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The Contribution of Mathematics to Graduates' Professional Working Life

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Abstract

Recent mathematics graduates are in an intermediate position between students and professionals. On the one hand, their experiences as mathematics students are still fresh in their minds: on the other, they have started to develop an appreciation of the role that mathematics plays in their working life. Here we investigate their views of the contribution that their mathematics education has made to their professional work through an analysis of a series of interviews. One notion that was expressed by each of the graduates was that studying mathematics has developed their ability to solve problems and think logically. However, their interpretation of these terms was quite varied. In this paper, we explore their experience of mathematics as problem solving and link this with their view of their professional role as mathematicians.

Introduction

Recent graduates of a mathematics degree who are currently working in a wide variety of contexts provide us with an important sources of information regarding how they experience and understand mathematics in the workplace. We carried out interviews with 14 such graduates, asking them about their views of mathematics, their previous learning of mathematics and the contribution of the mathematics that they had studied to their present work situation. An interesting feature of these interviews was that all respondents discussed the fact that mathematics had enhanced their abilities at 'problem solving' (using this or related language), yet they seemed to interpret these terms in a wide range of ways. It is this feature that we explore in the present paper, investigating their ideas of problem solving and linking them to their views of their professional role using a theoretical model that we have developed in the context of a range of professions (Reid & Petocz, 2004a). This exploration is an extension beyond our previous research where we focused on what students learn from undertaking a degree in mathematics, and what they take with them from their studies into their workplace. We have been exploring these and related questions for several years, based initially on a series of interviews with students in a mathematical sciences degree, analysed using a qualitative research approach. We have written about undergraduate students' conceptions of mathematics (Reid *et al*., 2003), their conceptions of learning (Reid *et al.*, 2005), and have also discussed the extension of our project into an international comparison of undergraduates' ideas that entails quantitative as well as qualitative components (Petocz *et al*., 2004). Research in higher education rarely moves beyond the institutional learning situation, but here we explore what graduates have taken from their previous learning and how they interpret what they have learned in their new work contexts.

From the point of view of *mathematicians*, the notion of solving problems is an essential component of mathematics, and one that has been linked with the notion of creativity (see Reid & Petocz, 2004b). Many books have been written about the process of problem solving (for example, Polya, 1957, 1962; Schoenfeld, 1985; Zeitz, 1999). The general viewpoint is expressed by Zeitz (1999, p. ix) when he writes: *"Problems and problem solving are at the heart of mathematics. …Someone who learns to solve mathematical problems enters the mainstream culture of mathematics."* Polya (1962, p. ix) introduces his book with a broad definition: *"Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable."* He presents a theory of 'heuristics', the study of means and methods of problem solving, that can be applied to problems in mathematics and beyond. He paraphrases Descartes' (1628) universal method for the solution of problems as: reduce any problem to a mathematical one, then to a problem in algebra and finally to the solution of a single equation. Schoenfeld (1992) has carried out an extensive programme using videotapes and interviews investigating mathematicians' and students' approaches to solving problems. He concludes that important factors in successful problem solving include cognitive resources, heuristics, control or metacognition and beliefs. These 'cognitive resources' can been seen in action as Burton (2004) reports on a series of interviews with research mathematicians and discusses their approach to problem solving. All mathematicians claimed to be working on problems, anywhere from one to 20, all the time. Burton found three characteristic styles of thinking about problems: visual (in pictures, possibly dynamic), analytic (in symbols, formalistic) and conceptual (in ideas, classifying). The first two styles, but not the third, have been previously reported and discussed in the mathematical literature. Very few of the mathematicians used all three of these styles, and one-third of them claimed to use only one style.

