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Teaching Common Fractions in Primary School: Teachers' Reactions to a New Curriculum

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This paper presents teachers' views about changes imposed by the introduction of a new curriculum in NSW (BOSNSW, 2002), which significantly increased the expectations of learning about common fractions for primary school children. While current research was used to develop a continuum of key ideas for the learning of rational numbers, early consultation suggested teachers were anxious about the new expectations. A range of resource materials was developed and professional development opportunities made available for teachers to support knowledge building and to provide practical advice about the teaching of common fractions. However, while teachers generally report an increase in confidence, there still appears to be concern particularly in relation to fraction equivalence.

The learning of fractions is an area of mathematics which children find particularly challenging (Moss & Case, 1999; Pearn & Stephens, 2004) and achievement by Australian primary school students in fractions is behind international averages (Thomson & Fleming, 2004). Students have considerable difficulties with fraction equivalence, the notion that different fractions can represent the same amount (Bana, Farrell, & McIntosh, 1997; Pearn, Stephens, & Lewis, 2003). Understanding fraction equivalence is important as it forms the foundation of understanding fraction addition and subtraction, and enables students to compare and order fractions (Kamii & Clark, 1995).

The introduction of the new *Mathematics K-6 Syllabus* (Board of Studies NSW, 2002) in primary schools in NSW has significantly increased the expectations in many content areas, particularly in relation to common fractions. The fraction content now includes: (a) four fraction sub-constructs (part/whole, measure, operator, and quotient); (b) fraction equivalence; (c) fraction addition and subtraction (d) comparison of fractions and (e) mixed numbers. Hence the difficulties experienced by students in learning fractions and the high expectations in achievement indicated by the new syllabus have put considerable pressure on teachers. To support teachers, the Department of Education and Training in NSW has developed resource materials (NSWDET, 2003) and provided professional development. For teachers from other systems, opportunities for professional learning have been provided through a range of providers and professional associations (for further information see Anderson and Moore, 2006).

The study reported here forms part of an investigation into teachers' knowledge, beliefs and practices in relation to the new mathematics curriculum in NSW. The first author has been involved in the delivery of a range of professional development opportunities for primary school teachers to support knowledge building in mathematics content and mathematics pedagogical content. The second author is undertaking postgraduate study with a focus on the learning and teaching of common fractions. Surveys and reflective writing tasks were used to collect data. This paper presents the data from two surveys with primary school teachers.

Background

The development of the new mathematics syllabus for NSW primary schools involved an extensive review of relevant literature (Owens & Perry, 2001), a symposium involving key researchers and a range of stakeholders, and consultation with teachers, consultants and systems personnel. This led to the development of draft documents, which were disseminated to all stakeholders including teachers, researchers and parents. At all stages of the

development process, teachers were involved in writing, reviewing and evaluating the material. Several areas of the draft documents, including common fractions, raised much concern with teachers suggesting the expectations were unrealistic. It is possible that this reaction resulted from poor mathematical content knowledge as well as lack of confidence and experience (Ball, Lubienski & Mewborn, 2001). To support teachers, written materials and professional development have been provided.

Common Fractions Research which informed the New Syllabus

A continuum of key ideas was developed for each of the key concepts in the new syllabus including Fractions and Decimals (see BOSNSW, 2002, p. 28). Several important aspects of the research were particularly relevant in informing the placement of content on the continuum or developmental sequence. One of these is the five interconnected sub-constructs or interpretations of fractions (Kieran, 1980), which are listed in Table 1 with examples from Lamon (2001). These interpretations are both mathematically and psychologically dependent. Understanding incorporates the ability to make connections within and between these different interpretations (Cathcart, Pothier, Vance, & Bezuk, 2006).

Table 1

Different fraction interpretations for the fraction, $\frac{3}{4}$ (Lamon, 2001)

Interpretations	Example
Part/whole	3 out of 4 equal parts of a whole or set of objects
Measure	$\frac{3}{4}$ means a distance of 3 ($\frac{1}{4}$ units) from 0 on the number line
Operator	$\frac{3}{4}$ of something, stretching or shrinking
Quotient	3 divided by 4, $\frac{3}{4}$ is the amount each person receives
Ratio	3 parts cement to 4 parts sand

Another aspect is the use of instructional materials in the form of external representations to model and represent mathematical concepts (Ball, 1993). Goldin and Shteingold (2001) describe the importance of external representation systems, which involve a combination of written and spoken symbols, manipulatives, pictures and real world situations as depicted in Figure 1 (Behr, Lesh, Post and Silver, 1983).

