ANALYSIS OF THE PROCESSING DEMAND OF COUNTING AND THE COUNTING SEQUENCE AS A PREDICTOR OF KNOWLEDGE OF PLACE VALUE IN THE EARLY YEARS OF SCHOOL.1

Gillian, M. Boulton-Lewis, School of Learning and Development, Faculty of Education, Queensland University of Technology, Kelvin Grove Campus.
Graeme, S. Halford, Department of Psychology, University of Queensland, St Lucia.

A sample of 55 children in years 1, 2 and 3 in three suburban schools in low, medium and high socioeconomic areas of Brisbane was tested for knowledge of the structure of the counting sequence and for other aspects of number including place value. Each child was interviewed individually early and late in the school year. The teachers were interviewed and asked to describe their objectives and strategies for counting and other aspects of number. Analysis of the processing load of levels of knowledge of the counting structure and place value was undertaken using Halford's structure mapping theory of cognitive development. Children's responses for counting and place value were analysed and categorized. This paper is a description of earlier research in children's counting, an analysis of the processing demand of levels of counting and place value, a description of the children's counting competence, the relation between the latter and teachers strategies, the relation between counting and place value knowledge and implications for teaching.

The focus of the research presented in this paper is to describe and explain levels of knowledge of the counting sequence on the basis of the processing load of the increasing complexity required at each level. Children's counting and teachers' strategies for teaching the counting sequence in the first three years of school are described and analysed. Because this research is part of a wider study, which aims to assess processing loads of representations and strategies used by teachers and children for a range of aspects of number in years 1 to 3, the relationship of levels of counting to performance with place value tasks is examined. Levels of counting knowledge as a predictor of performance on place value tasks and hence, by implication operations such as subtraction and addition, are considered.

SEQUENCE COUNTING

Fuson and Hall (1983) described three uses of counting. These were sequence counting (where number words are said aloud in the usual sequence), count or summative cardinal uses of counting words (where the child can explain relative numerosity on the basis of quantity), and measure uses of counting words (where the child can explain the generative rule on the basis of place value).

Fuson (1988) defined seven different uses of number words which are distinct but, obviously, interrelated. One of these uses is sequence counting, as described above, when number words are just said aloud (or later silently) in the usual sequence for a particular language, and in this case English. The defining characteristics of sequence counting are that the number words are used in sequence, they are used without objects, a conventional ordering already exists, there are developmental changes in their use, no referents are used, the count describes nothing (except perhaps movements or mentally visualised objects) and these sequence words can be taken as the sets for addition and subtraction operations. She argued, on the basis of her own and others research that by 4´ years most children can count objects in rows up to 20 with considerable accuracy (and hence possess knowledge of the counting sequence at least to that number).

Siegler and Robinson (1982) described levels of abstract counting where children count aloud without referents. This seems to be the same kind of behaviour as for sequence counting described by Fuson above. Siegler and Robinson identified three levels of abstract counting as follows. The first level included `children who did not count as high as 20, showed no obvious pattern in their stopping points, often omitted or repeated one or a few numbers, did not omit or repeat whole decades, used no nonstandard numbers, and did not count on to the end of the decade when given a three number running start'. The second level consisted of children whose counts ended between 20 and 99, usually stopped at a number ending in "9", often omitted or repeated entire decades, used relatively many nonstandard numbers, most of which involved concatenating decade numbers with 10, 11 or 12, and counted on to the ends of decades both within and beyond their counting range'. The third level consisted of `children who counted beyond 100, generally ended their counts at a "9" or a "0", occasionally skipped whole decades, occasionally formed nonstandard numbers by concatenating hundreds names and, when, asked to count on, both completed decades and went on to later decades within and beyond their previously exhibited counting range.

Siegler and Robinson suggest that children at the first level, who do not count as high as 20, may not know that there is any structure in the number string because it is not apparent in the numbers that they do know. We believe that this indicates that their counting behaviour is dependent on a rote learned sequence of number names in order. Children who count beyond 20 seem to be making use of some aspects of observed relationships and structure in the counting system. For example they try to generate decade numbers by concatenating tens and unit numbers. We believe that counting at the second level rests on some knowledge of a generative rule connecting the sets of digit and decade numbers. Children who count beyond 100 must note an additional rule involving the set of hundreds names and then concatenate these with the tens and digits numbers.

