

## Using an Inquiry Approach to Develop Mathematical Thinking

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This case study describes the use of an inquiry approach to develop mathematical thinking in year eight students. A series of guided inquiries were co-constructed by teachers and researchers as a component of a Professional Learning program in a rural Tasmanian secondary school. The inquiries covered the concept of proportional reasoning and content from the Chance and Data curriculum strands and were underpinned by the Teaching for Understanding framework (Blythe, 1998). Students received modelling and formative feedback on the inquiry process with an emphasis on providing written explanations of their mathematical thinking and their investigative processes. At the conclusion of the unit, students completed a fifth inquiry as a culminating assessment task. Student work samples were collected and assessed against a rubric to determine if an improvement had occurred over the course of the project. These results together with teacher observations demonstrate the capacity of this approach to influence student outcomes in terms of mathematical thinking.

### Introduction

Inquiry includes identifying and clarifying issues, and gathering, organising, interpreting and transforming information. It encompasses the processes of creatively, imaginatively and inquisitively thinking about possibilities, analysing, synthesising and evaluating proposed solutions; and explaining and justifying decisions. The skills of inquiry can be used to clarify meaning, draw appropriate comparisons and make considered decisions.

(Department of Education, Tasmania [DoET], 2002, p.14)

Using an inquiry based approach to learning and teaching is firmly underpinned by constructivist principles (Carter, 2004; Piaget, 1973). Students are able to make connections between existing and new knowledge, to work collaboratively, learn from each other, and take responsibility for their own learning. The importance of developing processes of inquiry in students has been formally recognised in the Tasmanian Essential Learnings curriculum (DoET, 2002; 2003). Inquiry was seen as central to the curriculum and necessary for teachers to consider when designing, implementing and evaluating learning and teaching programs (DoET, 2002). As a key component of the 'Thinking' essential learning, inquiry therefore overlaid all curriculum areas. This focus on inquiry has been maintained in the 2007 revision of the Tasmanian Curriculum (DoET, 2007).

An inquiry approach has been advocated by many mathematical researchers – having the potential to provide for a diversity of learners – in terms of engagement and challenge – by using interesting and authentic tasks (Carter, 2004; Diezmann, 2004; Heibert, 2003; Lovitt, 1999; Schoenfeld, 2002; Stein, Grover & Henningsen, 1996; Sullivan, 2001). In an inquiry mathematics classroom, students are actively engaged in the construction of knowledge (Carter, 2004, p.2). Listening, discussion and explaining mathematical thinking are important characteristics of an inquiry classroom as is a climate of cooperation between the teacher and students, creating a community of learners. “Inquiry approaches (e.g. open-ended questions, investigations, thought experiments) are advocated in mathematics because they engage students and suit a range of student capabilities. They also provide authentic opportunities for teamwork and co-construction of knowledge” (Diezmann, 2004, p.80).

A major influence on the acceptance of inquiry-based approaches to mathematics has been the National Council of Teachers of Mathematics *Standards* documents (1989; 2000). Following publication of these documents, there has been a growing body of research to provide evidence of positive impacts of inquiry on student learning in mathematics (Heibert, 2003; Schoenfeld, 2002).

#### *The Tasmanian context*

Curriculum reform[s] in Tasmania, in particular the introduction of the Essential Learnings (ELs), have provided a significant catalyst for pedagogical change. The reforms, however, have reached beyond pedagogy, encompassing a need for teachers to work in cross-curricular ways and to focus on designing assessment tasks that are authentic and support learning. Tasmanian teachers have been supported through these reforms by curriculum materials and professional learning programs. The former encompasses exemplar units, planning and assessment pro formas and related readings coordinated by the DoET (<http://www.ltag.education.tas.gov.au/default.htm>). The latter delivered predominantly through designated curriculum officers and assessment support leaders (from the DoET and Catholic Education Office).