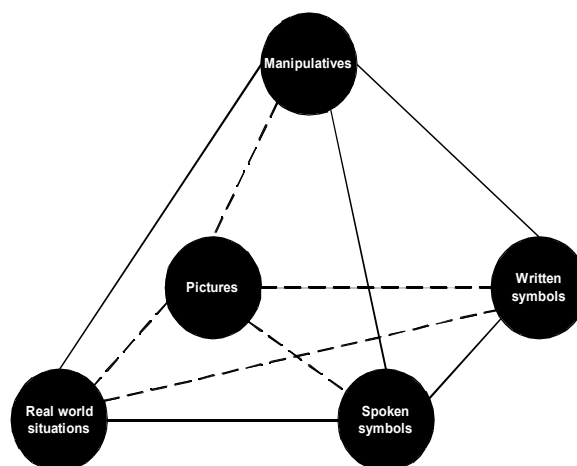


Figure 1. Modes of external representations (Behr, Lesh, Post & Silver, 1983, p. 102).

The modes of external representation are interrelated. Irrespective of whether initial instruction focuses on the combination of manipulatives and real world situations or written

symbols and pictures, or other combinations of external representations, students need to build their understanding across the various modes (Cathcart et al., 2006). Fundamental to effective teaching and learning is the interaction between external and internal representation, with students interpreting teachers' external representations in a way which makes sense to them (Goldin & Shteingold, 2001; Gould, 2005). As suggested by Halford (1993, p. 7), "To understand a concept entails having an internal, cognitive representation or mental model that reflects the structure of that concept. The representation defines the workspace for problem solving and decision making with respect to the concept".

The introduction of fraction notation is a critical aspect of learning about common fractions. In contrast to the previous syllabus when it was left until Grades 3 and 4 (NSW Department of School Education, 1989), the new syllabus recommends the introduction of fraction notation in Stage 1, which corresponds to Grades 1 and 2 (Board of Studies NSW, 2002). Mack (1995) argues for building on students' informal knowledge about fractions from an early stage including the introduction of symbols, and developing the concepts of whole numbers and fractions together.

It is the teacher's role to assist students in the development of correct representations. There needs to be a clear and direct mapping between the modes of external representation such as fraction name, the use of appropriate language and diagrams, and the formal symbol (Ball, 1992; English & Halford, 1995). Cramer, Post and delMas (2002) explored the sequencing of fraction lessons in which fractions were introduced as one representation with the translation to another representation. The effectiveness of the Rational Number Project (RNP) experimental curriculum was compared with the use of commercial mathematics textbooks (CMC). Over 1600 Grade 4 and 5 students participated in the study. Students using the RNP program had opportunities to explore fraction concepts using multiple representations in the form of fraction circles, chips, pictures, story problems, and written symbols, and outperformed the CMC students in follow-up assessment tasks. The RNP group improved their conceptual understanding, were able to transfer their knowledge to tasks not directly taught to them, and they were better able to order fractions and estimate sums and differences.

There is evidence that the development of students' understanding of fractions is greatly enhanced by students' developing their own representations of fraction ideas including pictorial, symbolic and spoken representations to clarify their thinking (Lamon, 2001; Streefland, 1991). By gaining an understanding of the relationship between students' use of external representations in demonstrating their conceptual understanding of fractions, more effective use of external representations can be incorporated into instruction.

Teachers' Knowledge about Fractions

It is recognised that teachers' mathematical knowledge as well as their pedagogical content knowledge are critical factors in determining classroom practices (Ball, Lubienski, & Mewborn, 2001; Ma, 1999). In their review of research, Ball et al. (2001) report there is considerable evidence that teachers are less confident and less successful in teaching rational numbers than whole numbers. Post, Harel, Behr and Lesh (1991) surveyed 218 practising middle school teachers and found that about 25% struggled with basic computation of fractions. In her comparison of Chinese and American teachers, Ma (1999) reports several differences in knowledge about mathematics, knowledge about teaching mathematics, and attitudes to ongoing learning. In particular, Chinese teachers were described as having "profound understanding of mathematics" (p. 126) even though they had fewer formal years of learning than their American counterparts. In relation to fractions, all of the Chinese teachers were able to accurately describe division of fractions using inverse operations whereas many of the American teachers made errors in trying to apply a rule.