The first level of counting described by Siegler and Robinson is probably rote learned sequence counting. Fuson (1988) also described a first level of counting behaviour such as that described by Siegler and Robinson where the first counting words are correct and stable and the subsequent words, although stable for a time are incorrect. Such behaviour suggests that the sequence is probably learned by rehearsal without understanding of structure or rules for generating the numbers.

The second level of counting described by Siegler and Robinson probably depends on a mix of sequence counting, some knowledge of relative numerosity of numbers on the basis of their position in the counting sequence, beginning knowledge of concatenation of tens and digits, and rote learned next relationships between such numbers as 29-30. Whilst the purpose of counting at this level is not summative cardinal counting (cf. Fuson and Hall, above) it would need to be derived from the same knowledge base. Such behaviour should rest on understanding of relative numerosity on the basis of quantity and its relation to the counting sequence.

The third level of counting described by Siegler and Robinson is one where the child is apparently using knowledge of the base ten structure of the number system, implicitly at least, to generate `new' numbers in the counting sequence. Full understanding of the generative rule (to 100 and beyond) should depend on knowledge of the number sequence words in order to 20, the decade words in order to 100 (and their relationship to the set of digit numbers), the rule (implicitly at least) that when counting within a decade the next number after a two digit number ending in 9 is the next decade number. The invented numbers, such as 120-20, that appear at this level are probably examples of attempts to generate numbers beyond 100, based on some knowledge of place value, without fully knowing the rule that after 100 the number sequence from 1 begins again. Sequence counting at this level should be derived from the same knowledge base as the measure use of counting described by Fuson and Hall. Alternatively use of the generative rule may only require knowledge of a few rules about the conventional counting sequence that can be learned by modelling and a great deal of practice. The latter is however unlikely because the memory load would be very high . If the latter is the case then there should be no significant relationship between knowledge of place value and counting.

Omission of decade numbers, at the second level identified by Siegler and Robinson, probably means that the child does not know the sequence well enough. At the third level it probably means that the child only has limited knowledge of place value as a basis for generating the decade numbers, that is, by taking the numbers one to nine, changing the spoken form a little for some, and adding on `ty' because each decade is a equal to 1-9 sets of ten and the `ty' means ten.

STRUCTURE MAPPING THEORY OF COGNITIVE DEVELOPMENT

In order to cognize a new concept a child must construct a mental model that reflects the structure of the concept. A familiar situation is often used as an analog to assist or guide this structuring process. Suppose a child is given an instance of the addition operation 2+3=5. A concrete analog can assist the child to recognize the relations between 2, 3 and 5 as shown in Figure 1. A suitable analog would comprise a set of 2 objects and a disjoint set of 3 objects. As shown in Figure 1 the number 2 (first as a verbal and then as a written symbol) is mapped into a representation of the concrete set of 2 things, the number 3 is mapped into the concrete set of 3 things, and the number 5 is mapped into the union of the disjoint sets of 2 and 3 things. As Halford (in press) has argued, the concrete analog provides a template for the structure of the task. It is used to organize the problem into an appropriate coherent mental model.

INSERT FIGURE 1

The mental model of a mathematical concept can be used as a guide to the development of procedures. However it is well established that once the procedures have been developed they may be used without invoking the mental model (Anderson, 1987; Greeno, Riley & Gelman, 1984; Halford, forthcoming). This corresponds to the fact that understanding is required when devising a way to perform a task but is not always entailed in performing it.

The above does not entail the fallacy that use of sets as analogs for arithmetic requires understanding of set theory. What is required is that the child sees the correspondence between the set relations and the arithmetic concept. However the set relations can, and should, be made visible; e.g. the fact that a set of 2 objects can be combined with a set of 3 objects to produce a set of 5 objects can be demonstrated concretely. The child does not need to know the concept of union, in a formal sense, but only needs to see the sets combined. Neither does the child need to know the definition of disjoint sets, but only needs to see that 2 non-intersecting sets are used.

