. This growing emphasis on teachers' allowing students to construct understandings, rather than on transmitting knowledge, arises from the constructivist movement. Constructivism is a notion apparently first recorded by the philosopher

Vico, who claimed that 'facts' are made by us and our way of experiencing, rather than existing independently in an objective world (von Glassersfeld 1983). While it would not be possible for a constructivist to claim this as a truth (for truths are also created by people), it did present a possible model of knowing, and of knowledge aquisition, through which pedagogy may be explored. The idea of knowledge construction as a major component of teaching and learning processes was further developed by Piaget (see, for example, Piaget 1937). He was influential in popularising the notion that it is 'operating on' the experiential world which leads to understanding it through the constructing of a relatively regular world from a flow of experience. Following this philosophy of learning, Piaget was also instrumental in the development of the use of real-world materials and experience-based problems in mathematics classrooms.

It would seem a logical and relatively simple move to introduce teaching styles which allow for constrction of mathematical knowledge. However, constructivism challenges many conventions of mathematics pedagogy. Essentially, constructivism implies that the imposition of complete concepts and rules through rote learning or via 'logical' explanations by teachers is to be replaced by activity which leads to students gradually adapting their image of the world (as well as their ways of measuring, describing and operating on reality) to their environment. While there is room in the constructivist classroom for generalisations, theorems and algorithms, these are developed (probably in a wide variety of ways) over time - largely by students themselves. Thus the traditionally quick transfer and repetitive practice of supposedly elegant ways of knowing can no longer be primary teaching objectives in mathematics classrooms.

While explanations - by pupils and teachers - have a proper place in a constructivist classroom, they are made within a meaningful context. If new knowledge is built on old, there is necessarilty a recognition of Wittgenstein's premise that explanation ceases to become meaningful, and therefore fails, when the student lacks the knowledge to interpret it with understanding. While the teacher may pass on a definition, formula or rule, understanding of it can only be given to someone who already has the necessary concepts, and who already is familiar with the 'language-game' in which it has a place (Wittgenstein, 1956). This has serious implications for our past assumptions and practices related to the sequential ordering of learning activities. Intervention in the learning process, by teachers, parents and authors of texts, also needs to be both careful and timely - a tall order when we are dealing with a large number of indivdual students.

A different pedagogical aim implies the necessity to assess different skills. This is unsettling for teachers because in the past traditional modes of assessment, largely measuring skill performance, have driven curricular practice. While there are many interpretations of constructivist philosophies, some more radical than others (see, for instance, Watzlawick, 1981; Cobb, 1986), the general goal of constructivist instruction is not the manipulation of numbers to find pre-determined answers, but 'the construction of increasingly powerful conceptual structures and the development of intellectual autonomy' (Cobb, 1988, p. 99). The latter is not generally the primary aim of present mathematics teachers. Assessment strategies will need modifying, for logically-correct solutions students have created themselves must be accepted as useful strategies until children are ready to perceive for themselves that other constructions are more efficient but equally as meaningful. Thus assessment needs to be more flexible, sensitive and responsive.

Curriculum content must also be less controlled in constructivist classrooms, for student-initiated topics will not follow the traditional pattern of well-processed content, packaged into sequential, manageable and familiar portions which ensure maximum stability, coverage and predictability. Yet a mathematics curriculum which is a regulated and basically linear progression of learning discrete mathematics facts, concepts, operations, techniques and skills is a widely-accepted premise in the Australian educational system - a security blanket which supports and is supported by formal patterns of classroom interaction.

Traditional pedagogical routines are comfortable in that they eliminate much of the challenge of the unexpected. Common social practices, writes Habermas, seek to eliminate the ambiguity and uncertainty that makes human action a personal statement; thus also working to some extent to depersonalise human interaction. Habermas (1971) illuminates this problem in his discussion of technique - standardised ways of acting to get to previously chosen ends. He feels that problems are generally treated in our society as technical situations, soluble by application of an 'engineering rationality'. He argues that orientation toward particular actions is guided by the fundamental interests of the educators, and is aimed at gaining 'certainty in the interaction' among human beings as well as control of the environment to guarantee this certainty.

With constructivism, not only content, assessment and classroom interaction are less certain: students and teachers are also presented with alternative pictures of mathematics itself. Instead of seeming like a fragmented, objectified and seemingly rationally ordered body of knowledge and skills to be learned and practised, mathematics becomes a set of temporary understandings, constructed through interaction with other students and the teacher and through actions on the environment. Instead of being divorced from reality and from inquiry, mathematics is experienced as a creative and dynamic, but not infallible, activity.

Chaos in mathematics education?

A curriculum movement which challenges common patterns of planning, lesson content, classroom interaction and assessment, as well as our ingrained notion of the nature of school mathematics, can create frightening outcomes. It can disrupt the traditional hierarchies of pedagogical decision-making to the extent where minor changes have unexpected consequences. It can disturb culturally and personally historical patterns of behaviour and habits of control which are learned so well during our own education, legitimated during teacher-training and supported by an expectation of administrators, parents and the pupils themselves (Mousley and Rice, 1989).