The good news is that when teachers are provided with opportunities to develop a deep understanding of mathematical ideas, their teaching changes, they are more confident and their students reap the benefits. Sowder, Phillipp, Armstrong and Schappelle (1998) worked with five middle school teachers for two years, focusing on rational numbers, quantity and proportional reasoning. They report that the teachers were more willing to try new mathematics with their students and they became less reliant on textbooks.

In recognition of the need to support the teaching of fractions, Lamon (2006) translated her research into practice, and provided resource materials for teachers with information about the mathematical content as well as student thinking, the nature of conceptual development, and instructional activities. She states “one of its underlying assumptions is that facilitating teacher understanding using the same questions and activities that can be used with children, is one way to help teachers to build the comfort and confidence they need to talk to children about complex mathematics” (p. xiv). The NSW Department of Education and Training has produced a book with background information, lesson ideas and interactive games on a CD Rom (NSW DET, 2003).

The increased expectations for the learning of common fractions from the new syllabus and the research which highlights concerns about teachers’ knowledge (e.g., Post et al., 1991) and confidence, suggest that teachers may need more support than is currently available. To explore the early impact of the new documents, to ascertain teachers’ levels of confidence, and use of existing resources, the authors used surveys to gather data from primary school teachers in NSW who attended several professional development sessions.

Methodology

Two surveys were used to gather data about teachers’ views and practices of common fractions in the new curriculum in NSW. The first survey was completed by teachers who attended the *Certificate of Primary Mathematics Education* (CPME), which was designed by lecturers from the University of Sydney with a particular focus on the implementation of the new mathematics syllabus. One study that informed the planning of the CPME as well as the design of the reflective surveys was the Garet, Porter, Desimone, Birman and Yoon (2001) large-scale national survey into the effects of different characteristics of professional development on teachers’ learning. The survey of 1027 mathematics and science teachers revealed a set of

... three core features that have significant positive effects on teachers’ self-reported increases in knowledge and skills and changes in classroom practice: (a) focus on content knowledge; (b) opportunities for active learning; and (c) coherence with other learning activities. It is primarily through these core features that the following structural features significantly affect teacher learning: (a) the form of activity (e.g., workshop vs study group); (b) collective participation of teachers from the same school, grade, or subject; and (c) the duration of the activity.

From the results of this survey, the CPME incorporated a focus on content from the new syllabus, opportunities for active learning through discussions with colleagues and opportunities to try teaching ideas between the weekly workshops, and a focus on the need to program and plan the new syllabus. The CPME included both a conference and workshops, encouraged more than one teacher from a workplace to attend, and involved ongoing meetings over a six-week period. Seventeen participants completed the CPME, and were invited to complete the first survey, which sought to obtain information about their knowledge and confidence as well as new knowledge gained during a workshop on fractions and decimals.

The second survey aimed to explore teachers’ views about the suitability of particular aspects of fractions for each stage of development in the new syllabus (see Appendix 1). It was distributed to three different groups of teachers who participated in professional

development opportunities conducted by the first author. Teachers were also asked which fraction ideas were most difficult to teach and to report their confidence in teaching fractions. The final two questions asked which resources they choose to use and what additional support they would like to assist them with the teaching and learning of fractions. Teachers’ responses to the Likert items on both surveys are reported in the next section. The open-ended questions were read and coded independently by each author to identify key issues and recurring themes. The following section reports the main results and discusses the findings for each of the surveys.

Results and Discussion

Survey Data

Survey Results from the CPME

For the first survey, 15 of the 17 teacher participants completed a reflection before and after the two-and-a-half hour workshop on Fractions and Decimals. The survey questions included a report on teachers’ knowledge and confidence, an indication of what had been learnt, and how the new knowledge had challenged their thinking. Table 2 indicates teachers’ ratings of their knowledge against four categories of “very good”, “good”, “satisfactory” and “unsatisfactory” both before and after the workshop. No participants selected the “unsatisfactory” category. Table 3 indicates teachers’ ratings of their confidence against four categories of “very confident”, “confident”, “somewhat confident” and “not confident” both before and after the workshop.