Mapping the concept into the mental model (structure mapping) entails processing loads, the size of which depend on the structural complexity of the task. Halford's structure mapping theory of cognitive development (1988; forthcoming) aims to account for cognitive development in terms of this structural complexity factor, together with the factors of capacity and knowledge. He has defined four levels of structure mapping the processing demands of which are known and described below. We present only a brief summary of the theory here as it applies to this research. A detailed description is to be found in (Halford, 1988; forthcoming).

The first level of structure mapping is that of Element mappings in which individual elements in one structure are mapped into individual elements in another structure, on the basis of similarity or convention; e.g. an image or word representing an object or event. Figure 2A is an example of an element mapping. In this figure a set of two sticks is the concrete analog for the mental model which is mapped into the numeral 2. The set of 2 objects thus is an analog for a mental model for the meaning of the numeral "2".

The next level is that of Relational mappings in which two elements and a relation between them are mapped from one structure to another; e.g. the relation between small numbers as represented by sets of objects. The child can see that a five-object set is greater than a four-object set by putting the sets into correspondence. Therefore a set of five and a set of four objects can serve as an analog for a mental model for the mathematical relation 5>4. In Figure 2B the five-object set is mapped into the number five and the four-object set into the number four. The relation greater than between the sets corresponds to the relation greater than between the numbers. This mapping is validated by the fact that the relation "greater than" between the sets corresponds to the relation ">" between the numerals.

The third level is that of System mappings in which three elements, with a set of relations between them, are mapped. An example is shown in Figure 2C, where a set of two objects and a disjoint set of three objects are combined to form a set of five objects. When the sets are mapped into numbers the union operation on the sets corresponds to the addition operation on the numbers. The mappings are validated by structural correspondence independent of similarity or convention. Neither the elements nor the operation need be the same in both structures. System mappings are therefore more abstract than element or relational mappings.

The fourth level is that of Multiple system mappings in which sets of four elements, with a set of operations defined on them, are mapped. An example of such a mapping is shown in Figure 2D. In this mapping the mental model consists of numerals and the operations defined on them, multiplication and addition. They provide a concrete example of the distributive law.

INSERT FIGURE 2

Each higher level of mapping permits more abstract concepts to be represented but imposes higher processing loads, due to the larger amount of information that is required to validate each mapping decision. Approximate ages can be related to these levels (Halford & Wilson, 1980; Halford, 1988). Most children should be capable of cognizing element mappings by the age of 1 year, relational mappings from 2 years, system mappings from 4 or 5 years and multiple system mappings from 10 or 11 years onwards. However lack of experience and hence knowledge in a particular domain may prevent a child from cognizing to the upper limit of his or her processing capacity.

STRUCTURE MAPPING AND LEVELS OF COUNTING

We have applied Halford's structure mapping theory of cognitive development (1988; forthcoming), as described above, to analysing the processing load of levels of counting. The first level of abstract counting, as described by Siegler and Robinson (1982), which is probably no more than basic sequence counting, can be conceived of as relational mapping for the following reasons. The first 10 or so numbers are probably learned on the basis of repetition and then as element mappings between sets of objects and the sequence of number names which have been heard and repeated so often that they are overlearned. Then on the basis of that knowledge the child probably continues to learn the subsequent names to 20, at least, by rote on the basis of some kind of next relationship between the number names. As a relational mapping this behaviour should be possible from about 2 years onwards.

The second level of counting described by Siegler and Robinson is probably a transition from cognizing at the level of relational mappings to that of system mappings. The child is relying mainly on rote learned next relations but is also beginning to observe and cognize the structure of the tens and units in the counting sequence. This kind of counting should occur between 2 and 5 years approximately.

The third level of abstract counting, described by Siegler and Robinson, which has some elements of the measure use of the counting sequence as proposed by Fuson and Hall (1983), is system mapping. This is because the child explains (sometimes imperfectly in a formal sense) and generates the counting sequence on the basis of a system of numbers that depend on powers of ten and place value. This should be possible, depending on other knowledge of the conventions of the number system, from 5 years onwards.

