AN INVESTIGATION IN THE GEOMETRIC UNDERSTANDING AMONG ELEMENTARY PRESERVICE TEACHERS

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INTRODUCTION

It is well-known that the elementary teachers should have sound knowledge of the mathematical concepts and processes they are required to teach in primary schools. It is also known that a firm grasp of underlying concepts is an important and necessary framework for the elementary (mathematics) teachers to possess...(when) teaching related concepts to children ...(and) many teachers simply do not know enough mathematics (Post, Harel, Behr, and Lesh; 1991). Thipkong and Davis (1991) found conspicuous weaknesses in both preservice and inservice teachers' knowledge of a number of mathematical topics they teach. They, therefore, stressed the importance of preservice elementary teachers and their college lecturers identifying likely areas of weakness so that steps can be taken to prevent or correct misconceptions, since it appears highly likely that such misconceptions adversely influence students' learning.

Fuys, Geddes and Tischler (1988) conducted research which focused on a model of geometry presented in 1957 by the Dutch educators P.M. van Hiele and his wife, Dina van Hiele-Geldof. In the clinical interviews

conducted by these researchers, many preservice primary teachers revealed that much of their prior learning of geometry had been by memorisation and rote (pp 154). In another study, Mayberry (1983) found through two one-hour interview for each student involving geometry tasks that preservice teachers often did not perceive properties of figures, frequently did not perceive class inclusions, relationships and implications. In a similar study for preservice primary teachers, Gutierrez, Jaime, and Fortuny (1991) used interviews and a test of reasoning in three-dimensional geometry and found some interesting results.

According to the van Hieles theory (van Hiele, 1957; van Hiele-Gedolf, 1957), students progress through a five-level sequence in a particular order and that students cannot reach a higher level without passing through the lower levels. The progress from one level to the next is more dependent on educational experience than on age or maturation, and certain types of experiences can facilitate (or impede) progress within a level and to a higher level (Fuys, Geddes and Tischler, 1984). In fact, many researchers have supported the hierarchical nature of the van Hiele levels within several populations, for example, see Mayberry (1983), Usiskin (1982), Fuys, Geddes and Tischler (1988), Senk (1989), Hang (1994), Perdikaris (1996), and others.

The student who has attained the basic level, i.e. Level 1 (Recognition) can identify names, compare and operate on geometric figures according to their appearance. At Level 2 (Analysis), the student operates on certain objects, namely classes of figures (which were products of Level 1) and discovers properties for these classes. At Level 3 (Informal Deduction) these properties become the objects that the student acts upon, yielding logical ordering of these properties. At Level 4 (Formal Deduction) the student proves theorems deductively and establishes interrelationships among networks of theorems. Finally, at Level 5 (Rigor), the student establishes theorems in different postulation systems and analyses/compares these systems. For further information about the van Hiele model, one may refer to Fuys, Geddes and Tischler (1988, chapter 2). It may be noted that van Hiele and several other researchers have named these five levels by level 0 to level 4. In fact, the 1 to 5 scale is becoming increasingly popular as some studies have allocated students below the level 1 as 'Level 0' or 'No Level' for these students. For example, see Mayberry (1983) and Usiskin (1982).

The major purpose of this study was to investigate whether the van Hiele model accurately describes the geometric thinking of the preservice primary teachers. Specifically, this study sought to address the following questions:

Does the van Hiele model characterise the thinking in geometry of the preservice primary teachers?
To what extent are the gender differences in performance in the van Hiele levels?
To what extent does a student's van Hiele level account for geometry-writing achievement?
To what extent the van Hiele levels depend on the students' background in school mathematics?