Table 2
Teachers’ rating of their knowledge and understanding (N=15)

	Before the Workshop	After the Workshop
Very good	2	4
Good	6	8
Satisfactory	7	3

Table 3
Teachers’ confidence about teaching fractions and decimals (N=15)

	Before the Workshop	After the Workshop
Very confident	1	1
Confident	6	9
Somewhat confident	8	4
Not confident	0	1

These tables indicate that the teachers report having a range of knowledge and understanding as well as varied confidence. Five of the fifteen participants reported that the workshop had increased their knowledge, while four teachers reported improved confidence and two reported a decrease in confidence. While these results are fairly reassuring for the workshop presenters, it is of concern that two teachers felt less confident after the workshop. Unfortunately neither teacher included contact details for follow-up interviews but this has led us to consider what might have occurred. We can only speculate that some teachers think they understand how to teach fraction concepts but the additional knowledge gained from the workshop convinced them that there is much more to it. Of course, there is no guarantee that new knowledge and improved confidence will be retained but at least some of these teachers felt more positively about fractions and decimals after the workshop than they had beforehand.

In relation to the question on what had been learnt, several themes emerged. Teachers noted the need to (numbers in brackets indicate how many teachers noted each point):

- focus on both continuous and discrete models (2)
- use a range of representations (4)
- take care when introducing the symbols (2)
- develop deep understanding (5)
- focus on misconceptions (3)
- ask students to explain their thinking (3)
- think about the types of questions which are posed during lessons (2).

Challenging the way they think about fractions was another desired outcome of the workshop. In response to this question, teachers commented on:

- students providing correct answers from incorrect reasoning
- the relationship between fractions and division
- the need for consistency in teaching about fractions
- their confidence in teaching the lower stages but not necessarily the later stages of primary school.

The final question on the survey invited teachers to provide any additional comments. Three respondents indicated that more support was needed for teachers in relation to teaching fractions. One commented:

I enjoyed fractions at school but the challenge is teaching it and allowing students opportunities to make meaning and develop an understanding that is appropriate to their age and level of development.

Second Survey Results

Of the 92 teachers who attended three different professional development opportunities conducted by the first author, 36 primary school teachers from fifteen schools in the Sydney, Central Coast and far Western regions of NSW responded to the surveys. All grade levels were represented with teaching experience ranging from two years to 35 years. Table 4 summarises background information about the respondents' level of experience and teaching grade level. Most teachers (59%) were currently working in either Year 5 or Year 6 classrooms and half of the teachers had 15 years or more teaching experience.

Table 4

Teachers' level of experience and teaching grade level (N=36)

Level of teaching experience	0 to 5 Years Experience	6 to 15 Years Experience	More than 15 Years Experience
Teaching Grade Level			
Early Stage 1 (Kindergarten)	0	1	1
Stage 1 (Years 1 and 2)	0	2	5
Stage 2 (Years 3 and 4)	2	4	3
Stage 3 (Years 5 and 6)	5	2	11

Table 5 indicates when each new key idea related to common fractions is introduced in the syllabus, and summarises the responses of the participants in relation to perceived level of difficulty for the Stage. It should be noted that not all respondents provided feedback on all Stages. It was anticipated that the responses would be on the basis of the experience each teacher has had teaching the fraction content from the new syllabus.

Over 40% of respondents indicated that equivalence between fractions in Stage 2 and 3 is too hard and almost 50% of respondents indicated that adding and subtracting simple fractions where one denominator is a multiple of the other and multiplying simple fractions by

whole numbers is too difficult. Over 25% of respondents indicated that model, compare and represent fractions with a range of denominators in Stage 3 was too hard. This feedback shows that teachers are quite concerned about the new content and as yet, they are not convinced that the syllabus writers have put the content in the right Stages for the majority of students.

Table 5

Fraction key ideas for Early Stage 1 to Stage 3 (BOSNSW, p. 28) and teachers responses to the survey question about level of difficulty (N=36)

Stage	Fraction Content – Key Ideas	Too easy	Appropriate for this Stage	Too hard
ES1	Divide an object into two equal parts	4	22	0
	Recognise and describe halves	4	22	0
1	Model and describe a half or a quarter of a whole object	2	22	1
	Model and describe a half or a quarter of a collection of objects	0	22	3
	Use fraction notation $\frac{1}{2}$ and $\frac{1}{4}$	0	20	5
2	Model, compare and represent fractions with denominators 2, 4, and 8, followed by fractions with denominators 5, 10, and 100	0	15	7
	Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths	0	12	10
3	Model, compare and represent commonly used fractions (those with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100)	0	20	11
	Find equivalence between thirds, sixths and twelfths	0	15	16
	Express a mixed numeral as an improper fraction, and vice versa	0	16	15
	Add and subtract simple fractions where one denominator is a multiple of the other	0	15	16
	Multiply simple fractions by whole numbers. Calculate unit fractions of a number.	0	15	16