Counting at all levels, based on rote learning and rules, makes a demand on memory as described below and will interfere with cognition of place value and use of algorithms for addition and subtraction operations unless well learned. On the other hand understanding the counting sequence on the basis of place value makes an initial demand on processing load but, when known well enough to be used automatically, should not make any extra demand when used with knowledge of place value in solution of written algorithms.

STRUCTURE MAPPING, COUNTING AND PLACE VALUE

Analyses of each child's upper limit for the counting sequence, of their explanations for the position of selected numbers in the sequence, aberrations such as repeating, omitting or inventing numbers, knowledge of relative numerosity of pairs of numbers, place value explanations for 2 and 3 digit numbers and use of counting and/or place value knowledge or strategies for subtraction operations, provide data that allow assessment to be made of the processing loads of the individual concepts and possible relationship of each of these tasks singly and combined. This allows determination as to how well and to what level each component aspect of number must be known for use in further more complex situations without an increase in the processing load.

Analyses of processing demands of the strategies and representations used by teachers and young children for place value and subtraction are discussed by Boulton-Lewis and Halford (submitted, a; submitted, b). In the first paper, dealing with place value, we argue that the construction of the system of tens on the basis of knowledge of ones is a problem of part whole relationships and that this requires knowledge of compositions or binary operations which are concepts at the system mapping level.

INSERT TABLE 1

In Table 1 we summarize the mappings at the relational and system mapping levels that we believe are necessary for full understanding of place value. Research in children's counting (cf. Gelman & Gallistel, 1978) suggests that they first make a one-to-one correspondence verbally between numbers in sequence and objects in a set and then learn the rule that the last number named represents the quantity of the set. Steffe and Cobb (1988) propose a more detailed sequence of counting types from their research, which progresses from perceptual through figural, motor, verbal to abstract unit items. At the last level the child's counting is independent of sensory motor materials and the utterance of a number word, e.g. "eight" implies the sequence of numbers preceding it as well as the items that would be co-ordinated with that sequence. At that point Steffe maintains that the child has an abstract conception of number. The child must then make mappings between those concepts and the written symbolic representation of the number. Knowledge of all the preceding mappings would allow a child to make a global or one-to-one response to numbers greater than 10 without knowing how many tens and how many units there were in the number. That is the child would have knowledge of a number (e.g 16) as a whole only or as a collection of ones, rather than as a whole made up of parts consisting of tens and units.

Bell (1990) describes the application of Steffe & Cobb's (1998) counting types to conceptual structures for the unit "10". They culminate in a "numerical composite unit" which allows the child to consider a perceptual collection of ten items as one unit, while maintaining its numerosity, followed finally by an "abstract composite unit" where the child can co-ordinate counting by 10s and 1s when counting on.

Once the mappings are well enough known for small numbers and a collection of ten can be thought of as a unit then the child must learn that 12 is equal to 1 ten and 2 units and that the 1 in 12, when it is written in symbols, is really a conventional way of representing the 10. Automatic understanding and application of all these mappings should ensure that the child knows that a number, such as 12, can be thought of both as 12 units and as a set of 10 plus 2 units. In order to understand the part whole relationship inherent in the place value representations of numbers greater than 9 the child must be aware of the additive property at least, if not the multiplicative property of the units regrouped as tens, hundreds and so on. When children give inconsistent canonical and non-canonical responses to numbers, as described by Miura et al. (1998) they probably have some knowledge of part-whole relationships between tens and units in numbers beyond 9 but do not yet recognize that it is usual to group by tens and then represent such groups by digits in a specified place in a number.

Figure 3 shows the mappings that must be made between the spoken word, actual objects regrouped as tens and units, and symbolic representations for full understanding of 16. All these mappings impose a processing load and unless each is learned well enough to be automatic then the total load will be too great and interfere with understanding of place value. When children who do not understand place value are expected to use materials as representations of numbers greater than 9, in subtraction and addition operations, the load is further increased and it is no wonder that they try to learn procedures by rote or invent `buggy' algorithms. We describe such consequential problems for subtraction in Boulton-Lewis and Halford (submitted, b).