METHOD

Population

The subjects for this study were 145 preservice primary (P) teachers enrolled under three different programmes: 71 in Diploma in Education (Dip (P)), 54 in BA/BSc with Diploma in Education (BA/BSc (P)) and 20 in Post Graduate Diploma in Education (PGDE (P)). The sample consisted of 121 females and 44 males. The entry requirement for 2-year Dip (P) and 4-year BA/BSc (P) is Cambridge A-Level or equivalent. PGDE (P) is a 1-year teacher training programme after BA/BSc or Honours. The students' mathematical background ranged from Elementary Mathematics at Cambridge O-Level to Further Mathematics at A-Level. Thus the entire population of 145 could be divided into 5 subclasses according to the highest level of mathematics they studied prior to a preservice programme: (i) 6 with Elementary Mathematics (EM), (ii) 13 with Polytechnic Mathematics (Poly Maths), (iii) 61 with Additional Mathematics (A Maths), (iv) 59 with Higher Mathematics (called C Maths), and (v) 6 with Further Mathematics (F Maths).

We notice that F Maths is much more in-depth and has more mathematical contents than 'C Maths' which in turn is more advanced than 'A Maths', 'Poly' Maths, and 'E Maths'.

The students ranged from eighteen to thirty years in age. No geometry lesson was given to the students in the recent past.

Instruments and Grading

In order to investigate the questions raised in the first section, we use two instruments. The first and the main instrument was centred on the van Hiele Geometry Test developed by the Cognitive Development Achievement in Secondary School Geometry (CDASSG) Project (Usiskin, 1982). This 25-item and 35-minute timed multiple choice paper-and-pencil test with 5 proposed answers per item and 5 items per level, was developed by Usiskin (1982, pp 242), to test the van Hiele theory. According to Usiskin, the items were written to correspond directly to statements from the van Hieles about characteristic behaviours students exhibit at each level. The CDASSG test was criticised by Wilson (1990) and Crowley (1990) and defended by Usiskin (1990). We decided to use this instrument because (i) the 25-item multiple-choice test matches the van Hiele Theory , (ii) the test is short and easy to administer (iii) its multiple choice format is easy to apply, and (iv) the test has been widely used by teachers and researchers to determine van Hiele levels in different populations.

Our second instrument in this study was an open-ended 10-minute "Written Geometry Test" (WGT) which consisted of the following:

Question 1: Write down a description of a parallelogram as you would to

a friend over the telephone.

Question 2: Write the description of a parallelogram using the minimum number of properties that would still allow the shape to be identified.

Question 3: Describe a square in terms of the word 'parallelogram'.

This test served as an alternative to a time-consuming and expensive method of conducting clinical interviews. It served as a tool to analyse students' thought processes and language structure. It also helped us to assign van Hiele levels to all the 'no fit' cases found with CDASSG classification system.

For the CDASSG test, we followed the same method of grading as recommended by Usiskin (1982). More specifically, we looked into the four different criteria for assigning students to levels; minimising Type 1 error (use a 4 of 5 criterion) or minimising Type 2 error (use a 3 of 5 criterion) was crossed with using or not using Level 5. We shall denote these criteria with using Level 5 by C4 Crit (4 out of 5 criterion) and C3 Crit (3 out of 5 criterion), and without using Level 5 by M3 Crit and M4 Crit. If a student met a criterion for mastery of each level up to and including level n and failed to meet the criterion for mastery of all the levels above level n, the student was assigned to level n under that criterion. If the student could not be assigned to a level in this manner, he/she was said to not fit (or "nofit") under the selected criterion.

For WGT, we assigned the van Hiele levels by looking into the written responses of the above three questions, when taken together, and using the following rough guideline:

No level	Level 1	Level 2	Level 3	Level 4
0	2030	60	90	100

Use of effect size and power analysis in test of significance

In 1986, Oakes stated that it is "extraordinarily" difficult to find a statistician who argues explicitly in favour of the retention of significance tests" (p. 71). We must abandon the statistical significance test (Schmidt, 1996; Menon, 1993). On the other hand, because of the influence of meta-analysis for multiple studies, the practice of computing effect sizes for individual studies has become more frequent in some research literature.

