Table 3

Determinants of Learning Quality (No. Correct)

1. Internalizing (A) x Instrumental (B) x Conditions (C) x Occasions (D)
2. Achieving (A) x Conditions (B) x Occasions (C)

		(a) Third Wave	(b) Day Care
1.	A	ns	ns
	B	ns	<.05 (2)
	AB	ns	ns
	C	ns	ns
	AC	ns	ns
	BC	ns	<.05 (3)
	ABC	ns	ns
	D	<.01 (1)	<.01 (1)
	AD et seq.	ns	ns
2.	A	ns	ns
	B	ns	ns
	AB	ns	ns
	C	<.01 (1)	<.01 (1)
	AC	ns	<.01 (4)
	BC et seq.	ns	ns

(N = 43)

Notes:

Third Wave
(1) Fewer Correct on second occasion

Day Care
(2), (3)

		Immediate	Delayed
Fact Cond.	Hi. Inst.	10.9	7.7
	Lo. Inst.	6.8	4.8
Mean. Cond.	Hi. Inst.	7.8	5.2
	Lo. Inst.	7.3	4.9

High Instrumental, better recall of facts, both immediately and delayed. Effect much stronger under factual conditions.

(4)

	Immediate	Delayed
Hi. Ach.	9.1	5.3
Lo. Ach.	7.4	5.0

High Achievers recall more facts immediately; effect fades within a week.

INVESTIGATIONS INTO PIAGET'S MODEL OF FORMAL THOUGHT

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The Growth of Logical Thinking from Childhood to Adolescence (*GLT* Inhelder and Piaget, 1958) is addressed to the identification of and explanation of qualitative changes in the thought processes of the child during the stage of formal operational thinking. The two reviews which *GLT* received in *Br. J. Psychol.* (Bruner, 1959; Parsons, 1960) were in retrospect, uncannily predictive of the attention that Piaget's stage of formal operations would attract. Bruner's review claims that *GLT* is "the most important yet to appear" (*ibid.* p. 363) and that "Piaget's formal descriptions of....thinking by interpropositional or formal operations" explain adolescent thinking. He urges "that it does so remarkably well" (*ibid.* p. 368). However, in the second review, logician Charles Parsons (*op. cit.*) considers Piaget's logico-mathematical model of adolescent thinking to be inadequate. "Although Piaget uses the standard notation of propositional calculus, it turns out that he vacillates somewhat in his interpretation of the notation, and, his use of it obscures some features of inferences undoubtedly made by the subjects which logicians call logical." (*ibid.* p. 75). His primary difference of interpretation involves a number of inconsistencies and ambiguities which follow from the premise that Piaget uses the symbols "p", "q" etc. to stand for propositional functions and not merely for propositions as logicians would require.

Subsequent appraisals of Piaget's formal stage of logical operations have followed a remarkably similar bent, namely, many investigators have explicitly or implicitly accepted the Piagetian model while others, generally with particular interests and/or competencies in mathematical logic have variously criticized the inadequacies of Piagetian logical model.

Parsons asserts that Piaget's formula $p \cdot q$ (conjunction) should be replaced by using the predicate calculus from quantification theory which contains the extension $\exists$.e.g. $(\exists x)(Fx.Gx)$ replaces Piaget's conjunction formula $(p \cdot q)$ and should be read "Something is both F and G". The requirements of quantification theory would necessitate a similar change in all other representations of logical operations. e.g. Piagetian shorthand $p v q = (p \cdot \bar{q}) \vee (\bar{p} \cdot q) \vee (p \cdot q)$ would be replaced by $(\exists x)(x \epsilon P.x \epsilon Q).(\exists x)(x \epsilon P.x \notin Q).(\exists x)(x \notin P.x \epsilon Q).-(\exists x)(x \notin P.x \notin Q)$. It is little wonder that Piaget claims to have used his interpretation to represent symbolic logic "intuitively" (*ibid.* p. 78).

Particularly, Parsons refers to his own erroneous interpretation of Piaget's comments on $p \supset q$ and $q \supset p$ (*GLT*, pp. 299-300) (Parsons, 1960, p. 78). Close reading of the section in *GLT* indicates that Piaget's interpretation that $p \supset q$ and $q \supset p$ are not equivalent is identical to Parson's interpretation (Parsons, p. 78). Parsons claims that equivalence of $p \supset q$ and $q \supset p$ is *not* true under the official interpretation. Importantly, Parsons concludes his criticism of Piaget's model of formal thinking by claiming: "It is puzzling that Piaget does not use quantification theory, since by means of it the ambiguities can be easily untangled." (*ibid.* p. 78).