Leonhard Euler, one of the greatest mathematicians of all time, expressed the close connection between mathematics and problem solving in a letter of 1736 concerning his investigation of the Königsberg bridge problem:

> Thus you see, most noble Sir, how this type of solution bears little relationship to mathematics, and I do not understand why you expect a mathematician to produce it, rather than anyone else, for the solution is based on reason alone, and its discovery does not depend on any mathematical principle. Because of this, I do not know why even questions which bear so little relationship to mathematics are solved more quickly by mathematicians than by others. (quoted in Hopkins & Wilson, 2004)

Professional mathematicians would agree with this sentiment even today, and later in this paper we will see some of our graduates expressing very similar ideas.

Recent graduates are in an intermediate position between students and professionals, in that they have completed their studies but have not yet taken on their full professional role – they could be regarded as novice rather than expert practitioners (Rosenfield, 2002; Wenger, 1998). They are in a unique position to comment on aspects of their studies and the transition to their profession. However, graduates seem to be most often asked about their earning power and overall rating of their departments and universities, rather than their retrospective views on their recent studies. At Griffith University, Australia, Crebert *et al*. (2004) discussed graduates' (and employers') perceptions of the role of the university, work placements and subsequent employment in developing generic skills and abilities, including problem solving. Data were collected via a survey of graduates who had participated in work placements during their studies and through two small focus groups. The results highlighted some mismatch in graduates' and employers' expectations of the qualities needed by a graduate: while employers felt that some graduates were not adequately prepared for the realities of work, some employees thought that they were being held back by the limitations of the employers and the workplace. In the area of problem solving, employers felt that graduates should have more experience of open-ended problems that need a solution within the constraints of limited time and resources. However, no mathematics graduates were included in the sample. In the context of Australian Technology Network (ATN) universities, Bowden *et al.* (2000) provided case studies of the integration of generic capabilities within specific disciplinary contexts. Their report assumed that that students would be able to take their capabilities into their work. They postulated that "problem solving for graduate practice requires a focal awareness of the relevance of one's repertoire of skills within a specific context and the confidence and ability to devise an appropriate response."

In the discipline of mathematics itself, Houston and Mulholland (2002) discussed the development at the University of Ulster, Northern Ireland, of a mathematics curriculum that includes "entrepreneurial studies". The program seeks to prepare mathematics graduates with a range of skills including innovative thinking and creativity, communication and team-building ability, and a range of business-related skills. Components such as group projects, peer tutoring and job placements were incorporated into the course, informed by feedback from students, employers and graduates. A panel discussion at a Mathematics Association of America meeting (Campbell *et al*., 2000) investigated the projected skills that graduates might need a decade later. They pointed to the general decline in numbers of mathematics graduates, suggesting that pedagogical approaches may be able to reverse the trend (see the *Curriculum Guide*, Mathematical Association of America, 2004). Note that the references in this paragraph present a view of mathematics graduates' skills and abilities from the viewpoint of educators rather than the graduates themselves: the latter seems to be uncommon in the literature.

The Interviews

The aim of our study was to investigate recent graduates' views of mathematics, learning mathematics and using mathematics in their workplace. Interviews were carried out with 14 graduates of a degree in mathematical sciences, a three or four-year degree including a wide range of mathematically-based subjects, and allowing specialisations in statistics, operations research, computing, mathematical finance and applied mathematics. Invitations to participate were sent to a list of recent graduates (all within five years of graduation and most within one year) and those who expressed interest were given further details. The study was approved by the appropriate ethics committee and all participants gave written informed consent. Interviews were carried out at convenient locations, including the university, the workplace and even a local café. Two of the interviews were carried out by e-mail with participants in different parts of the world (one in the US, the other in the UK). Thus, the raw material of our study was the transcripts from these 14 interviews, a text consisting of over 60,000 words. As opposed to students in a contained learning environment, graduates are situated in a range of work contexts that provide different opportunities for undertaking mathematical work. Our graduates included people working in finance, insurance, IT and statistics, as well as people still in the academic arena, two undertaking doctorates (one after a period of work in the finance industry) and one who was working as a lecturer, having completed a doctorate.