The Mathematics in Reform Based Learning Environments (MARBLE) professional learning program began in 2005. An Australian Research Council project, the project involved a research team from the University of Tasmania, the Department of Education Tasmania and the Catholic Education Office working with middle years’ teachers with a goal to improve student outcomes in numeracy. An important background to the project was the curriculum reform that was taking place and a shifting of the position of numeracy in the curriculum. As an identified key element, Being Numerate was recognised as a cross-curricular understanding, and one that is important for development in all students.

Being numerate involves having those concepts and skills of mathematics that are required to meet the demands of everyday life. It includes having the capacity to select and use them appropriately in real life settings. Being truly numerate requires the knowledge and disposition to think and act mathematically and the confidence and intuition to apply particular principles to everyday problems.

(DoET, 2002, p.21)

As a result of these reforms, teachers of mathematics were encouraged to adopt different planning practices. Using the features of the Teaching for Understanding (TfU) framework (Blythe, 1998; Wiske, 1998) and, where possible, planning cross-curricular and collaborative units were encouraged. With content not prescribed, consideration of what was to be taught, and importantly, why it was being taught gained importance. This was followed by a need for teachers to closely match purposes with appropriate teaching and learning activities, with an emphasis on Inquiry and Reflective Thinking (two overarching Key Elements).

At the commencement of the MARBLE professional learning program, participating teachers completed a profile to assist the research team in their planning. The findings indicated that teachers recognised the importance of inquiry-based learning particularly to give students opportunities to work on authentic tasks and engage in problem solving (Brown, Watson, Beswick & Fitzallen, 2007). When describing their own practice, teachers were predominantly concerned with the need to engage learners. To this end there were a number of teachers who indicated that they attempted to use pedagogies that involved students in group work, activities, games or inquiries. Yet, the profile data indicated that the ELs framework emphasis on Inquiry had not been widely adopted in the planning of this particular group of teachers. Providing more experience in planning and implementing inquiry was seen as an important component to include in the professional learning program.

Using data gained from teacher and student profiles, the MARBLE team constructed a framework for the professional learning program, with the work of Shulman (1987a; 1987b) providing a theoretical underpinning. In particular, four of Shulman's essential teacher knowledges were drawn upon:

- mathematical content knowledge  
(building teachers' knowledge of the mathematics they need to teach);
- pedagogical content knowledge  
(modelling and building understanding of pedagogical content knowledge appropriate to middle years numeracy);
- knowledge of students as learners  
(using student profiles to identify students' mathematical thinking and to plan to meet the identified issues);
- curriculum knowledge  
(in the context of the curriculum reform, to support teachers in adopting practices and building confidence in implementing innovation).

These knowledges have been considered in conjunction with factors identified as being important for successful professional learning:

- factor 1: involvement of teachers in the identification of what they need to learn and the process to be used (Hawley & Valli, 1999);
- factor 2: facilitation of collaborative problem solving (Hawley & Valli, 1999);
- factor 3: engagement with the content of the mathematics curriculum taught, in ways that challenge and deepen teachers' own mathematical understandings, assisting them to make significant changes in their classroom practice (Schifter, 1998).

The framework that resulted from a consideration of these key elements together with data from participant surveys is reproduced in Table 1. Consistent with the

professional learning principles, the MARBLE program was co-constructed with participants (factor 1), following sharing of summaries of the data obtained from the profiles. The program encompassed both whole of cluster professional learning days led by the research team (factor 3), and individual school case studies, where each school was assigned a researcher to be involved in a project of its own choice (factor 2). The individual case studies have focussed on a particular need of the school or teacher involved. A summary of the professional learning activities in the program to date is included in Table 1.

Table 1.