In a constructivist classroom, teachers cannot continue to be minor technicians within an industrial process, the overall goals of which are set in advance in terms of rational needs (Sheffler, 1965). On the contrary, we becomes partners in processes of mathematical exploration. But the notion of teachers as fellow learners is an unsettling one because we are then deprived of the familiar and important role of leading children along a well-ordered path towards socially-legitimated ends. We also lose the reassurance of having pupils respond in uniform ways - at least in so far as knowing that our students are able to perform particular operations by using accepted procedures.

With constructivism, teachers feel tensions caused by the need to teach socially-valued mathematics concepts, facts and skills to a wide variety of pupils who have very disparate interests, needs, skills and understandings and who often bring to the classroom a wide range of social and cultural credentials. What we have with constructivist approaches to teaching, then, is a dynamic field of uncertainty, so many teachers feel that recent innovations in mathematics education are leading to a state of chaos.

Scientific chaos

Our notion of 'chaos' as behaviour out of control has been challenged by the non-linear mathematics of dynamical systems - an important field of mathematical exploration over the last eighty years. Mandelbrot and other mathematicians, in exploring the iterative equations which produce fractal shapes, introduce random elements to create conditions of scientific 'chaos'. They are thus able to create mathematical models of many natural phenomena: their work has contributed to our understanding of patterns which occur in seemingly random elements in biology, meteorology, fluid dynamics, geography, botany, chemistry, laser physics, population growth and geology.

When working with this type of chaos, we are exploring a new kind of order, one with regularities of an unexpected nature (see Gleik, 1987) that do not have the linear and predictable 'reality' of mathematics as we know it. A small change in input can create spectacular, seemingly unruly outcomes. But while the formation of a shape appears chaotic, repetition tames the patterned results around foci ('attractors') so that new, unimagined fractal patterns are produced. Thus out of disorder, a new and possibly useful (as well as sometimes very beautiful) order is formed.

Perhaps a similar new order will be negotiated in constructivist classrooms. Careful observation of many classrooms in which practitioners have developed styles of teaching which facilitate the realisation of different curricula for individual students - through encouraging children to design, carry out and evaluate many of their own mathematics activities - leads to the realisation that these people are involved in the creation of a new kind of order. While their classrooms are necessarily busier, noisier and apparently more uncontrolled than traditional mathematics classrooms, these teachers use strategies which effectively link student-initiated activities with the more traditional, socially-valued curriculum. It was a search for some common strategies ('attractors'?) which led to the preparation of this paper.

Methodology

During a 'Language in Mathematics Classrooms' research project, with a focus on the analysis of discourse in fifteen mathematics classrooms, some

teachers were observed working in classrooms in which the activity could be considered relatively chaotic. Four of these teachers are the foci of this paper - teachers who, despite teaching a range of ages (Years 1, 5, 6 and 8) had similarities in their pedagogical styles which encouraged student-led development of mathematical understandings. These four teachers were selected because in each lesson observed in their classrooms there was little prior indication for the teacher of where mathematical activities might lead and few assurances regarding learning outcomes. Each teacher seemed to be operating with a strong constructivist philosophy.
In observing the classrooms, audiotapes and field notes were made. When students were working, one group was audiotaped and tapes were later transcribed. For the purposes of this paper, classroom transcripts were analysed in a search for strategies the teachers used to control and keep track of what was learned.

Results and discussion

Encouraging diverse understandings

Each of the four teachers allowed a lot of freedom in decision-making regarding how a particular idea would be explored. The Year 1 teacher encouraged complete freedom of activity while the other three teachers announced a topic from a relatively standard curriculum and then allowed to students to create their own starting points, methods of procedure and ways of presenting their work.

Sandi (Grade 1) (The children bring their own problems to school.) Well, some of you have started well. Is there anyone without a problem today? There are rice and containers on the wet table for you.

Mary (5/6) (Continuing from discussion in 'Morning Talk'.) Now there are enough Toblerone boxes for five groups. After you have made your groups, I want you to find your own ways of calculating the capacity of the boxes.

Gordon (6) (Percentage is to be a new topic.) I'll give you about thirty minutes to record all you can about 'sixty percent' and then I'll ask any person in your group to explain what you've written or drawn.

Sue (8) (Introducing pre-calculus idea.) Now here is your task. You must pick one of these topics, or another one, and explore it in two ways. Graph how it changes and work out how you would explain the rate of change. The rate of change - how fast it changes. It might be a steady change, or it might vary, and you need to be able to measure the rate of change at any time. Okay? … You know you can present work any way you like - next Tuesday.

Each link in the chain of communication of what these activities entailed involved the students in fitting the teachers' suggestions into frameworks of understanding which they had evolved during schooling and other experience. For the students in these four classes, mathematical content was expected to be negotiated between the members of the group. There was room for adaptation of the topic to fit previous learning, resources and time available as well as for the creation, by students, of a variety of learning contexts.