Our study was planned using a phenomenographic orientation, that is, a hermeneutic, interpretive methodology. Phenomenography is a qualitative approach to research that looks at how people experience, understand and ascribe meaning to a specific situation or phenomenon (Marton & Booth, 1997; Marton & Saljo, 1976), in this case, their experience of using the mathematics that they had previously learned in their workplace. In a phenomenographic study, the questions posed are designed to encourage the participants to think about why they experience the phenomenon in certain ways and how they constitute meaning of the phenomenon. The results are reported in an *outcome space*, a hierarchical set of logically related categories, from the narrowest and most limited to the broadest and most inclusive. These categories display the qualitatively different ways in which people in a particular group view the phenomenon. Thus, our initial interview questions were: *What do you think mathematics is about?, How would you describe your experience of learning mathematics? What have you taken with you from your experience of studying mathematics at university?, What were your ideas about working as a qualified mathematician when you were at university?* and *How do you use mathematics in your work now?* These questions were followed by further probing questions, both general ones such as: *Can you give me a practical example of that?* and specific ones such as: *Why do you think it's important that people do learn maths?*

In this paper we are focusing on our respondents' ideas about the role of mathematics in problem solving, and what this contributes to their professional working life. The analysis that we have carried out uses a combination of phenomenographic, thematic, comparative and interpretive methodologies. We read through the complete set of transcripts several times and noted the particular theme of problem solving that seemed to be present in some form in all transcripts. We then identified an outcome space representing the range of ways that our participants viewed the notion of problem solving, made comparisons with our previous analyses of data from undergraduates (Reid *et al.*, 2003, 2005) and interpreted the results from the point of view of a previous theoretical model of the relationship between conceptions of learning and perceptions of professional roles (Reid & Petocz, 2004a). The illustrative quotes that we use in the following section have been labelled with pseudonyms. Our analyses were made considerably easier by using the qualitative data analysis package NVivo (QSR International, 2002) to code, examine and interrogate emergent themes and ideas.

Results

First, all our respondents put forward the idea that studying maths had helped them in the general area of problem solving, either using this specific term or an alternative wording, often as the first response they made, but sometimes elsewhere in the course of the interview.

> Kath: [What do you think mathematics is about?] I think it's about learning techniques to apply to problems, it's about the way you approach problems and think about, yeah, think about techniques needed to solve the problems.

> Leo: [Okay, so could you just describe that way of sort of tackling things?] It's just the whole problem solving thing like with maths, you know what I mean, being able to tackle a problem, going through it step by step, you know, in how to solve it, you can apply those steps in any other situation in life, so working through problems, problem solving, being able to sort of quickly analyse something or yeah stuff like that.

> Lucy: [What do you think mathematics is about?] Problem solving, for me it's problem solving, you've got a problem and I would think about things in a very logical fashion, which I think mathematics teaches you to do, and is a very reasonable area in that it's all based on reasoning, I'm not making any sense, that's what I think, for me it's problem solving.

> Tony: [What do you think mathematics is about?] … So, it's something that you would basically use to help you get to a solution that you are after so, most people don't really want to integrate something or do something mathematical just for the sake of doing something mathematical. But if you are trying to get from A to B, you'd want to use some sort of mathematical functions or some sort of maths to help you get to that point.

Using a phenomenographic approach to the analysis, we identified an outcome space, a hierarchy of three qualitatively distinct conceptions of problem solving displayed by these mathematics graduates. From the narrowest to the broadest, we summarise these conceptions as (A) specific mathematical problems, (B) problems in a work context, and (C) problems in life generally. In the following paragraphs, we describe these three conceptions in more detail, and illustrate them with brief quotations from the transcripts.

(A) Problem solving refers to specific mathematical problems, often in a school or university context, possibly the sort of problems that occur in textbooks. Linda's quote makes this explicit, while Vida's quote links 'questions' and 'problems' in a study context.