In order to answer meaningfully the "how much" or "what extent" type of questions, we find the effect sizes (ES) and make use of power analysis. Effect size means the degree to which the phenomenon is present in the population or the degree to which the null hypothesis is false (Cohen, 1988). It is also known that when null hypothesis is false, it is false to some specific degree, i.e. the effect size is some specific non-zero value in the population. The larger this value, the greater the degree to which the phenomenon under study is manifested. The effect size index (Cohen 1988, p 67) is given by

The above formula for is one directional (one-tailed) test. We also note that is the standardised mean difference for the sample. It is simply related to the t statistic by

.

The value of necessary for a chosen significance level is called , the significance criterion value of . Now using the power tables 2.3.1 to 2.3.4 in (Cohen, 1988), we can find its power for the value of n = . If then the difference between two sample means is statistically significant at given a. On the other hand, if then the difference between sample means is not statistically significant.

Results

Scalogram Analysis

In Table 1, we summarise information for all subjects corresponding to various criteria for the CDASSG test. We notice that about 66% of students were classifiable into a level on the easier criterion C3 (3 out of 5 correct) and about 77% on the stricter criterion C4 (4 out of 5 correct). Compare with 71% and 88% respectively under C3 and C4 criteria found by Usiskin (1982) for his sample of 2361 students in secondary school geometry.

Table 1: Descriptive statistics for all subjects (N=145) corresponding to various criteria

Criterion	Assigned VHL		Nofit		Mean	S.D.	Median
	Count	%	Count	%			
C3 Crit	96	66.2	49	33.8	2.729	.900	3.000
C4 Crit	111	76.6	34	23.4	1.784	.846	2.000
M3 Crit	122	84.1	23	15.9	2.648	.738	3.000
M4 Crit	122	84.1	23	15.9	1.787	.855	2.000
Adj-M3 Crit	145	100	-	-	2.469	.842	3.000
Adj-M4 Crit	145	100	_	_	1.800	.787	2.000

We first look into the question of hierarchical aspect of the van Hiele theory. We note that each student's score could be written as an ordered quintuple. For example, 10011 means that the student met a criterion on levels 1, 4 and 5 but not on levels 2 or 3. Also, notice that 10011 has 2 errors, where error is defined as the smallest number

of changes one would have to make for a pattern in the quintuple to agree with a perfect scale pattern.

Using Guttman scalogram procedure (Torgerson, 1967, pp 307), we find the measure of errors for the entire scale, called the coefficient of reproducibility (Rep) which is defined by the formula

Here, Rep gives the fraction of the scores that are correctly placed in the response patterns. Using the above formula, we have

Under C3 criteria:
Rep (5) = 98.5, using all five levels.
Rep (4) = 99.0%, using the first four levels.

Under C4 criteria:
Rep (5) = 98.7%, using all five levels
Rep (4) = 99.4%, using the first four levels.

Since Rep = 90% or better is considered as the standard for scalability (Torgerson, 1967, pp 323) and our values are much higher than 90%, we thus conclude that the van Hiele levels as tested by the CDASSG test form a hierarchy.

The Modified Theory

It is interesting to see the response patterns of some students; for example, the responses under C4 criterion, of 11 students (7.6%) were: 11101 (3), 11001 (5), 10101 (1), 10001 (1), and 00001 (1). Though these students satisfied '4 out of 5 criterion' in Level 5, yet they could not satisfy this criterion in one or more in the lower levels. Thus no student could be assigned Level 5. This is acceptable because Level 5 is considered as most controversial. In fact, our observations are in accordance with those of the CDASSG test designers (Usiskin, 1982 pp 13) who found the behaviour defining the Level 5 "quite vague". Also, Usiskin (1982) found that K-R reliability coefficient for Level 5 was .10 in the fall and .30 in the spring. These observations motivated us to exclude Level 5 from further consideration.

Table 1 further shows that under modified criteria M3 and M4, i.e., with Level 5 omitted, the percentage of 'no fit' cases reduces from 33.8% (under C3 Crit) and 23.4% (under C4 Crit) to about 16% (under both M3 and M4). For example, 5 students with response patterns 11001 under C4 Crit were classified into Level 2 under a modified criterion M4.