*Summary of MARBLE Professional Learning Activities for Teachers*

<i>Focus of Professional Learning</i>	<i>Mathematical content knowledge</i>	<i>Pedagogical content knowledge</i>	<i>Knowledge of students as learners</i>	<i>Curriculum knowledge</i>
<i>Whole of Cluster Professional Learning</i>	Fractions Measurement Ratio Problem solving <i>TinkerPlots</i> (Data collection, handling, representation, interpretation) Mental computation Place value Accuracy Space Decimals Percentages Proportional reasoning Quantitative literacy (in media)	Fractions Pi Chance and Data; (Designing surveys, collecting data, representing data, interpreting data) Problem solving Numerator language Mental computation strategies	Division Fractions Applying rubrics to students' responses Progression statements	Coordinating the mathematics curriculum Assessment: Formative and summative including use design and use of rubrics Making inter-disciplinary connections with science; SOSE Planning units of work – connecting understanding goals with teaching, learning and assessment
<i>School Case Studies</i>	<i>TinkerPlots</i> Constructing a school scope and sequence Student produced resource kits	Mental computation strategies <i>TinkerPlots</i>	Mental computation and problem solving strategies	<b>Implementing an Inquiry*</b> Whole-school numeracy audit

\* The case study which forms the focus of this paper.

(adapted from Watson, Beswick, Brown & Callingham, 2007)

The inclusion of case studies, initiated by teachers in specific schools was an attempt by the researchers to reflect best practice in professional learning. The Australian Government report, *Teachers as Learners* (Ewing, Smith, Anderson, Gibson & Manuel, 2004) recognised the creation of a community culture, collaboratively designed with teachers as key members and focussed around priorities identified by

the schools as being important to the success of school based projects. It also conformed to two of the key principles for professional learning in Mathematics teachers outlined by Clarke (1994); addressing issues of concern raised by teachers themselves and involving groups of teachers rather than individuals.

The focus of the case study described in this paper was the use of inquiry to explicitly develop students' abilities in explaining their mathematical thinking and investigative processes. The catalyst for the work was an inquiry modelled in a whole of cluster professional learning session (My favourite ratio – An inquiry about pi [Brown, 2006]). This inquiry was completed by all teachers at the Professional Learning session, its planning (underpinned by the Teaching for Understanding framework [Blythe, 1998]) discussed and an assessment rubric fleshed out and applied.

Recognising the possibilities for using an inquiry approach in their year 7 and 8 classes, teachers at Valleyfield High worked with researchers from the MARBLE team to design a term's work with an inquiry focus. The unit comprised a series of mathematical inquiries, each underpinned by the Teaching for Understanding framework. The assessment criteria for each inquiry focussed on each of the four dimensions of understanding (Ritchhart, 1999). These dimensions are: Knowledge (the content or What?); Methods (the processes or How?); Purposes (Why?) and Forms (how is it communicated?).

## **Methods**

The intention of this project was to establish whether using an inquiry approach to mathematics can improve students' capacities to use mathematical language to explain thinking, and whether this in turn improves student outcomes in mathematics. Consistent with a case study approach, the project was sited in a single school which conforms to Stake's description of a 'bounded system' that is 'complex' and 'dynamic' (Stake, 1995). The study took place at Valleyfield High School, a government 7-10 school situated in a rural area in Southern Tasmania. Although confined to a single case, it was hoped that the data obtained would provide researchers with an indication of possible results of implementing this approach, and more importantly a model for investigating this question on a broader basis.

The whole of cluster MARBLE professional learning sessions for teachers provided the impetus for writing the unit around inquiries. Valleyfield mathematics teachers were then supported with an in-school professional learning around inquiry. Following this session, the leading teacher then took on significant responsibility to complete the planning and share the documentation with all teachers of junior secondary mathematics teachers (years 7 and 8).

The focus of the case study was a series of five mathematical inquiries that required students to explicitly explain their thinking and investigative processes. To establish baseline data, students completed an initial independent inquiry. The teacher then structured a series of guided inquiries to develop students' mathematical thinking and ability to explain mathematical processes. These guided inquiries included teacher modelling and collaborative investigations with formative feedback given to students as they progressed through these inquiries. Concurrent with the inquiries, an emphasis

on written explanations was embedded into all mathematical tasks completed during the case study period, irrespective of whether or not they had a direct relationship to the inquiries. Inquiry 5, taken from the whole of cluster professional learning session mentioned previously, served as a culminating assessment. The content of inquiries is outline in Table 2.