Since Parsons' review, a number of other critics (e.g. Weitz, 1971; Ennis, 1975; Brainerd, 1976) have similarly attacked the validity the Piagetian model of formal operational thinking has on a number of logical issues. Not only have the latter two authors criticized Piaget on logical grounds but each claims that formal stage logical thinking is evident in the majority of much younger children.

While Weitz (1971) worked within the framework of the Piagetian model, he and his collaborators (Weitz et al, 1972) claim that many of Piaget's logical operations cannot be expressed in a common language form. To the extent that the present author has succeeded with the operationalization of each of the sixteen binary operations, Weitz' claim is fallacious (ref. Bond, 1976).

Undoubtedly, the most vigorous and longstanding critic of Piaget's model of adolescent intelligence is R.H. Ennis (1969, 1975). In a recent paper (Ennis, 1975), he devotes some 40 pages to a well referenced attack on Piaget's use of propositional logic with particular reference to the following points: (a) paradoxes (inherent in the Piagetian model) and (b) lack of overgeneralization safeguards (formulation of laws) (*ibid.* pp. 16-22). Ennis suggests that although Piaget's logical system avoids the currently controversial paradoxes of material implication there are a number of other "problems engendered by this logic" (*ibid.* p. 16). Further, Ennis correctly asserts that the Piagetian system suggests that the subjects formulate laws after limited manipulation of restricted variables in a controlled environment. Parsons' deliberation on this point indicates that the problem is certainly not novel or restricted to Piagetian propositional logic: "The fact the problems involve *laws* raises another question. It is an old philosophic problem to justify the assertion of general laws on the basis of the observation of particular cases. It would certainly be too much to expect the authors to have the solution to this problem." (1960, p. 79)

Brainerd (1976) provides a most readable attack on the validity of Piaget's use of propositional logic as a model for adolescent intelligence. His "principal thesis is that if the logic of relations and the logic of classes are acceptable models for some earlier level of cognitive development, propositional logic cannot possibly be an acceptable model for adolescent cognition". (*ibid.* p. 40) In as much as Brainerd's article is couched in language more relevant to psychologists and educators (than to logicians) and its approach may be seen to be also representative of the philosophy behind the criticism of Parsons and Ennis, the remainder of the first part of this paper will deal more specifically with Brainerd's critique.

<u>Brainerd's Thesis</u>

In order to mount an argument against the validity of Piaget's use of logic as a model for adolescent development, Brainerd (as does Parsons and Ennis) resorts to arguments derived from some commonly known principles of mathematical logic. In particular he refers to the logically necessary relationship between the three branches of mathematical logic: (1) propositional logic; (2) logic of relations; (3) logic of classes. Most logicians would also admit to Brainerd's contention concerning the priorities of logic; namely, that "propositional logic is the most fundamental of the three branches" and "its primitive ideas, axioms, rules of inference, and theorems are employed in both the logic of relations and the logic of classes, *but the converse is not true*". (*ibid.* p. 41). He further illustrates his assertion by drawing an analogy between the logically determinate relationship expressed above and "the more familiar determinate relationship between arithmetic and algebra".

> We all remember from our school days that arithmetic forms a logically indispensible foundation for algebra. There is not a single concept, operation, or theorem in algebra that is not an extension of some corresponding notion of arithmetic....none of us would presumably make the mistake of regarding algebra as more basic than arithmetic....To say that propositional logic represents a more "advanced level of logical complexity than the logic of relations or classes is similarly absurd. It is like saying addition is logically more complex than multiplication...."
>
> (*ibid.* p. 41)

With regard to the implications of this argument for the use of the three brances of logic as models for different levels of cognitive development, Brainerd asserts (as many others do) that Piaget uses propositional logic as a model for adolescent intelligence (formal-operational stage) and uses the logic of relations and the logic of classes as models for middle childhood (concrete-operational stage). "I must confess that I am at a complete loss to explain why Piaget, in constructing his models for the concrete-operational and formal-operational stages, chooses to reverse the order of logical determination." (*ibid.* pp. 41-42). The corollary, then, Brainerd goes on to claim, is that since the logic of classes and the logic of relations comprise the formal models of the concrete-operational stage, Piaget's model of the formal-operational stage cannot be valid and one should expect that the concepts of propositional logic would appear in the thinking of preschool and kindergarten children.

Psychologists and educators may be excused for assuming that the Brainerd article (along with those of Ennis and Parsons) indicates a very serious flaw in the Piagetian explanation of cognitive development, especially when both Brainerd and Ennis provide evidence that children in their early school years can satisfactorily solve propositional logic problems based on axioms and rules of inference.

The Piagetian Logical Model

The most important flaw in the Brainerd argument (and consequently, that of Ennis and of Parsons) is that his critique is based on an analysis of axiomatic (or pure) logic. This is most obvious in that he continually refers (as does Ennis) to axioms and rules as being "clearly central in propositional logic" (*ibid.* p. 43). Those who are familiar with the Piagetian logical model will realize that Brainerd's description is a gross misrepresentation of that model as it is described, appropriately enough, in *Logic and Psychology* (Piaget, 1957) a short treatise expressly devoted to an explanation of the relationship between psychological and logical structures.