Written Geometry Test (WGT)

On analysis of the response patterns, and by using the criteria for grading WGT we identified 7% students at No Level and 26% who were operating at Level 1. These students could not see the properties of the figures (Level 2 concept). Their expression was usually poor. They thought of shapes as they could see them and they could not 'see' why. Their responses included: (i) A parallelogram is quite similar to the shape of diamond except that its length is longer than the breadth....., (ii) It looks like a compressed rectangle, (iii) A parallelogram is a rectangle which is inclined at 45 degrees, (iv) It looks like a door, (v) It looks like a square.

Table 2: Percentage of subjects at each VHL under WGT and adjusted modified criteria

Criterion	No Level	Level 1	Level 2	Level 3	Level 4
WGT	7.3	26.3	56.2	10.2	-
Adj-M4 Crit	6.2	23.4	55.2	14.5	0.7
Adj-M3 Crit	2.1	8.3	38.6	42.8	8.3

Many of the 56% students who were assigned Level 2 in WGT could give only vague answers and, in fact, they were still using Level 1 type language. Though these students did know that a parallelogram has opposite sides parallel and equal, yet they could not discuss relationships between properties of a given figure or question (Level 3 concept). Many of these students thought that "a parallelogram must be slanted and because a square is not slanted, it would not be a parallelogram". Some of them stated: (i) A parallelogram is actually a slanted rectangle, (ii) It (parallelogram) is an oblique rectangle, (iii) A square is like a parallelogram with four equal lengths and four straight lines, (iv) A square is parallel on all 4 sides, all the angles made up 360 degrees, (v) A square is a parallelogram moved to face the front (not tilted), (vi) A square is a subset of parallelogram, (vii) A square is not a parallelogram because its four sides are equal.

The remaining 10% students were operating at Level 3 under WGT. These students accepted the class inclusion concept. But, most of them could not think in terms of minimum properties (a Level 4 concept). For example, their responses included phrases such as: (i) A square is a 90 degree angled parallelogram, (ii) a square is a parallelogram with 4 right angles, (iii) a square is a unique type of parallelogram except that all the sides are equal and that the diagonals are equal in lengths and so on.

Adjusted Modified van Hiele Levels

We notice from Table 1 that even after omitting Level 5, we have about 16% 'no fit' cases. So, we determined the van Hiele levels of no fit cases under modified criteria MC3 and MC4 by looking at their responses both in CDASSG test and WGT. For example, a 'no fit' response patterns 1101 under MC4 was found at Level 2 in WGT and we decided to assign Level 2 to this student.

Table 2 lists that on easier criterion Adj-M3, 3 out of 5 correct (i.e. 60% mastery), about 51% students were classified at Level 3/Level 4. On the other hand, by using the harder criterion Adj-M4, 4 out of 5 correct (i.e. 80% mastery), 15% of students were at Level 3/Level 4.

Which is a better criterion: Adj-M3 or Adj-M4 ? How much? In order to answer such questions meaningfully, we find the effect sizes and use power analysis technique. Note that the first two rows in Table 3 prove that for a given level of significance and common sample size , the van Hiele levels as determined by Adj-M3 Crit are significantly higher with power more than 99.5% than those found by Adj-M4 or WGT. On the other hand, at a1 = 0.01 we find that there is no significant difference (with power 28%) between the van Hiele levels determined by the Adj-M4 Crit and WGT. However, for better results we need to increase the sample sizes and hence n'. Nevertheless, the effect sizes and powers in the first three rows of Table 3 indicate that the van Hiele levels determined by the Adj-M4 for the CDASSG test represent more accurately the van Hiele levels of the students. These findings, therefore, have led us to use only the criterion Adj-M4 for the CDASSG test for the rest of our data analysis.