INSERT FIGURE 3

We believe that the generative rule for counting must be understood in the same way as full understanding of place value in numbers such as 16. Therefore use of the generative rule with understanding should require use of binary operations and be at the system mapping level. If the child, at the first or second level described by Siegler and Robinson (1982), tries to solve addition and subtraction algorithms, which require knowledge of place value, by counting then success with the algorithm will be problematic and the processing load of the task will be too great for understanding of the process to result.

METHOD

Sample

This consisted of 55 children, 18 in year 1, 19 in year 2, and 18 in year 3 in three suburban schools in Brisbane in low to medium (school 1), medium to high (school 2) and high (school 3) socioeconomic areas and the 9 teachers of those classes. The mean ages in months at the end of the year were, in year 1 (81, 77, 73), in year 2 (93, 92.5, 90.5) and in year 3 (103.5, 108, 103) in classes in each of schools one, two and three respectively. The mean age of the entire sample was 91 months. The children were selected to represent a range of mathematical performances.

Design

Each teacher was asked to identify in advance her objectives for counting and place value for the children in her class for the year. She was also asked to describe her strategies for teaching and the representations that she would use. Each child in the sample was interviewed individually before and after a period of instruction to determine level of counting and explanations for relations and rules and for knowledge of place value. The pre and post test interviews were conducted late in terms 1 and 3 and separated by 10 weeks approximately. All pre and post test interviews of the children were videotaped. The videotaped material was viewed, transcribed by the interviewer and then summarized and analysed in consultation with the authors. Children were tested for other selected aspects of number in a similar way. Results of research for place value, subtraction and other number concepts will be described elsewhere. However, for the purpose of assessing the relationship between counting and place value, levels of response for tasks were determined for each child.

Tests

Counting The testing procedure for counting, adapted from that used by Siegler and Robinson (1982), was as follows. Each child sat with the interviewer in front of a videocamera and was asked to count as far as possible from 1 without stopping. (highest correct count unaided). When the child stopped he or she was asked what number was next. if the child could not respond then the last number the child said was repeated as a prompt to encourage the child to continue. If the child counted beyond 120 he or she was stopped and asked how far she could count. If the response was e.g. 1000 then the child was asked which number was next.

The child was then asked, depending on the stopping point above, to count on from a given number. If the stopping point was less than 20 the child was prompted to start at 31, 11, 41, and 21. If the point was between 20 and 99 then 71, 31, 91, 51, and if the point was beyond 100 then 31, 171, 91, 151, 291, and 991 (as a check on scores for the first part of the test).

The child was also asked about next numbers depending on original stopping points. If the stopping point was less than 20 then he or she was asked what number comes after 10 and why, after 18 and why, after 29 and why; if the stopping point was after 20 then after 29, after 40, after 99, after 300, 1099, and 1999 and why (explanations of the structure of the counting sequence).

Place value The pre and post tests for place value were as follows. Children were tested for knowledge of place value in 8, 10, 12, 16, 18, 20, 30, 50, 80, 23, 27, 34, 56, 92, 101, 128, 265, 671, 998. These numbers were chosen because; 8 could be used as a training item as well as a test of whether the child knew that there were no sets of ten in this number; 10 because it represents a single set of ten or 10 units; 12 to 18 because these are combinations of tens and units and cause difficulties for children because of the way they are spoken in the English language; 20 to 80 because they are multiples of ten with no units and because they are often confused with the teens numbers and vice versa; 23 to 92 because they are combinations of tens and units of increasing size; and 101 and above to determine whether children could represent and explain numbers beyond 100.

A range of materials was provided for children to use as they chose. These included Multibase Arithmetic Blocks (MAB) in base 10 (tens and units for years 1 and 2 plus some hundred blocks for year 3), matchsticks (or iceblock sticks) singly and in bundles of ten, counters, a metre ruler, and the numbers, as above written on cards.