> Linda: [What do you think mathematics is about?] For me, yeah, maths is about solving problems. That's, I mean if I go back to high school mathematics, which is probably what got me interested in maths, especially in the last year or so, it's just, actually I like solving problems, and maths is one way to solve problems I guess. [Could you tell me a bit more about what you mean by solving problems?] Well I guess, I don't know, the, I mean the example I always think of in my head is I remember in HSC three unit maths, is a problem where you've got to manipulate, I guess it's just algebraic manipulation, pretty simple stuff, and it's when you are introduced to factorials and stuff, and you've actually got to do a bit of manipulation and follow a method until you can sort of see what answer you are trying to find, and you are going down a path and you've got to find the solution and it's just finding different ways to get to the end.

> Vida: Well, maths is one of those things where it's very practical, you've got to actually go in and do the questions, as opposed to say just reading about it. I mean you can read all you like but you won't know how to do questions or you won't know how to solve problems until you actually go and have a go at it.

(B) Problem solving refers to problems in a work context generally, including but not limited to mathematical problems. In the quotes below, both Kath and Vida set their discussion in terms of workplace problems that are not necessarily mathematical ones.

> Kath: I think what I've derived from mathematics more than a set way that people use mathematics in the workplace, I've actually derived more of an idea that it's a framework to think about problems and not, you don't see calculus in the workplace or anything like that, well I haven't in my particular position, I mean I'm not generalising it and saying that no one else will, but I think as far as the workplace has been for me it's just, it's been a starting point for me to think about problems and for me to analyse things.

> Vida: Okay, my job doesn't really rely heavily on maths or mathematical knowledge, but what I use, or what I perceive to be using are the skills that I've picked up, you know the analytical skills, the research skills, the ability, you know, like I said to pick up something, a report or whatever it is that you have got and see whether it does make sense. Although other people, although other professionals write them, sometimes it's not always obvious or clear what they are trying to say or trying to do, so I think that's where the logic and the analytical skills come into it. I'm able to sort of look at something and piece the puzzle if you like.

(C) Problem solving refers to an approach to problems in life generally, not necessarily limited to the workplace, or to obviously mathematical problems. In the following quotes, both Matt and Adele show this explicitly, while Vida implies that she is looking beyond work.

> Matt: So logic and reasoning were ingrained into my family. I guess that's part of the reason I like statistics and research. It's very much solving real world problems. Statistics also attracted me as it could be applied to answer important questions. In the medical area where I use my stats, I got into it because it could be used to help people by helping to solve important medical questions.

> Adele: [What do you think mathematics is about?] It's about learning how to be logical, how to do problem solving, so in addition to learning all the formulas and stuff, I would say it is an approach to help us to structure our thinkings and thoughts, so that structure allows us to deal with problem solving in daily life. … I mainly see mathematics as a way to help me learn how to think in problem solving.

> Vida: [Would you say that learning maths has changed you?] Studying maths for three years, has it changed me? Might've, it may have changed the way I approach certain projects or certain obstacles in my life. I don't know, it's not something you think of or think about, but I find myself, or you try to be a bit more objective with problems that you face, I don't know if that's just me or if it's something that I've learnt, it's hard to answer that one.

In common with other phenomenographic outcome spaces, the conceptions that we have identified and described are hierarchical and inclusive (Marton & Booth, 1997, p.125–6). Those graduates who describe the narrower, more limiting views of problem solving mathematics or learning seem unable to appreciate features of the broader, more expansive views. However, those graduates who describe the broader, more holistic views are aware of the narrower views, and can discuss and use them when necessary. Vida's quotes at all three levels illustrate this. This is the reason why we as educators value the broader, more holistic conceptions.

Our participants also offered specific comments on their methods of problem solving. A common theme involved the classical technique of breaking a problem down into components, which could then be attacked in various ways. Some relevant quotes from three graduates are given below:

> Adele: Problem solving, it's more like okay you've got a problem in front of you with maybe restrictions, constraints and stuff like that, and you will need to look at how you come to a solution, maybe you see a problem in the middle and you've got to just eliminate say, look outside the box in a way, so you come over a problem and you can say 'okay, let's try to solve it doing what you were taught or what you experienced too', but you come and get stuck and you have got to say, you know, how do you fix that up.