Table 2.

*Summary of the five inquiries completed by students in the case study*

Inquiry	Title	Content	
1	Proportional Reasoning	An inquiry into similarity and proportion in 2 dimensional shapes	Independent
2	Is this information reliable?	An inquiry into probabilities (experimental and theoretical probability)	Guided Inquiry
3	Organising Information	Probability, data collection, presentation of data	Guided Inquiry
4	Myth Busters	An inquiry into misconceptions in probability *	Guided Inquiry
5	My favourite ratio	An inquiry into Pi (what is pi?, how can it be calculated? and why is it useful?)	Independent

\* Adapted from Lovitt and Lowe, 1993.

*Sources of data*

Initial planning for the inquiry unit was carried out with the intention of all Year 7 and 8 classes at Valleyfield High completing the work. In actuality not all teachers chose to conduct the inquiries. This paper therefore describes the case of a single Year 8 class with a specialist mathematics teacher who conducted the unit in the manner originally planned. Consistent with qualitative research paradigms (Miles & Huberman, 1994) multiple data sources were utilised in this study. These were: student work samples; two teacher interviews (immediately following the project and early the following year); and researcher journal. A researcher was present at the initial planning stage of the inquiry unit and made classroom observations during the unit.

The student work samples were collected from four representative students. These students were purposefully chosen by the teacher through meeting two requirements; firstly that they had completed all five inquiries and secondly that they represented a range of ability levels. Each student provided five inquiries with these being assessed against the rubric reproduced in Table 3. The use of rubrics for assessing inquiries has previously been outlined by Diezmann (2004) drawing on the work of Popham (1997) and Taylor and Bidlingmaier (1998).

Table 3.  
*Assessment Rubric used to assess inquiries*

Criterion	On the way... D	Getting there... C	Moving well... B	Really flying... A
<b>Knowledge</b> <i>Can explain concept to an audience</i>	Has given an explanation but is not strictly correct, or is confusing	Has given a definition or explanation which is basic, or one which has been directly copied	Has given an explanation which has evidence of original thought, either in the definition or in diagrams or additional information	Has given a clear definition which is easy to understand and goes beyond a basic definition. The definition should have evidence of original thought. Additional information which clarifies or adds interest is included
<b>Methods</b> <i>Can carry out an investigation</i>	Provides evidence that an investigation has been carried out according to teacher directions and with help from the teacher	Provides evidence that an investigation has been carried out, e.g. measurements have been taken and recorded and calculations have been completed	Provides evidence that an investigation has been carried out systematically, e.g. measurements being taken with appropriate instruments, checked against approximations (no way-out measures), clearly recorded	Investigation carried out systematically. Explained, recorded accurately and presented in an appropriate format. Measurements are accurate, appropriate units used, calculations are correct. A conclusion to the investigation is provided
<b>Purposes</b> <i>Can identify and give examples of how the knowledge can be used; and can apply knowledge to a new problem or situation</i>	Demonstrates little understanding of the purpose of the concept being investigated  And/or evidence of considerable teacher assistance	Can give a rudimentary explanation of the concepts and some indication of application/importance  Evidence of teacher assistance	Beginning to demonstrate understanding of concepts and application/importance and can express in their own words and related to their own investigation	Work reflects a good understanding of the concepts and application/importance. Can express in own words and drawing examples from own work and can see limitations
<b>Forms</b> <i>Can communicate the results of the inquiry to an audience</i>	Can communicate information or idea but it lacks clarity. With explicit instruction, can use organisers to present results	Can communicate information or idea using some mathematical language and accepted forms/conventions when modelled (e.g. tables)	Can communicate mathematics information accurately in a prescribed way and can use diagrams/tables/graphs accurately. Logically sequenced. May make some reference to real life examples	Can communicate information in a variety of ways, independently use tables graphs to communicate findings, discuss mathematical ideas using mathematical language. Can draw parallels with real life examples