The construction of the current model is the result of Piaget's attempts to discover whether by using the operational techniques of logic he could uncover or construct structures which would correspond with the operational structures which are evident in the thinking of children. However, Piaget considers that "axiomatic logic is useless for the particular purpose we have in mind" (1957, p. 23), giving as one of his major reasons:

> The order inherent in axiomatization reverses in certain respects the genetic order of the construction of operations.

> For example, from the axiomatic standpoint the logic of classes is to be deduced from that of propositions, whilst from the genetic point of view propositional operations are derived from the logic of classes and relations.
>
> (*ibid.* p. 24)

Piaget and Brainerd are clearly in agreement that the logic of propositions is clearly inappropriate as a model of cognitive development at the formal stage and it is patently incorrect to assert, as Brainerd has, that Piaget has used one of the branches of modern logic, namely propositional logic, as that model. Ennis' criticisms of Piaget's work as being inadequate (as propositional logic) is now more readily seen as being less than appropriate.

Measurement: Propositional Logic or Formal Operational Ability?

A number of investigators (Ennis, 1975, 1976; Macnamara, 1977; Suppes, 1965; Brainerd, 1976) claim to have demonstrated propositional reasoning in preadolescent children, and thereby "have given cause for questioning the Piagetian tenet that qualitative changes in verbal reasoning take place during development" (Bereiter, 1978). In light of the foregoing discussion here and elsewhere (Bond, inpress), it is clear that findings of the type of Ennis and Brainerd in particular are largely irrelevant to the validity of the Piagetian model of formal operational thinking. Certainly they do not carry the import claimed for them by their proponents.

Clearly then, it is inadequate to use the measures of propositional logical ability commended by Brainerd and Ennis if one wished to attempt to guage whether children are able to use the system of logic which Piaget claims is representative of the psychological operational structures of the formal stage. Typical of the rather simple propositional logic problems used by these investigators are those contained in Amend's *Twenty Propositional Logic Problems* described by Brainerd (1976, p. 44). In each of these problems, a target rule or axiom describing the relationship between two propositions is presented (e.g. Suppose *P* implies *Q*.). Information about one of those propositions is also presented (e.g. Suppose *P* is true.) and the child is required to decide on the validity of the other proposition (e.g. Is *Q* true?) Ennis (1976) and Brainerd (1976) both claim, and most reasonably too, that very young children are able to solve these problems correctly.

A more recent and thorough-going investigation into children's ability to reason correctly with various types of propositional logic problems (Bereiter *et al*, 1978) provided "results that are not inconsistent with other results in so far as they can be compared" (*ibid*, p. 12). However by clever manipulation of the presentation of the evidence the children could use from which to draw their conclusions, "the results of the experiments show a striking and systematic change in the verbal reasoning behaviour of children across the age range 7 to 13" (*ibid.* p. 23). Bereiter further claims that the results are consistent with some aspects of the Piagetian approach and "consequently previous research has not given children the opportunity to show what is distinctive about the way they reason" (*ibid.* p. 24).

Inhelder and Piaget (*GLT*, 1958) developed abstract scientific reasoning tasks in order to assess the use of the formal logical abilities of adolescence. Brainerd clearly objects to that approach and claims the Piagetian tasks to be inappropriate; analogous to "devising an addition test consisting of items such as 5,662,397 + 4,881 + 925,711 and devising a multiplication test consisting of items such as 2 X 1" (*ibid.* p. 24). In light of these, and other objections, it might be advantageous to adopt a more clearly defensible measure of formal operational competency. This would involve the logical analysis of each of the formal operational schemata identified by Piaget (*GLT*, 1958) and the construction of questions to measure each of the logical operations directly.

In contrast to the approach adopted by Ennis and by Brainerd, Piaget clearly requires that the formal operational child should be able to formulate the logical relationship or target rule after observing patterns of occurrence or non-occurrence of two variables[2]:

> They know when they see an elementary association $p.q$ or $p.\bar{q}$, etc., that it may be included in any one of several combinations ($p*q$, $p[q]$, or $p \supset q$, for example), and they can verify its truth or falsehood more or less systematically. Conversely, when they assume a complex combination such as $p \supset q$ as a hypothesis, they know how to verify it by going back to its elements $p.q$ and $\bar{p}.q$ or by looking for a counterproof in the falsehood of $p.\bar{q}$.
>
> (*GLT*, 1958, p. 304)

Clearly, even if Amend's problems were based on the Piagetian logical model then the problems then are too simplistic to be representative of the Piagetian conception of competency with the formal logical operations.