The child and the interviewer discussed the materials. The interviewer briefly explained any unfamiliar materials in case the child wanted to use them. The child was shown the first number and asked to read it; then to put materials underneath the number to show how much it was; the materials were covered and the child was asked how many tens and how many ones there were in the number; and asked to explain why the number was written in the way it was. Each child started at a number appropriate to his or her year level, proceeded through the numbers and stopped after difficulty with three tasks in succession.

RESULTS

Teacher objectives and strategies

The teachers in year 1 gave the following responses when interviewed. In school 1 the children were expected to count to 20 backwards and forwards and also by odd and even numbers. The teacher used stepping activities such as a floor ladder, rhythmic activities, rhymes with numbers to 10, actions, single items including the children themselves, and single sticks and bundles of ten. Some discussion of the structure of the counting sequence may have occurred when bundles of ten sticks were used. The activities in school 2 were similar but also included discussion of relations between numbers to 10, counting games, counting with calculators and counting when measuring. In school 3, as well the preceding activities children were encouraged to count beyond 20 to 100 in informal activities.

In year 2 the teachers responded as follows. In school 1 the children were expected to count forwards and backwards to 20, by 2, 5 and 10 to 99 and to identify, continue and create (generate) counting patterns to 99. The objectives in school 2 were similar to those in school 1 and the strategies were like those for year 1 in the same school. In school 3 the expectations were much the same as in the other two schools but there was extra activity with discs on a counting board.

In year 3 in school 1 children were expected to count forwards and backwards to 999 by 1, 2, 5, 10, 20, 25, 50, and 100. They counted with calculators and used objects to represent numbers to 999. They also engaged in activities where they estimated, analysed, explained and classified numbers to 999. In schools 2 and 3 the activities were the same except that in school 3 children were encouraged to count and explain numbers beyond 1000 and also used calculators to count by atypical numbers such as 6 and 8.

Categories for counting and place value

The counting behaviour of each child on the pre and post test was analysed as follows; The highest correct count unaided on pre and post test (HCUPRE & HCUPOS); the level of counting (1,2 and 3) (cf. Siegler and Robinson 1982) as described above, on the basis of stopping points, invented numbers, omission of decades and of numbers less than 20, and repetition of numbers (LEVPRE & LEVPOS); and explanations (3 categories) for the structure of the counting sequence (EXPRE & EXPOS). Explanations were coded as 1 if irrelevant or if the child could not explain, 2 if the child used a sequence explanation such as `it's next when you count' or convention explanation `that's how you count', and 3 if the child gave a place value or generative explanation . Note was also made of whether the child used fingers, head nodding or other aids to counting.

Each child's response to place value tasks for pre and post tests (cf. Boulton-Lewis and Halford, submitted b) was categorized as one of three kinds on the basis of their most frequent explanation (PVPRE & PVPOS).

These categories were 1 if the child gave an irrelevant or no explanation, 2 if the child gave a global or one-to-one response, and 3 if the child gave an explanation that clearly evidenced knowledge of place value.

Statistical analyses

The means of the highest unaided counts (HCUPRE, HCUPOS) by school by year are shown in Table 2. It can be seen that, except for the post test in year 3 in School 1, the mean counts for each year in each school were higher on the post test and that the mean counts generally in years 1 and 2 were higher than one would expect from previous research and from teachers' expectations for each year level (i.e. to 20 in year 1, and to 100 in year 2). In fact in schools 1 and 3, early in the school year, most children had already met or passed the teachers' objectives. This suggests that perhaps children should be encouraged to discuss the structure of the counting sequence and challenged to generate the counting sequence as far as possible. The post test results for year 3, school 1 are interesting. The teacher encouraged the children to count to 1000 but they seemed to think that to count a little beyond 100 was enough. The children in school 3 were generally a little younger than those in the other schools however as they progressed through years 2 and 3 their counting became more competent than children in the same years in the other schools. We stated above that cognizing the structure of the counting sequence was a task at the structure mapping level and should be possible for most children from 5 years onwards. The children in school 3 probably learned this structure faster because of the extra challenge provided by their teachers and perhaps by their home environment. A T-test for the entire sample for HCUPRE and HCUPOS produced a t value significant at >.01. All children then in all schools made significant gains in their knowledge of the counting sequence.