Encouraging group understandings
Group discussion, compared with whole-class discussion, provides more opportunity for students to elaborate their ideas and cognitive experiences can be mediated through language. It also allows judgements and reasoning to be opened up to contradiction by others in a relatively non-threatening environment. The freedom each of the observed teachers created allowed for the construction of shared understandings within groups. However, each made it clear that learning for indivduals was the reponsibility of the group as a whole.
Sandi Robert, tell me about your problem.

Mary But how did your group decide there wouldn't be ten square millimetres in a square centimetre, Aslan?

Gordon: Whoa! Wait a minute. This is excellent. Explain the 'three-fifths' bit to me, Simon.

Sue And if you work in groups, I will ask any one of you to teach the class what you have found.

Essentially, the recognition of shared understandings developing within groups is not far from the teachings of relativists like Popper (1962) who claim a 'mind-dependence' for all knowledge. Popper implies that what is true is true only for the individual or group who makes a particular claim; so the knowledge that each individual has, or each group shares, is completely relative to that individual or group. However, he acknowledges both mind-dependent individuality of knowledge and a more objective knowledge which results from the products of social interaction.

Encouraging interactive understandings
In each classroom, there were formalised opportunities for the sharing of findings between groups. Linking of common ideas was an important component of this experience.

Sandi: So we have had two lots of people talking about middles today. That's interesting. I wonder if any other people used the idea of a middle … half way … in between … average … in the middle?

Mary: We have three separate groups claiming that the Toblerone box has exactly half the capacity of a box the same size, but with square ends. Like this (Draws on board). Jan, you tell me first. How did your group find it is a half? I think it looks more like a third.

Gordon: This is good, Green. I saw Blue do a similar thing with a circle. Didn't you Annie?

Ideally, teachers find a fine balance between 'personal knowing' and 'socially-valued knowing', but this requires an acute perception of the pluralistic nature of knowledge and a recognition of the value of individual, but perhaps incomplete, mathematical understandings as well as of shared understandings.

Linking understandings with traditional expectations

Each of the four teachers demonstrated a valuing of individual perspectives but managed to draw this knowledge close to the traditional curriculum through the use of more formal terms and an indication of the wider nature and purposes of the knowledge.

Student (Grade 1): The tree. See, there were lots of leaves on the tree. This tree here. Then we knocked lots off. Now these leaves … here … were … up here. Our problem is how many.
Sandi: (Nods.) So you're going to find out what part of all the leaves - out of all the ones that were on the tree - are now on the ground? You are looking for a fraction - a part. Is that right? Do you want me to write your problem?

Student (5/6): So what you need to find is the net weight.
Mary: Yes. The weight of only the contents. There are other sorts of weights we need to know about. (Writes 'Net weight'.) Net weight. How can we find out the other weights that are used in packaging and trucking? They are important in many businesses and for your maths. How can we find out?

Student (6): It's three out of five parts. Look. (Points to drawing of one hundred people.) This lot (Points to top row) is a tenth. A tenth of all these (the hundred). Ten equal rows. Sixty percent is six rows.
Gordon: Sixty out of a hundred. Good. We write it this way. (Writes ' 60% = f(60,100) ')

Student (8): When you say 'explain the rate' do you mean be able to measure it, or just explain why it is quicker or slower?
Sue: Both. If it's quicker, you might need to know just how much quicker as well as why. Take this one for instance (pointed to 'engine speed') - you know why the motor has to run faster when you drive up a hill, but some people like engineers have to be able to predict how much faster. It should be possible to use a graph to make that prediction. (Some quiet discussion and laughter.) Right. Now have a talk with your friends about the type of graph that is most appropriate to show a rate of change, and which axis you would use for each variable. You have learnt about that, and its important to get it right so that others can intepret our graphs accurately.

Conclusion

Despite the fact that our culture has a body of highly-valued mathematical knowledge that is traditionally legitimated (Kuhn, 1970) and therefore implicitly defined as worthy of passing on, the children in any particular maths classroom have had a variety of family, school, peer-group, media-based and personal experiences. Individuality of personal subjective understandings of mathematics is thus to be respected.

In spite of its diversity, however, our society does have a powerfully common expectation that children will leave school competent in certain mathematical skills. This works strongly to prevent moves in classrooms toward constructivist approaches to teaching and learning in any curricular area. There is always the lurking logic that pupils who have not studied a particular content area will be less facile than those trained specifically to deal with it - the motivation behind timetabling, planning of sequential learning activities, artificial separation of curricula and of context areas, achievement testing and many of the other pedagogical habits ingrained in our schooling systems. Teachers feel a need to keep some control over the construction of understandings that takes place in classrooms.

However, observation in these four classrooms demonstrates that it is possible to maintain a constructivist philosophy while leading children towards facility with a pluralistic body of knowledge. This leads me to argue that constructivism does not have to represent chaotic disorder: clear expectations that all students will participate in the construction of individual, group and socio-cultural knowledge could bring a new type of order to mathematics classrooms.

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