Gender Differences

Row 4 of Table 3 shows that at = 0.10 and n' = 40, the level of geometric understanding of males is significantly higher with power 83% than those of females. This is further exhibited in Chart 1. However, Chart 1 indicates that there is no significant difference between the performance of male and females students in Diploma (Primary) programs. This may be due to the fact that this program attracts only a few male students (about 16%) and many of them have a little interest in mathematics.

Chart 2 also shows that the performance of male students is better than female students except for those who did C Maths . Note that there were no male students with E Maths and so there is no question of comparison in this case.

Comparison of the performance between different programs

Chart 1 shows that the performance of PGDE (P) was far better than those of the BA/BSc (P) students. This was expected because firstly the entry requirement for PGDE (P) is BA/B.Sc (P) and secondly over 50% students in PGDE (P) did F Maths or C Maths in their A-level.

The above observations are further confirmed from the 95% confidence interval (1.56, 2.56) of means for PGDE (P) with those of the other groups as listed in the last column in Table 4. However, the performance in WGT of BA/B.Sc (P) with 95% CI as (1.66, 2.07) appears to be slightly better than those of the PGDE (P) with 95% CI as (1.01, 2.05).

Table 3: Effect sizes, Power and Significance Testing at a1 = 0.10

Row

Null hypothesis H0
n'
ds
dc

Effect size ds is:
In favour of

1

145
.82
.5
H. SS with P >99.5%
Adj-M3 VHL

2

140
.97
.15
H. SS with P>99.5%
Adj-M3 VHL

3
W = 0

140

.21

.15

SS with P= 67%

adj-M4 VHL

4
M - F = 0
40
.50
.29
SS with P = 83%
Males

5
B-D = 0
61

.04
.24
Not SS with P = 9%
BA/BSc (P)

6
PG-D = 0
31
.54
.33
SS with P = 79%

PGDE (P)

7
PG-B = 0
29
.47
.34
SS with P = 69%
PGDE (P)

8
C-E = 0
11
.72
.57
SS with P = 72%
C Maths

9
C-P = 0
21
.68
.40
SS with P = 81%
C Maths

10
C-A = 0
60
.25
.24
SS with P = 53%
C Maths

11
F-C = 0
11

.69
.57
SS with P = 61%
F Maths

Table 3 (rows 5 to 7) confirms, as expected, that geometric understanding of PGDE (P) students are significantly better than their counterparts in BA/B.Sc (P) or Diploma (P). However, row 5 in Table 3 also proves that there is no significant difference (with very low power) in performance between BA/B.Sc (P) and Diploma (P) students

Chart 1: Boxplot of adjusted van Hiele levels against Gender and Program

Table 4: 95% CI for Mean for Subjects According to Subgroups (N=145)

Subgroup	Mean		S.D.		Median		95% CI for Mean	
(n)	WGT	adj-VHL	WGT	adj-VHL	WGT	adj-VHL	WGT	adj-VHL
Dip (P) (71)	1.61	1.73	0.69	0.72	2.00	2.00	(1.44,1.77)	(1.53,1.86)
BA/BSc (P)(54)	1.86	1.76	0.72	0.80	2.00	2.00	(1.66,2.07)	(1.55,1.98)
PGDE (P) (20)	1.53	2.15	1.01	0.93	2.00	2.00	(1.01,2.05)	(1.56,2.56)
All (145)	1.69	1.8	0.75	0.79	2.00	2.00	(1.57,1.82)	(1.67,1.93)

Effect of Background in School Mathematics

Chart 2 and effect sizes given in Table 3 (rows 8 - 11) indicate that the geometric understanding of a student depends on his/her background in school mathematics. For example, at = 0.10, there are statistically significant differences in the mean performances of those who studied C Maths, E Maths, A Maths, and Poly Maths. We see from Chart 2 that those who studied F Maths have better geometric understanding than those who studied C Maths, A Maths, E Maths or Poly Maths.

Chart 2: Boxplot of adjusted van Hiele levels against school maths and gender

Conclusion

The successful attainment of Level 3 (Informal Deduction) should be the minimum goal for all preservice primary teachers because as claimed by van Hiele the Level 3 is considered as the "essence of geometry".