Several open-ended questions were used in the survey to gather more information about fraction concepts which are difficult to teach, confidence in teaching fractions, resources which are used to plan and program, and additional support or assistance which would help teachers to plan and program. Teachers' responses highlight the difficulties that children experience and the issues teachers face as they try to engage children in meaningful learning (summarised in Table 6). Understanding fraction equivalence was recorded as an aspect, which is difficult for students to understand. This is related to the notion of ordering fractions, and operations with fractions. Several also commented on the issue of developing understanding of the relationships between fractions, decimals and percentages. As one teacher of Year 6 indicated in response to the aspect which was difficult to teach:

The whole lot as the children haven't had enough groundwork from previous years! It was better this year compared to last year, so things will improve as long as the syllabus STAYs the same.

As teachers were given three years to fully implement the new syllabus and this is only the third year, it is not surprising that this comment was made. It is anticipated that as teachers become more familiar with the changes and learn new approaches to helping students learn fractions, they will become more confident.

Table 6

Teachers' responses to aspects of fractions which children find difficult to learn and are difficult to teach (numbers in brackets indicate the number of teachers who included this response)

Question	Responses
For the class you are teaching this year, what aspect of fractions content have the children found difficult to understand?	ES1 – none S1 – half and quarter of collections of objects (3); written notation; S2 – equivalence (3); ordering fractions (2); converting fractions to decimals (2); S3 – too difficult; equivalence (6); mixed numerals (3); fractions to decimals and percentages (6); addition and subtraction with different denominators (3); problem solving with fractions; fractions as part/whole
What aspect of the fractions content have you found difficult to teach?	ES1 – None (2) S1 – None; equivalent fractions; finding collections of objects S2 – equivalence; none (2); percentages/decimals/fractions S3 – equivalence (4); the whole lot as children have not had enough early experiences; fractions to decimals (4); mixed numerals (2); improper fractions; ordering fractions; finding a fraction of a number

Another question asked teachers to comment on “how confident do you feel teaching fractions”, and the final two questions on the survey asked teachers to indicate the resources they use and the additional support they would like. The results for this survey were similar to those recorded on the earlier survey with about half of the respondents at each stage indicating that they were “very” confident or that they had “good” knowledge of fractions. Of the remainder, most indicated they were “fairly” or “reasonably” confident. One teacher of Stage 6 indicated “mildly”, another suggested they “like this the least”, and a third recorded a “need to revise”. Table 7 presents a summary of teachers’ responses to the resources they use and requests for the additional support.

Table 7

Teachers' responses the resources they use and the additional support they would like

Question	Responses
What resources do you use to help plan and program fractions lessons with your students?	Hands on materials (20) including fraction “cakes” Interactive Whiteboard (2) Hands on games (4) Textbooks (6) Computer programs (4) including learning objects Paper and pencil Flip or flash cards (3) Fraction wall (3) MAB blocks (4) Food (5) Number lines OHPs Pretests Sample Units of Work
What additional support or assistance would you like to help plan and program for teaching fractions to children in primary school?	ES1 – none S1 – more resources S2 – syllabus needs to be modified; assessment advice; OHP aids; hands on materials (2); audio visual material; S3 – time (3); resources (2); concrete materials; games (3); open-ended questions and problems; inservice courses; sample units of work; fractions/decimals relationship materials

Reassuringly, most teachers reported using a range of manipulatives to support the teaching and learning of common fractions; this at least supports one of the modes of external representations noted in Figure 1. These included fraction cakes, MAB blocks, food, and

counters. Others reported using models like the fraction wall and number lines, with interactive games and learning objects used by some teachers to support learning, particularly in the upper grades of primary. For additional support, teachers requested resources, materials, and units of work. Interestingly, only one Stage 3 teacher indicated he would like more professional learning opportunities, and only one teacher of Stage 2 wanted the syllabus to be changed as it was perceived to be too difficult for her class.

One concern about these data is the reported use of textbooks by six of the respondents. It is possible that teachers who are not confident rely on textbooks to support their teaching (Sowder et al., 1998). However, a limited examination by the authors of some of the currently available textbooks in NSW indicates a lack of opportunities for students to externalise their internal representations and understanding. Pictures are pre-divided and some are also shaded. Questions relating to ordering fractions or finding an equivalent fraction are usually presented in symbolic notation and only require an answer. The types of questions presented limits children's thinking and use of the full range of representations of fractions.