The first and fifth inquiries were completed individually and independent of the teacher. Each of these was assessed against the four assessment criteria. In the case of the three guided inquiries, an average assessment for each criterion was arrived at by comparison of each of these inquiries with each other against the rubric. To increase validity, both researchers (who are experienced teachers) marked the work sample, compared results and reached a consensus. To facilitate analysis, individual student results were tabulated to allow comparisons to be made with respect to outcomes for each of the four criteria.

The focus of the qualitative data from the Teacher interviews and researcher's journal was to gain insight into the extent to which students were willing and capable of explaining their mathematical thinking and investigative processes. To obtain an overall picture of this data, responses and entries were listed and read through in their entirety. A categorisation then occurred (Miles & Huberman, 1994). The qualitative data was also used to contextualise and comment on the findings from the student work samples.

## Results and Discussion

Work samples from each of the participating students were assessed against the rubric presented above. Results for each criterion were tabulated and are presented in Tables 4-7.

Table 4.

*Student results for each criterion - Student A: Jason*

Inquiry	Criterion 1	Criterion 2	Criterion 3	Criterion 4
1 (independent)	C	B	C	C
2-4* (Teacher directed)	B	B	C	B
5 (independent)	B*	B	B*	B*

Table 5.

*Student results for each criterion - Student B: Kelly*

Inquiry	Criterion 1	Criterion 2	Criterion 3	Criterion 4
1 (independent)	A	B	B	B
2-4*(Teacher directed)	A	A	A	A
5 (independent)	A	A	A	A*

Table 6.

*Student results for each criterion - Student C: Kym*

Inquiry	Criterion 1	Criterion 2	Criterion 3	Criterion 4
1 (independent)	D	C	D	D
2-4* (Teacher directed)	C	C	C	C
5 (independent)	D	C	C*	C*

Table 7.

*Student results for each criterion - Student D: Tiffany*

Inquiry	Criterion 1	Criterion 2	Criterion 3	Criterion 4
1 (independent)	B	A	B	B
2-4* (Teacher directed)	A	A	A	A
5 (independent)	B	A	A*	A*

\* Represents an improvement from the initial inquiry.

There was clear evidence of improved student performance, from the first inquiry, over the course of the guided inquiries. Although not all students showed improvements in all criteria, all students improved in at least two of the criteria, and remained the same in the others. All four students improved in Criterion 4 (communicating the results of the inquiry). Three out of four students improved their results in Criterion 1 (knowledge), suggesting that the guided inquiry and modelling provided by the teacher was beneficial in promoting mathematical knowledge. This was also the case for Criterion 3 (identifying and explaining purposes).

The final independent inquiry was set as a culminating assessment task. It also served to indicate whether the improvements noted in students' performances in the guided inquiries were sustainable without teacher and peer input. Although two of the students did not perform as well on one criterion as they had in the guided inquiries, the students generally performed at either the same or a higher level. When original student performances on each of the criteria were compared to performances on the culminating inquiry, each of the students had demonstrated some improvement. There was no evidence of students performing more poorly than they had in the original inquiry. It should be noted that in the case of two students, Tiffany and Kelly, they had achieved an A rating in the first inquiry and this was sustained through to the final inquiry. Teacher and researcher observations noted that there was an increasing capacity and willingness to explain thinking as the inquiries progressed.

Teacher interview data gives some insight into how the students reacted to the inquiry unit. From her account, it was initially extremely difficult to get students to accept that giving explanations for mathematical thinking was necessary. The teacher indicated that "it took three to four weeks" and significant modelling, that was consistent and reinforced, before there was an overall acceptance of the need to provide explanations. However, as the unit progressed there was a considerable shift in the way in which students were approaching their mathematical tasks; "by the end of term two I no longer had to remind them to explain their processes or thinking".