> Alistair: Like I don't use that calculus, but I definitely use the same methodology, like I'd break it down into bits, I'd work it out, like I'd work from the middle or something and work my way out or something like that, or even vice versa, you work from the end sometimes and just, you know came back.

> Josefine: Um, well with mathematics personally I've found that it helps me, instead of, when you get a big problem, you don't just panic and go 'oh my God, how am I going to do this?' because of, I guess, the routines that you've gone through or the procedures that I've been taught with mathematics, I've learnt to just break it down into smaller parts and solve each of those parts separately. And so you sort of, the big picture is working out what the separate parts are and then the detail is going in and working out how that little part works and then you do the next little part and you put it all together at the end.

Another theme was the notion of quantifying a problem, translating it into equations which could then be solved. This sounds very like Descartes' (1628) universal method, as the following quotes show:

> Matt: So, by thinking about the maths behind something, it makes more sense to me. The easiest example I can think of reducing to an equation is think of a pedalling a bike. We all know that the faster you pedal the faster the bike will go. But why that's true is based on maths, you use this to work out how much 'pedal power' you would need to go at a certain speed. … So, basically maths can help you answer questions even in nature, which would be one of the reasons you would want to reduce something to an equation. Also, the problem never looks as difficult to me once it's in an equation.

> Phil: When you are trying to solve these problems, you look at the problem that you are faced with, a particular problem and you try and isolate what are the key variables, what are the key factors that are driving the problem, and if they drive the problem, they probably drive the solution in some way as well. So what you need to do is arrange these variables, if you will, into some sort of relationship.

One graduate, Linda, raised an interesting point about the influence of the workplace on the method of problem solving, both in terms of the pressure of time and the necessity for communicating the solution:

> Linda: [Okay, did communicating non-technically about maths, does that change the way you see maths itself?] It can change the way you approach problem solving. For example, there may be technically sort of, there may be two ways to solve a problem and one may be very technical and sort of the optimal way of solving a problem and the other one may be a little more rough guess or more an estimate, and quite often you haven't got the time to do the optimal method anyway in the workplace, certainly in the environment that I worked in, so you've only got the time to do the estimate and also when you do do that sort of thing it's easier to work through and show people what you have done, if you've taken, if you've taken a simplistic method.

It is an interesting counterpoint to the many discussions of problem solving by mathematicians that recent graduates seem to be able to talk fluently and comprehensibly about this important mathematical activity.

The Professional Entity

In the context of tertiary education, students look beyond classes and curriculum towards their future professional life. Their perceptions of their future profession influence their approach towards learning at university. In the same way, lecturers' perceptions of their professional discipline influence their teaching approach as they seek to prepare students for their future roles. For both groups, the way they view their profession influences the way they carry out their part in the pedagogical enterprise. Our investigations of this link evolved from research in music education (Reid, 1997), and have been extended into disciplines as varied as design (Reid & Davies, 2003), statistics (Reid & Petocz, 2002), law (Reid, 2003) and now into mathematics (Reid *et al*., 2003, 2005) . The recognition that views of professional work and learning, and the relationship between them, had similarities across disparate disciplines, and the research on which this was based, led to the notion of the 'Professional Entity' (Reid & Petocz, 2004a). A recent European study exploring students' ideas about education, professional identity and work (Hult *et al*., 2003) adds support to our ideas.