The mean levels for place value (LEVPRE & LEVPOS) by school by year are shown in Table 3. Children in schools 1 and 2 were operating at level 2 in year 1 (i.e. they gave global or one-to-one responses most frequently). The children in school 3 gave irrelevant or no responses at the beginning of the year but were giving the same kind of explanation at the end of the year as children in the other two schools. By the end of year 2 children in all schools gave place value responses for the items that they attempted. A T-test for the entire sample for LEVPRE & LEVPOS gave a t value significant at >.01. Children in all the schools made a significant increase in their ability to understand and explain place value.

HCUPRE & HCUPOS (that is the scores for the highest unaided count) were recoded as HCURPRE and HCURPOS where 1 was a stopping point less than 20, 2 was a stopping point from 20 to 100, and 3 was greater than 100. Crosstab analysis was used to summarize the most typical explanations for the counts less than 20, from 20 to 100 and greater than 100 (EXPRE & EXPOS). As described above an explanation was coded as 1 if it was irrelevant or the child could not explain; 2 if the child used an explanation based on sequence such as `it's next when you count' or convention `it's just like that' or `that's how you count'; and 3 if the child gave a place value or generative explanation. These Crosstabs are shown in Tables 4 and 5. It can be seen that on the pretest at level 1 of counting the most frequent explanation for the counting sequence was convention with one irrelevant explanation; at level 2 it was convention with still some irrelevant explanations and one generative explanation; at level 3 it was still predominantly convention with an equal number of irrelevant and generative explanations. By the post test no children were counting at level 1. At level 2 the most frequent explanations were irrelevant or convention with almost as many generative explanations. At level 3 of counting the most frequent explanation was convention followed by generative explanations. There were still some irrelevant explanations or children who could not explain the counting sequence at this level. This seems to indicate that the move from irrelevant to convention to generative explanations follows an increase in the level of the highest count rather than precedes it. It shows a close relationship between the highest level of counting and the ability to explain the structure. It would seem that the child can first generate the sequence without being able to explain it fully and once having done that can then begin to explain the process. A T-test for EXPRE and EXPOS for the entire sample gave a value significant at $p = 0.5$. This means that there was a significant overall improvement pre to post test in all years in levels of explanations of the counting structure.

Partial correlation coefficients were computed for the sample as a whole, for LEVPRE and PVPRE controlling for year, age, and year and age together; and for LEVPOS & PVPOS controlling for the same factors. These correlations for level of counting and place value were significant ($p <. 05$) when age was controlled on the pretest. For the posttest they were significant ($p < .01$) when year, age and year and age were controlled. There was therefore a close relationship between levels of sequence counting and knowledge of the place value in numbers regardless of age on the pretest and of age and year in school on the post test.

LEVPRE, EXPRE & HCUPRE were used as predictor variables in stepwise multiple regression analysis for the dependent variable PVPRE for the entire sample. Multiple regression analysis was also computed with LEVPOS, EXPOS & HCUPOS as predictor variables and PVPOS as the dependent variable. LEVPRE accounted for the greatest proportion of the variation in PVPRE ($R2 = .34$). The inclusion of EXPRE increased the variation to .40, and HCUPRE made a difference of only .01. On the post test LEVPOS again accounted for the greatest proportion of the variance in PVPOS ($R2 = .36$). The inclusion of HCUPOS and EXPOS increased the by only .03. These figures suggest that on both pre and post test the level of the child's counting, as described by Siegler and Robinson (1982), is more likely to predict knowledge of place value than how far the child can count or how well the child can explain the counting structure. There is however a large proportion of the variance in place value knowledge unaccounted for by these variables. The F value for these analyses was significant for all these analyses which shows a linear relationship between the variables.