The use of such modelling by the teacher to support student's developing understanding or practice in inquiry has been recognised as critically important by Borasi and Fonzi (1998). Modelling, particularly when combined with explicit articulation of thinking is very powerful (Carter, 2004), it positions the teacher as a co-inquirer and bridges the divide between telling the students exactly what to do, and expecting students to discover their own mathematical procedures.

The assertion that inquiries can provide for students with a range of abilities (Diezmann, 2004; Lovitt, 1999) was well supported by the data from this study. The teacher noted that even those students with low levels of literacy were able to provide explanations, although they were more limited in terms of word length than the more literate students. The researcher had noted a similar observation, adding that their explanations were often more concise and accurate. The teacher also recounted that several students who had not traditionally been successful in mathematics could provide clear explanations of concepts despite continuing inaccuracies in computation.

To gain some insight into the sustainability of the noted increase in student ability to provide explanations of mathematical thinking, a short interview was conducted with

the teacher the following year. Teaching a number of these students in a combined year 9/10 extended mathematics class, she reported that the year 9 students were noticeably more willing to explain their answers and use mathematical language than the year 10 students who had not undertaken this inquiry focussed unit.

## **Conclusion**

The results from this study are extremely supportive of the benefits of introducing inquiry into the mathematics classroom. It needs to be recognised, however, that as a case study conducted with a single class, these results are not generalisable. Nevertheless, this research has provided evidence that an inquiry focus, together with significant teacher modelling can bring about increases in student mathematical outcomes and in their capacity to explain mathematical thinking. The emphasis on mathematical thinking in a situation where students were constructing their own knowledge appeared to be very powerful in assisting student understanding of the mathematical concepts involved. This approach was advocated by Carter (2004) and mirrors what has been found in the United States where teachers have used the National Council of Teachers of Mathematics *Standards* (1989; 2000).

The capacity for inquiries to engage and challenge learners with a range of abilities is borne out in this case study. The teacher and researcher noted that all students were able to improve their willingness and capacity to explain their thinking, and the work samples indicated that even the more reluctant students did engage with the tasks. The improvement in student outcomes with respect to understanding the purposes and application of the mathematical concepts being studied strongly supports the use of authentic tasks in the mathematics classroom (Diezmann, 2004; Lovitt, 1999).

Although not specifically focussed upon, the characteristics of the teacher in this case study do need to be raised. As a specialist mathematics teacher, she had a solid base of content knowledge that could be drawn upon in the planning and implementation of the inquiry unit. She was also consistent in her approach, modelling and explicitly using explanations for all classroom work over an extended period. The need for inquiry programs to be 'implemented with fidelity and over a reasonable length of time' to improve student results has been previously noted as essential by Hiebert (2003, p.20) and could well have been a major contributing factor in this study.