The Professional Entity is a unifying way of thinking about students' and teachers' understanding of professional work, organised in a hierarchy of three levels. At the narrowest, Extrinsic Technical level students and teachers describe a perception that professional work consists of technical components that can be used when the work situation demands it. In mathematics, this is manifested by a view that mathematical work is concerned with gathering mathematical techniques for use in different situations. At the Extrinsic Meaning level, students and teachers hold that professional work is about understanding the important characteristics of the professional discipline. In mathematics, this is shown by the view that mathematical work is focused on finding meaning in numbers, equations or datasets, and using them to build models of aspects of reality. The broadest conception is the Intrinsic Meaning level, where students and teachers perceive that their professional work is related to their own personal and professional being. In mathematics, this corresponds to viewing the work of a mathematical scientist as creating and modifying views of the world based on mathematical evidence.

The graduates interviewed in the present study are in the intermediate position of having recently been students (and being asked to reflect on their experiences as students) and being relatively inexperienced professionals. Their views of the relationship between their learning and their profession highlight another facet of the Professional Entity model. In terms of the three levels of conceptions of problem solving in mathematics, we can suggest that recent graduates' views of their profession are reflected in their conceptions of problem solving. Graduates who see their profession at the Extrinsic Technical level, viewing mathematics in terms of mathematical techniques might view problem solving as another of these techniques to be applied to specific mathematical problems (conception A). Those who see their profession at the Extrinsic Meaning level may view problem solving as a way of finding meaning in their work context, applying the idea to a broader range of work problems, not limited to purely mathematical ones (conception B). Finally, graduates who see their profession in terms of the Intrinsic Meaning level view mathematics as a way of understanding the world, and hence apply the notion of problem solving to their personal as well as professional lives (conception C). Although speculative, this analysis is not contradicted by anything in our transcripts and suggests directions for further research.

Conclusion

Writing about statistics teaching, Sowey (1995) paraphrased an aphorism of the psychologist B. F. Skinner that "education is what remains when the facts one has learned have been forgotten". Sowey claimed that in his experience, and the experience of his students, what remains when the facts have been forgotten is a sense of the *structure* and *worthwhileness* of the subject, the discipline of statistics. He pointed out that that there had been very little detailed investigation of this theme: the present paper makes a contribution in the context of an education in the mathematical sciences. It seems quite clear from the transcripts of the interviews that for many of the mathematics graduates in our study what has remained is an ability to solve problems:

> Lucy: In my undergraduate, I don't know what I would have described maths as, but it probably wouldn't have been the problem solving aspect of it. For me now, that's what we do, that's what I do and the people I work with do, we go and we have a problem and we attempt to solve it and we do that by using techniques available to us or creating new ones.

Such a view supports a pedagogical approach where specific mathematical techniques are less important than appreciation of overall ideas about the nature and usefulness of mathematics, components of Sowey's 'structure' and 'worthwhileness'.

Employers often seem to have an understanding of the benefits of a mathematics background in terms of problem solving abilities, which gives a mathematics graduate a certain status in the world of work:

> Alistair: When you are looking for a job to be able to say that you've got a degree, a science degree in mathematics, straight away they open the door, no matter what the job is. … People really listen to you when you are just starting out, even without the experience they are like 'well you have got no experience but, you know, you've been studying pure mathematics, you must have, you must know what you are doing', which is not always true, but they don't realise that, which is great.

However, some employers seemed to be unable to get the full benefit from the skills and abilities of their recently appointed graduate employees, a finding also noted by Crebert and colleagues (2004):

> Lucy: So my experience of working mathematically hasn't been that great, mainly because I think they thought they needed a maths person, and they did, but they weren't giving someone with the maths anything to do, so yeah that was my feeling anyway. … It just felt silly that these people thought that this was mathematics and it was just odd, I just didn't know how to explain to them that there is a lot more than a t-test, but anyway.

Finally, Euler's characterisation of mathematicians as problem solvers is clearly reflected in these statements from two of the graduates in our study:

> Maya: The training the mind to be more logical is where studying mathematics helps in parts of life that don't seem to have anything to do with maths.

> Alistair: Like if something's hard you don't just sort of drop it and let it go, you work out how to do it, and that's what you have to do if you want to, I suppose, succeed or excel. You have got to solve the problems that other people can't.

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