This latter interpretation is that adopted by the present author in the development of a pencil and paper test of formal operational ability entitled *BLOT - Bond's Logical Operations Test* (Bond, 1977). Sixteen items on the Test assess the uses of the sixteen binary logical operations identified by Piaget (and mistakenly entitled propositional logic, by others). Typically, an item describes an everyday situation and identifies the occurrence or non-occurrence of the base combinations $p.q$, $p.\bar{q}$, $\bar{p}.q$ and $\bar{p}.\bar{q}$; the student is required to select from a number of alternative (multiple-choice) answers which describe possible relationships between p and q.

<u>On the Validity of the Piagetian Model: a By-Product of Test Development.</u>

Both Brainerd (1976) and Ennis (1976) have argued against the validity of the Piagetian model of logical operational ability on a theoretical (or logical level) and then used the results of empirical investigations (with tests of propositional logic) to show the ability of elementary school children to handle those problems. Inasmuch as the *BLOT* purports to be a measure of formal stage logical operational ability, administration of that Test should provide a substantially different description of the distribution of this ability. The results which follow have been extracted from the author's past and on-going research into formal operational ability (Bond, 1976).

<u>Sample</u>

The sample consisted of 966 children between the ages of 11 and 17 years, comprising the total final-year enrolment at a country primary school and a stratified sample of about one-half of the students at a large provincial secondary school. Because of the techniques adopted in the selection of the secondary school sub-sample, the grades 7-10 group may be seen to represent a larger population, whereas he termination of of compulsory school attendance at age 15 years would suggest that the grade 11 and 12 groups might consist of more able students.

Insert Table 1 here.

Results

The results most relevant to this discussion appear as Table 2 and it is not until the age of 13-14 years that over 50% of the children can correctly solve more than 50% of the 16 logical operational problems. It is not suggested that the 50% level of correct solutions to these problems is an adequate measure of formal operational ability - it is more in keeping with the standard adopted by Brainerd.

Discussion

The theoretical defence of the Piagetian logical model in the first part of this paper is substantiated by the results of the empirical investigation. Of importance to the interpretation of these results is the author's claim that the *Bond's Logical Operations Test* is a substantially reliable instrument; a test-retest administration of the *BLOT* to a randomly selected group of 91 students yielded a Pearson-Product-Moment Correlation Coefficient of $\bar{r}$ = ·91(p<·001) for an interval in excess of six weeks. Of more significance, however to both the standing of the *BLOT* and the validity of the Piagetian logical model as an explanation of the cognitive abilities of adolescents are results derived from the investigation to establish concurrent validity of the *BLOT* with three Piagetian tasks. The very high Spearman Rank Order Correlation of r_s = ·93(p<·0005) between ranks assigned by using the *methode clinique* and by using the *BLOT* for a selected stratified sample of secondary school students (N = 30), meticulously selected to reflect the whole range of abilities measured by the *BLOT*, represents the strongest evidence that could be adduced at the ordinal level to suggest that formal operational tasks of Piaget and the *BLOT* are measuring essentially those competencies based on the Piagetian logical model of formal operational ability. (Bond, 1976)

Conclusions

It is obvious then that the logical model on which Brainerd (1976) bases his theoretical critique and on which he and Ennis have based their respective propositional reasoning tests is not the logical model constructed by Piaget to explain formal operational ability and consequently the results of their research and the arguments concerning the logical inconsistencies of the order of stages (and the like) are irrelevant to the Piagetian framework. The theoretical stand taken by this author is supported by results from empirical investigations using the *Bond's Logical Operations Test* and this provides a substantiation of the validity of the Piagetian model of formal thought.

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1 We may use the symbols *p* and *q* to represent any two propositions (or, say, variables in an experiment) which can be either true $(p;q)$ or false $(\bar{p};\bar{q})$. Combining these two at a time we obtain $p.q$; $p.\bar{q}$; $\bar{p}.q$ and $\bar{p}.\bar{q}$. As each of these pairs may itself occur or not occur, we may derive sixteen possible arrangements. The logical operation or "target rule" (e.g., $p \supset q$, *p* implies *q*; $p[q]$, occurs whether or not *q* occurs) explains the relationship between the propositions.

Table 1

Student Performance on Logical Operational Problems

Grade/Age Level	Students scoring >50% n	%
Grade 7 (ages 11-12)	33	49.25
End of Primary School		
Grade 8 (ages 12-13)	93	52.54
Grade 9 (ages 13-14)	181	73.28
Grade 10 (ages 14-15)	160	73.06
End of Compulsory Schooling		
Grade 11 (ages 15-16)	155	95.68
Grade 12 (ages 16-17)	91	96.81
Total	713	72.98
Sample Size	966	