We have found that the van Hiele level of geometric thinking depends on the level of mathematics studied in the school. Most of those who studied up to Elementary Mathematics or Polytechnic Mathematics were operating at Level 1 or Level 2. In fact, some of these students could not be assigned any level. On the other hand, many of those who had studied Higher Mathematics (i.e. C Maths) or Further Mathematics were assigned Level 3.

Our findings confirm what Hoffer (1981) reported that 'the subject that is always universally disliked is geometry in high schools'. Those preservice teachers who operate at Level 1 have their perception of geometry figures based only on standard positions. They included incorrect properties in their descriptions. For example, "one side is longer than another" for a parallelogram, it is "always slanted at 45 degrees". Most students do know that a parallelogram has opposite sides parallel and equal. But, they cannot discuss relationships between properties of a given figure or questions. They could not see that all squares are parallelograms because their perception about shapes are made on the basis of "looking like", not on the basis of properties. Thus their perception of figures at Level 1 (or no level) could not help them in the development of the properties of figures in Level 2. It appears that many of the preservice primary teachers in their schooling could not have the types of meaningful and effective experiences necessary to enable a person to acquire skills appropriate to van Hiele levels 1 to 4.

Looking at the response patterns of Nofit cases, it is quite clear that the students could pass higher levels without passing Level 1 or 2 by learning rules or definitions by rote or by applying routine algorithms that they don't understand. These students are weak in problem solving because they lack spatial visualisation and they are still at Level 1. It is also a fact that rote learning or applying routine algorithms without understanding is no level. The findings also suggest that many students reason at Level 1 type language. These future teachers, if not provided help by their lecturers, might have serious communication problems when dealing with their pupils and text-books at higher levels.

There may be several reasons for many students who are still operating at Level 1 type of geometric thinking. The students' responses in WGT also revealed that even students at Level 2/Level 3 use the Level 1 type language. In fact, poor performance in geometry in preservice training or at the university level could be due to poor preparation at the primary or secondary level. It appears that those who are not yet operating at Level 3 might have learnt the school geometry where (i) too much emphasis was placed on the deductive aspects of the subject, (ii) a little emphasis was placed on the underlying spatial abilities with very little hands-on activities, (iii) not sufficient time spent on geometry when they were in primary or secondary schools, (iv) properties of shapes were taught in isolation (e.g. properties of a square separate from those of a parallelogram).

The present study clearly shows that for the CDASSG Test, the stricter criterion of '4 out of 5' correct gives more accurate van Hiele level for the student than the easier criterion of '3 out of 5'. In order to settle 'nofit cases' it is important for a researcher to either conduct interviews as done by Mayberry (1983) for the undergraduate preservice teachers or use an alternative method such as an open ended 'Written Geometry Test" conducted for the present test. However, we agree with Wilson (1990) that there is a need to improve the reliability of written instruments to assess van Hiele levels. Usiskin (1990) also felt that both the CDASSG test and the van Hiele theory

itself could be improved. For example, one may develop 125-item multiple-choice geometry test that is consistent with the van Hiele theory and has both descriptive and predictive power and is easy to administer. This study was also limited by the small sample size. Moreover, there is a need to design an open-ended 'Writing Geometric Test' covering several 2-D and 3-D geometric concepts because the present test was limited by three questions on only two concepts.

We conclude that the background of many preservice primary teachers in teaching geometry is weak or non-existent. We should help students develop vivid images and co-ordinate these images with their conceptual knowledge (Battista and Clements, 1991). In fact, they should be provided the opportunity to develop their own spatial sense. They should be provided with various geometric activities by using geoboards, geoboard dot papers, pattern blocks, wooden or plastic cubes, tangrams, pentominoes, and tessellation. The preservice teachers must understand the role of definition, the difference between a definition and a description, and the use of undefined terms. Information Technology can also be used to gain a deeper understanding of geometric structures.

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