Conclusion

It is evident from the results that teachers are still concerned about the higher expectations for the learning of common fractions in this new curriculum. Many still believe that the content is too difficult for students in the grade they are teaching. However, for those attending a workshop, which focused on the teaching and learning of fractions, their knowledge and confidence reportedly improved. This provides support for the efforts of professional learning providers but more is clearly needed. Focused learning, which enables teachers to explore the mathematical content including the importance of the sub-constructs or interpretations of fractions, as well as the modes of external representations is necessary. While teachers appear to use a range of manipulatives and real-world situations to support student learning, it is unclear how they link the representations together. Further investigation is necessary to explore teachers' knowledge of the sub-constructs and how they plan to use the external representations in lessons.

The two surveys used in this study provided valuable information about teachers' views on the new syllabus in NSW, as well as their level of knowledge about common fractions and their confidence in teaching this aspect of the syllabus. Given that several respondents included their name on one of the surveys for further participation in the research, they were refreshingly honest about their lack of confidence and concern about teaching fractions. It is anticipated that with further experience and increased knowledge, teachers' confidence will increase. It is also possible that some of the teachers reported a confidence that relies on the use of algorithms rather than using the full range of strategies to develop deep understanding.

While a small number of teachers were surveyed in this study, they represented teachers from small and larger primary schools as well as teachers from a wide geographical area in NSW. They represented a range of teaching experience although most were teaching in grades 5 or 6. The respondents were those who were attending professional development courses and hence were already seeking to develop their knowledge and understanding of the teaching and learning of mathematics to primary school children. A more representative sample of teachers from NSW could include those who are not attending a professional learning program as this might provide a better indication of the impact of the new syllabus and the level of knowledge about common fractions. In addition, interviewing teachers would enable the authors to gather more in-depth information about primary school teachers' knowledge and confidence. This will be pursued in the next phase of the research.

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Appendix 1 *Teaching Fractions in Primary School: What are the Issues?*

This brief survey is designed to obtain initial feedback on the **Fractions** content in the new *Mathematics K-6 Syllabus*. There was considerable change to the content in this new syllabus, and teachers were initially concerned that it was too difficult for children in primary school.

We are interested in your views about Fractions in primary school.

How do you feel about the Fractions content in the new syllabus?

Please provide some feedback by answering the following questions.

1. What is your current role at school? _____
2. If you are currently teaching, which year group? _____
3. How long have you been teaching? _____
4. On the basis of your experience teaching from the new syllabus, please indicate with a ✓ whether you think the Fractions content listed below is “too easy”, “appropriate” or “too difficult” for the Stage indicated.

Stage	Fraction Content – Key Ideas	Too easy	Appropriate	Too hard
ES1	<ul style="list-style-type: none"> • Divide an object into two equal parts • Recognise and describe halves 			
1	<ul style="list-style-type: none"> • Model and describe a half or a quarter of a whole object • Model and describe a half or a quarter of a collection of objects • Use fraction notation $\frac{1}{2}$ and $\frac{1}{4}$ 			
2	<ul style="list-style-type: none"> • Model, compare and represent fractions with denominators 2, 4, and 8, followed by fractions with denominators 5, 10, and 100 • Find equivalence between halves, quarters and eighths; fifths and tenths; tenths and hundredths 			
3	<ul style="list-style-type: none"> • Model, compare and represent commonly used fractions (those with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100) • Find equivalence between thirds, sixths and twelfths • Express a mixed numeral as an improper fraction, and vice versa • Add and subtract simple fractions where one denominator is a multiple of the other • Multiply simple fractions by whole numbers. Calculate unit fractions of a number. 			

5. For the class you are teaching this year, what aspect of Fractions content have the children found difficult to understand?

6. What aspect of the Fractions content have you found difficult to teach?

7. How confident do you feel teaching Fractions?

8. What resources do you use to help plan and program Fractions lessons with your students?

9. What additional support or assistance would you like to help plan and program for teaching Fractions to children in primary school?

Thank you for your assistance in answering these questions.

If you are interested in participating in further research into ***Teaching Fractions in Primary School***, please include your name and an email or telephone contact below.

Name: _____ Contact details: _____