DISCUSSION

It has been suggested in the literature that children's proficiency with counting and knowledge of the structure of the counting sequence should be an indication of their knowledge of numbers generally. Siegler and Robinson (1982) cite Pollio and Whitacre (1970) as reporting `that the length of preschoolers' counting strings is an excellent predictor of their ability to establish 1-1 correspondence, to divide objects into equally numerous sets, to insert the missing number into a series, and to count on from an arbitrarily chosen point within the number string'. We have argued above in support of the connection between knowledge of the structure of the counting sequence and knowledge of place value in numbers, particularly when children begin to explain the counting sequence using generative and place value explanations because such explanations make the same processing demand as those for place value. Our results show that levels of counting in particular, as classified in this research, are significant in predicting knowledge of place value at all levels. These levels also accounted for more than 34% and 36% respectively of the variance in knowledge of place value on pre and post tests.

The significance of counting as predictor of knowledge of place value (and by implication the use of place value knowledge in algorithmic operations) means that it is very important to teach counting in such a way that children not only know the sequence as far as possible but begin, as soon as they can within the limits of their processing capacity to explain the structure and to understand the generative rule. It suggests too that teachers should challenge children to count as far as they can rather than limiting them to an arbitrary stopping point such as 20 in year 1, particularly when most children can count beyond that limit when they start school.

It also suggests that concepts of number should be taught to children so that they begin to understand all aspects of number as an integrated system of relations rather than as a set of more or less unrelated activities and `bits' of knowledge. Where slightly more flexible objectives and procedures have been adopted, as in school 3 it can be seen that the children's knowledge of counting and place value is better. This could of course be attributed perhaps to a more stimulating home background, however we would argue on the basis of other research (Halford, 1988; Halford, forthcoming; Halford & Wilson, !980; Boulton-Lewis, Neill & Halford, 1987) that the average capacity of all children to learn about these aspects of mathematics in the early years of school should be the same and the only difference between the groups should be in the amount of knowledge about numbers that they have acquired.

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TABLE 1
Requisite relational and system mapping knowledge to cognize place value

Relational mappings

1. relation between number names in counting string and objects in order

2. counting rule dependent on relation between quantity in set and last number name said

3. relation between written and spoken number names

System mappings

4. knowledge that 12 units are equal to 10+2 units

5. knowledge that the composition of 12 is conventionally represented by a 1 in the tens place and by 2 in the units place

TABLE 2
Mean counts for HCUPRE and HCUPOS by school and year

		HCURPRE	HCUPOS
SCHOOL 1			
	YEAR 1	49	89
	YEAR 2	101	116
	YEAR 3	267	121
SCHOOL 2			
	YEAR 1	44	103
	YEAR 2	77	220
	YEAR 3	141	766
SCHOOL 3			
	YEAR 1	47	82
	YEAR 2	153	420
	YEAR 3	690	888

TABLE 3
Mean levels for place value on pretest (LEVPRE) and posttest (LEVPOS) by school and year

		LEVPRE	LEVPOS
SCHOOL 1			
	YEAR 1	2.2	2.6
	YEAR 2	3.0	3.0
	YEAR 3	3.0	3.0
SCHOOL 2			
	YEAR 1	2.3	2.6
	YEAR 2	2.6	3.0
	YEAR 3	3.0	3.0
SCHOOL 3			
	YEAR 1	1.8	2.5
	YEAR 2	3.0	3.0
	YEAR 3	3.0	3.0

TABLE 4
Crosstabulation of HCURPRE by EXPRE

HCURPRE (Count / Row %)	EXPRE 1	2	3	Row Total
1		1	4	5
		20.0	80.0	9.3

2	5	19	1	25
	20.0	76.0	4.0	46.3
3	5	14	5	24
	20.8	58.3	20.8	44.4
Column	11	37	6	54
Total	20.4	68.5	11.1	100.0

TABLE 5
Crosstabulation of HCURPOS by EXP0S

HCURPOS (Count / Row %)	EXPOS 1	2	3	Row Total
2	5	5	4	25
	35.7	35.7	28.6	24.4
3	7	20	12	39
	17.9	51.3	30.8	73.6
Column	12	25	16	53
Total	22.6	47.2	30.2	100.0

COUNT90.DOC

[1]Funded by the Australian Research Council.