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## References

- Blythe, T. (1998). *The teaching for understanding guide*. San Francisco: Jossey-Bass.
- Borasi, R., & Fonzi, J. (1998). Teaching practices that support an inquiry approach to mathematics instruction. Retrieved June 4, 2007, from [http://www.rochester.edu/radiate/C/clc\\_sttx.htm](http://www.rochester.edu/radiate/C/clc_sttx.htm)
- Brown, N. (2006). My favourite ratio: An inquiry about Pi. Professional learning support materials for MARBLE. Unpublished.
- Brown, N., Watson, J., Beswick, K., & Fitzallen, N. (2007). Numeracy in a reform-based learning environment. In *Australian Association for Research in Education 2006 International conference proceedings*, Adelaide, SA, November 27-30, 2006 [CD Rom].
- Carter, A. (2004). Autonomy, inquiry and mathematics reform. *The Constructivist*, 15(1), 1-15.
- Clarke, D.M. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele, & A. F. Coxfors (Eds), *Professional development for teachers of mathematics. Yearbook of the National Council of Teachers of Mathematics* ( pp. 37-48). Reston VA: NCTM.
- Department of Education, Tasmania. (2002). *Essential learnings framework 1*. Hobart: Author.
- Department of Education, Tasmania. (2003). *Essential learnings framework 2*. Hobart: Author.
- Department of Education, Tasmania. (2007). Tasmanian Curriculum update for parents. Retrieved May, 1, 2007 from [http://www.education.tas.gov.au/dept/about/minister\\_for\\_education/curriculumupdateparents2](http://www.education.tas.gov.au/dept/about/minister_for_education/curriculumupdateparents2).
- Diezmann, C.M. (2004). Assessing learning from mathematics inquiry: Challenges for students, teacher and researchers. In: *Proceedings Mathematics Association of Victoria Conference* (pp 80-85). Melbourne: Mathematics Association of Victoria.
- Ewing, R., Smith, D., Anderson, M., Gibson, R., & Manuel, J. (2004). *Teachers as learners: Australian Government Quality Teacher Program Action Learning for school teams project evaluation report*. Sydney: Division of Professional Learning, Faculty of Education & Social Work, University of Sydney.
- Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development. In L. Darling-Hammond, and G. Sykes (Eds), *Teaching as the learning profession*. San Francisco: Jossey-Bass Publishers.
- Heibert, J. (2003). What research says about the NCTM standards. In J. Kilpatrick, W. Martin, & D. Schifter (Eds), *A research companion to principles and standards for school mathematics*. Reston VA: National Council of Teachers of Mathematics.
- Lovitt, C., & Lowe, I. (1993). *Chance and data investigations (Volume 1)*, Mathematics Curriculum and Teaching Project. Carlton, Vic: Curriculum Corporation.
- Lovitt, C. (1999). Investigations as a central focus for a mathematics curriculum. Paper presented at the Australian Association of Mathematics Teachers, 1999 Virtual Conference.
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: Sage.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston VA: Author.
- Piaget, J. (1973). *To understand is to invent*. (English translation by Roberts, G.). New York: Grossman.
- Popham, J. (1997). What's wrong – and what's right – with rubrics. *Educational Leadership* 55(2), 72-75.
- Ritchhart, R. (1999). Uncovering the dimensions of disciplinary understanding in mathematics (and other subject areas). In L. Hetland, & S. Veenema (Eds.), *The Project Zero classroom: Views on understanding* (pp. 35-45). Cambridge, MA: Harvard Project Zero.
- Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education*, 1, 55-87.
- Schoenfeld, A. (2002). Making mathematics work for all children: Issues, standards, testing and equity. *Educational Researcher* 31(1) 13-25.
- Shulman, L. S. (1987a). Assessing for teaching: An initiative for the profession. *Phi Delta Kappan*, 69(1), 38-44.
- Shulman, L. S. (1987b). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1-22.
- Stake, R.E. (1995). *The art of case study research*. Thousand Oaks, CA: SAGE.

- Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal* 33(2), 455-488.
- Sullivan, P. (2001). Challenge, creativity, care, and concept development: Open ended questions can help. In C. Vukobratovic, J. Horwood, & J. Roullet (Eds), *A mathematical odyssey* (pp. 57-64). Melbourne: Mathematical Association of Victoria..
- Taylor, C. S., & Bidlingmaier, B. (1998). Using scoring criteria to communicate about the discipline of mathematics. *The Mathematics Teacher* 91(5), 416-425.
- Watson, J., Beswick, K., Brown, N., & Callingham, R. (2007). Student change associated with teachers' professional learning. In J. Watson, & K. Beswick (Eds), *Mathematics: Essential Research, Essential Practice*. (Proceedings of the 30<sup>th</sup> Annual conference of the Mathematics Education Research Group of Australasia, Hobart, Vol 2, pp 785-794). Sydney, NSW: MERGA.
- Wiske, M. S. (1998). *Teaching for understanding: Linking Research with practice*. San Francisco: Jossey Bass Publishers.